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Neutron Rich Hypernuclei in Chiral Soliton Model

V. B. Kopeliovich[1)]

Institute for Nuclear Research of the RAS, 117312 Moscow, Russia

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The binding energies of neutron rich strangeness $S = -1$ hypernuclei are estimated in the chiral soliton approach using the bound state rigid oscillator version of the $SU(3)$ quantization model. Additional binding of strange hypernuclei in comparison with nonstrange neutron rich nuclei takes place at not large values of atomic (baryon) numbers, $A = B \leq\sim 10$. This effect becomes stronger with increasing isospin of nuclides, and for the "nuclear variant" of the model with rescaled Skyrme constant e. Binding energies of ${}^{8}_{\Lambda}He$ and recently discovered ${}^{6}_{\Lambda}H$ satisfactorily agree with data. Hypernuclei ${}^{7}_{\Lambda}$H, ${}^{9}_{\Lambda}$He are predicted to be bound stronger in comparison with their nonstrange analogues ^{7}H, ^{9}He; hypernuclei ${}^{10}_{\Lambda}$Li, ${}^{11}_{\Lambda}$Li, ${}^{12}_{\Lambda}$Be, ${}^{13}_{\Lambda}$Be etc. are bound stronger in the nuclear variant of the model.

1. Studies of nuclear states with unusual properties — nontrivial values of flavor quantum numbers (strangeness, charm or beauty), or large isospin (so called neutron rich nuclides) are of permanent interest. They are closely related to the problem of existence of strange quark matter and its fragments, strange stars (analogues of neutron stars), and may be important for astrophysics and cosmology. Recently new direction of such studies, the studies of neutron rich hypernuclei, got new impact due to discovery of the hypernucleus ${}^{6}_{\Lambda}$H (heavy hyperhydrogen) by FINUDA Collaboration [1] which followed its search during several years [2].

Theoretical discussion of such nuclei took place during many years, beginning with the work by R.H. Dalitz and R. Levi-Setti, [3–6], in parallel with experimental searches [2, 7–9]. It has been noted first in [3] that the Lambda particle may act as additional glue for the nuclear matter, increasing the binding energy in comparison with nucleus having zero strangeness. Here we confirm this observation within the chiral soliton approach (CSA). Moreover, this effect becomes stronger for the neutron rich nuclei, with increasing excess of neutrons inside the nucleus.

The important advantage of the CSA proposed by Skyrme [10], in comparison with traditional approaches to this problem, is its generality, i.e. the possibility to consider different nuclei on equal footing, and considerable predictive power. (Some early descriptions of this model can be found in [11]). The drawback of the CSA is its relatively low accuracy in describing the properties of each particular nucleus. In this respect the CSA cannot compete with traditional approaches and models like shell model, Hartree–Fock method, etc. [3–6].

The quantization of the model performed first in the $SU(2)$ configuration space for the baryon number one states [12], somewhat later for configurations with axial symmetry [13] and for multiskyrmions [14], allowed, in particular, to describe the properties of nucleons and Δ-isobar [12] and, more recently, some properties of light nuclei, including so called "symmetry energy" [15][2)], and some other properties of nuclei [17].

The $SU(3)$ quantization of the model has been performed first within the rigid rotator approach [18] and also within the bound state model [19]. The binding energies of the ground states of light hypernuclei have been described in [20] within a version of the bound state chiral soliton model [21], in qualitative, even semi-quantitative agreement with empirical data [22]. The collective motion contributions, only, have been taken into account in [20] (single particles excitations should be added), and special subtraction scheme has been used to remove uncertainties in absolute values of masses intrinsic to the CSA [23, 24]. This investigation has been extended to the higher in energy (excited) states, with baryon number $B = 2$ and 3, some of them may be interpreted as antikaon-nuclei bound states [25]. Some of the states obtained in [25] are bound stronger than predicted originally by Akaishi and Yamazaki [26, 27]. These states could overlap and appear in experiment as a broad enhancement, in qualitative agreement with data obtained by FINUDA collaboration [28] and more recently by DISTO collaboration [29].

To estimate the total binding energies of neutron rich hypernuclei we are using here one of possible $SU(3)$ quantization models, the rigid oscillator version of the bound state model [21] which seems to be the simplest

[1)]e-mail: kopelio@inr.ru

[2)]Recently the neutron rich isotope ^{18}B has been found to be unstable relative to the decay ${}^{18}\mathrm{B} \to {}^{17}\mathrm{B} + n$ [16], in agreement with prediction of the CSA [15].

one. In section 2 our approach, the CSA, is shortly described, section 3 contains the formulas summarizing the CSA results for strange hypernuclei and numerical results for the binding energies of neutron rich hypernuclei with neutron excess $N - Z = 3$ and 4, atomic number $A \leq 17$. Final section contains conclusions and discussion of perspectives.

2. The CSA is based on few principles and ingredients incorporated in the *truncated* effective chiral lagrangian [10–12]:

$$L^{\mathrm{eff}} = -\frac{F_\pi^2}{16}\mathrm{Tr} l_\mu l_\mu + \frac{1}{32e^2}\mathrm{Tr}[l_\mu l_\nu]^2 + \frac{F_\pi^2 m_\pi^2}{8}\mathrm{Tr}\left(U + U^\dagger - 2\right), \tag{1}$$

the chiral derivative $l_\mu = \partial_\mu U U^\dagger$, $U \in SU(2)$ or $U \in SU(3)$ – unitary matrix depending on chiral fields, m_π is the pion mass, F_π – the pion decay constant known experimentally, e – the only parameter of the model in its minimal variant proposed first by Skyrme [10].

The chiral and flavor symmetry breaking term in the lagrangian density depends on kaon mass and decay constant m_K and F_K ($F_K/F_\pi \simeq 1.22$ from experimental data):

$$L^{\mathrm{FSB}} = \frac{F_K^2 m_K^2 - F_\pi^2 m_\pi^2}{24}\mathrm{Tr}\left(U + U^\dagger - 2\right)\left(1 - \sqrt{3}\lambda_8\right) - \\ - \frac{F_K^2 - F_\pi^2}{48}\mathrm{Tr}\left(U l_\mu l_\mu + l_\mu l_\mu U^\dagger\right)\left(1 - \sqrt{3}\lambda_8\right). \tag{2}$$

This term defines the mass splittings between strange and nonstrange baryons (multibaryons), modifies some properties of skyrmions and is crucially important in our consideration. The whole lagrangian given by (1), (2) is proportional to the number of colors of underkying QCD, $L^{\mathrm{eff}} \sim N_c$, which is one of justifications of the model.

The mass term $\sim F_\pi^2 m_\pi^2$ changes asymptotics of the profile f and the structure of multiskyrmions at large B, in comparison with the massless case. For the $SU(2)$ case

$$U = \cos f + i(\mathbf{n}\boldsymbol{\tau})\sin f, \tag{3}$$

the unit vector $\mathbf{n}$ depends on 2 functions, α, β, τ_k are the Pauli matrices. Three profiles $\{f, \alpha, \beta\}(x, y, z)$ parametrize the 4-component unit vector on the 3-sphere S^3. The topological soliton (skyrmion) is configuration of chiral fields, possessing topological charge identified with the baryon number B [10] (for the nucleus it is atomic number A: $B = A$). The important feature of the CSA is that multibaryon states including nuclei and hypernuclei can be considered on equal footing with the $B = 1$ case.

Minimization of the mass functional M_{cl} provides 3 profiles $\{f, \alpha, \beta\}(x, y, z)$ and allows to calculate moments of inertia Θ_I, Θ_F, the Σ-term (we call it Γ)

Table 1

Characteristics of classical skyrmion configurations which enter the nuclei – hypernuclei binding energies differences. The numbers are taken from [30, 31]: moments of inertia Θ, Σ-term Γ and $\tilde{\Gamma}$ – in units GeV^{-1}, ω_S – in MeV, μ_S is dimensionless (see next sections for explanation). All these quantities have the lower index B which is omitted for the sake of brevity. Parameters of the model $F_\pi = 186$ MeV; $e = 4.12$ [20, 30, 31]

B	Θ_I	Θ_J	Θ_F^0	Θ_S	Γ	$\tilde{\Gamma}$	μ_S	ω_S
1	5.55	5.55	2.05	2.636	4.80	14.9	3.155	307
6	25.4	178	13.1	16.64	29.0	38.0	3.125	287
7	28.9	221	14.7	18.64	32.3	44.0	3.009	283
8	33.4	298	17.4	22.15	38.9	47.0	3.125	288
9	37.8	376	20.6	26.25	46.3	47.5	3.269	292
10	41.4	455	23.0	29.35	52.0	50.0	3.289	293
11	45.2	547	25.6	32.74	58.5	52.4	3.340	295
13	52.1	737	30.5	39.07	70.2	56.8	3.372	296
14	56.1	865	33.7	43.15	78.2	58.9	3.460	299
16	63.2	1110	38.9	50.07	91.5	62.8	3.517	302

and some other characteristics of chiral solitons which contain implicitly information about interaction between baryons. In Table 1 we present numerical values of the moments of inertia and other quantities taken from [30–32] where analytical expressions for them can be found as well.

Table 2

Same as in Table 1 for rescaled (nuclear) variant of the model with constant $e = 3.0$ [15, 32]

B	Θ_I	Θ_F^0	Θ_S	Γ	$\tilde{\Gamma}$	μ_S	ω_S
1	12.8	4.66	5.893	10.1	19.6	6.407	344
6	62.6	30.7	38.60	64.7	50.6	6.728	334
7	69.6	34.9	43.75	72.5	54.4	6.500	330
8	79.9	41.3	51.97	87.4	58.2	6.785	334
9	88.9	47.1	59.43	101	61.7	6.927	337
10	97.4	52.6	66.40	113	64.9	6.957	336
11	106	58.5	73.88	126	67.9	7.038	337
12	114	63.8	80.65	138	70.8	7.049	337
13	122	69.5	87.94	151	73.6	7.102	338
14	132	76.3	96.81	168	76.3	7.289	341
15	140	82.3	104.5	182	78.8	7.353	342
16	148	88.1	112.0	196	81.2	7.402	343

The moment of inertia Θ_S given in Tables 1 and 2 is certain combination of Θ_F^0 and sigma term Γ:

$$\Theta_S = \Theta_F^0 + \frac{1}{4}\left(\frac{F_K^2}{F_\pi^2} - 1\right)\Gamma. \tag{4}$$

The strangeness excitation energies ω_S given in Tables 1, 2 are somewhat overestimated, especially for nuclear variant of the model — this is an artefact of the CSA. However, this overestimation is cancelled in the nuclear binding energies differences considered below.

The characteristics given in Tables 1, 2 have the following scaling properties: Θ_I, Θ_J, Θ_F, Θ_S, Γ, $\tilde{\Gamma} \sim N_c$; μ_S, $\omega_S \sim N_c^0 \sim 1$. The properties of the $B = 2$ toroidal skyrmion, not included in Tables 1, 2, have been considered in details previously, see [25] and references therein. The rational map approximation [33] simplifies considerably calculations of various characteristics of multiskyrmions presented in Tables 1, 2.

3. The observed spectrum of strange multibaryon states (hypernuclei) is obtained by means of the $SU(3)$ quantization procedure and depends on the quantum numbers of multibaryons and characteristics of skyrmions presented in Tables 1, 2. Within the bound state model [19–21] the antikaon field is bound by the $SU(2)$ skyrmion. The mass formula takes place

$$M = M_{cl} + \omega_S + \omega_{\bar{S}} + |S|\omega_S + \Delta M_{\rm HFS}, \qquad (5)$$

where strangeness and antistrangeness excitation energies

$$\omega_S = N_c(\mu_S - 1)/8\Theta_S, \quad \omega_{\bar{S}} = N_c(\mu_S + 1)/8\Theta_S, \quad (6)$$

$$\Theta_S = \Theta_F^0 + \frac{1}{4}\left(\frac{F_K^2}{F_\pi^2} - 1\right)\Gamma, \quad \mu_S = \sqrt{1 + \bar{m}_K^2/M_0^2},$$

$$M_0^2 = N_c^2/(16\Gamma\Theta_S) \sim N_c^0, \quad \bar{m}_K^2 = m_K^2 F_K^2/F_\pi^2. \quad (7)$$

The hyperfine splitting correction to the energy of the baryon state, depending on hyperfine splitting constants c_S, $\bar{c}_S$, observed isospin I, "strange isospin" I_S, the isospin of skyrmion without added antikaons $\mathbf{I}_r$ and the angular momentum J, equals in the case when interference between usual space and isospace rotations is negligible or not important [21], see also [31, 32]:

$$\Delta M_{\rm HFS} = \frac{J(J+1)}{2\Theta_J} +$$
$$+ \frac{c_S I_r(I_r+1) - (c_S - 1)I(I+1) + (\bar{c}_S - c_S)I_S(I_S+1)}{2\Theta_I}. \qquad (8)$$

The hyperfine splitting constants are equal

$$c_S = 1 - \frac{\Theta_I}{2\Theta_S\mu_S}(\mu_S - 1), \quad \bar{c}_S = 1 - \frac{\Theta_I}{\Theta_S\mu_S^2}(\mu_S - 1). \quad (9)$$

Strange isospin equals $I_S = 1/2$ for $S = \mp 1$. We recall that body-fixed isospin $\mathbf{I}^{bf} = \mathbf{I}_r + \mathbf{I}_S$ [21, 31, 32]. $\mathbf{I}_r$ is quite analogous to the so-called "right" isospin within the rotator quantization scheme [18]. When $I_S = 0$, i.e. for nonstrange states, $I = I_r$ and this formula goes over into $SU(2)$ formula for multiskyrmions. Correction $\Delta M_{\rm HFS} \sim 1/N_c$ is small at large N_c, and also for heavy flavors [19, 31].

The mass splitting within $SU(3)$ multiplets is important for us here. The unknown for the $B > 1$ solitons Casimir energy [23, 24] cancels in the mass splittings. For the difference of energies of states with strangeness S and with $S = 0$ which belong to multiplets with equal values of (p, q)-numbers $(p = 2I_r)$, we obtain using the above expressions for the constants c_S and $\bar{c}_S$ (it is the first subtraction):

$$\Delta E(p,q;I,S;I_r,0) = |S|\omega_S + \frac{\mu_S - 1}{4\mu_S\Theta_S}[I(I+1) - I_r(I_r+1)] +$$
$$+ \frac{(\mu_S - 1)(\mu_S - 2)}{4\mu_S^2\Theta_S} I_S(I_S+1). \qquad (10)$$

For the difference of binding energies of the hypernucleus with strangeness $S = -1$, isospin $I = I_r - 1/2$ and the nonstrange nucleus with isospin $I = I_r$ (the neutron excess $N - Z = 2I_r$) we obtain (the second subtraction):

$$\Delta\epsilon = \omega_{S,1} - \omega_{S,B} - \frac{3}{8}\frac{\mu_{S,1} - 1}{\mu_{S,1}^2\Theta_{S,1}} + \left(I_r + \frac{1}{4}\right)\frac{\mu_{S,B} - 1}{4\mu_{S,B}\Theta_{S,B}} -$$
$$- \frac{3}{16}\frac{(\mu_{S,B} - 1)(\mu_{S,B} - 2)}{\mu_{S,B}^2\Theta_{S,B}}. \qquad (11)$$

At $I_r = 1/2$ we obtain from (11) Eq. (9) of previous paper [20]. The term $\sim (I_r + 1/4)$ in Eq. (11) is responsible for the additional binding of neutron rich hypernuclei in comparison with $S = 0$ neutron rich nuclei. The results of calculations are presented in Tables 3 and 4.

Experimental data on total binding energies of nonstrange neutron rich nuclides presented in first numerical columns of Tables 3 and 4 are taken from [34]. The experimental value of binding energy of hyperhydrogen shown in Table 3, $\epsilon({}^6_\Lambda{\rm H}) = 10.8$ MeV is the sum of the binding energy of ${}^6_\Lambda{\rm H}$ relative to ${}^5{\rm H} + \Lambda$, measured in [1], $\epsilon({}^6_\Lambda{\rm H}) = (4.0 \pm 1.1)$ MeV and the binding energy of ${}^5{\rm H}$ measured in [35], $\epsilon({}^5{\rm H}) \simeq 6.78$ MeV.

The value of the binding energy of ${}^8_\Lambda{\rm He}$ shown in Table 3 is the sum of the Λ separation energy $7.16 \pm \pm 0.70$ MeV measured in [7] and the total binding energy of the ${}^7{\rm He}$ nucleus, $\epsilon({}^7{\rm He}) \simeq 28.82$ MeV. The values marked with *, $\Delta(\epsilon)^{th*}$ and $\epsilon^{th,*}$, here and in Table 4 denote the theoretical values obtained in the rescaled (nuclear) variant of the model with Skyrme constant $e = 3.0$. This variant allowed to satisfactorily describe mass splittings of nuclear isotopes, including neutron rich nuclides, with the mass numbers between ~ 10 and 30 [15]. The binding energies of the ground states of

Table 3

The total binding energies and binding energies differences $\Delta\epsilon^{th}_{2,3/2} = \epsilon_{3/2} - \epsilon_2$ between hypernuclei with isospin $I = 3/2$ and nonstrange isotopes with $I = 2$, $N - Z = 4$ (in MeV) for the original variant, $e = 4.12$, and for the variant with rescaled constant, $e = 3$ (numbers with the *). Experimental values of binding energy are available only for ${}^8_\Lambda$He [7] and ${}^6_\Lambda$H [1]

A–$_\Lambda A$	$\epsilon_2^{\exp}$	$_\Lambda\epsilon_{3/2}^{\exp}$	$\Delta\epsilon^{th}_{2,3/2}$	$\epsilon^{th}_{3/2}$	$\Delta\epsilon^{th,*}_{2,3/2}$	$\epsilon^{th,*}_{3/2}$
^{6}H–${}^6_\Lambda$H	5.8	10.8	9.0	14.8	11.2	17
^{8}He–${}^8_\Lambda$He	31.4	36.0	3.4	34.8	8.9	40
^{10}Li–${}^{10}_\Lambda$Li	45.3		−4.7	40.6	4.8	50
^{12}Be–${}^{12}_\Lambda$Be	68.6		−9.3	59.3	2.7	71
^{14}B–${}^{14}_\Lambda$B	85.4		−15.0	70.4	−1.7	84
^{16}C–${}^{16}_\Lambda$C	111		−18.5	92.3	−3.9	107

hypernuclei with moderate atomic numbers can be described within this variant of the model better than in the original variant ($e = 4.12$) [20] (these results will be presented in next publications).

Table 4

Same as in Table 3 for odd atomic numbers A, hypernuclei with $I = 2$ and nonstrange isotopes with $I = 5/2$, $N - Z = 5$. Experimental data on hypernuclei binding energies are not available, still

A–$_\Lambda A$	$\epsilon_{5/2}^{\exp}$	$\Delta\epsilon^{th}_{5/2,2}$	ϵ^{th}_2	$\Delta\epsilon^{th,*}_{5/2,2}$	$\epsilon^{th,*}_2$
^{7}H–${}^7_\Lambda$H	8	15	23	16.4	24
^{9}He–${}^9_\Lambda$He	30.3	0.1	30	7.0	37
^{11}Li–${}^{11}_\Lambda$Li	45.64	−5.0	41	5.0	51
^{13}Be–${}^{13}_\Lambda$Be	68.1	−9.0	59	3.0	71
^{15}B–${}^{15}_\Lambda$B	88.2	−16	72	−2.0	86
^{17}C–${}^{17}_\Lambda$C	111.5	−17	94	−2.7	108

The value 8 MeV for the binding energy of ^{7}H is preliminary result published in [36]. We did not include the correction to the binding energies difference depending on the spin of the nucleus J by following reasons. First, this correction is small in any case because the moment of inertia Θ_J shown in Table 1 is large, generally $\Theta_J \sim B^2$ and $\Theta_J > B\Theta_I$. Besides, in some cases of interest spins of nucleus and hypernucleus coincide, and in any case the spins of neutron rich hypernuclei are not known presently. The decrease of values of $\Delta\epsilon^{th}_{5/2,2}$ with increasing atomic number may be connected with limited applicability of the rational map approximation for describing multiskyrmions at larger baryon (atomic) numbers.

4. To summarize, we have calculated the difference of total binding energies of neutron rich hypernucleus with atomic, or baryon number A, strangeness $S = -1$, charge Z (i.e. containing Z protons), isospin $I = (N - Z - 1)/2$, and the zero strangeness nucleus with same atomic number A, Z protons and $N = A - Z$ neutrons, which has isospin $I = (N - Z)/2$. Within the chiral soliton approach this quantity contains the smallest uncertainty.

We performed calculations for two values of the Skyrme constant, $e = 4.12$, and $e = 3.0$ (the rescaled, or nuclear variant) which allowed to describe the mass splittings of nuclear isotopes with atomic numbers up to ~ 30 [15]. The total binding energies of the ground states of hypernuclei with $A \geq\sim 7$ are described better with rescaled constant e than it was made previously in [20] with the standard value $e = 4.12$. Both variants of the model provide close results for ${}^6_\Lambda$H and ${}^7_\Lambda$H, but for greater atomic numbers the difference becomes considerable. Results of the rescaled nuclear variant seem to be more reliable for greater atomic numbers, $A \geq\sim 10$. Further study of the dependence of our results on the only parameter of the model, the Skyrme constant e, is desirable.

Calculations performed in present paper may be extended easily to hypernuclei with arbitrary excess of neutrons in nuclei. Just the advantage of the CSA is that it provides a general look at nuclei with different excess of neutrons and great variety of atomic numbers. We hope that results presented here may be useful for planning of future experiments aimed to find new neutron rich hypernuclei.

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