

Two-dimensional plasmonic eigenmode nanolocalization in an inhomogeneous metal-dielectric-metal slot waveguide

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We show that transverse, with respect to the propagation direction, local narrowing of a metal-dielectric-metal plasmonic slot waveguide leads to a two-dimensional surface plasmon nanolocalization and can squeeze the plasmon eigenmode into a spot with a characteristic size of about several tens of nanometers. We demonstrate that the simultaneous waveguide tapering and decreasing transverse narrowing scale make possible an enhancement of the plasmon propagation distance in comparison with the homogeneous waveguide. We also find the fundamental limit of 2D plasmon nanolocalization, which is of the same order as the depth of penetration of the electromagnetic field into a metal which is actually independent of the field frequency in the near infrared domain.

1. Introduction. Light localization and light control at nanometer scales are currently one of the most rapidly developing branches of contemporary nanophotonics, which is caused by a number of possible applications in different fields including medicine [1], biology and chemistry [2], nanolasing [3], and integrated optics [4, 5]. Surface plasmon is widely discussed as an electromagnetic excitation enabling to provide light nanofocusing [6–9]. Lately, the linear and nonlinear plasmon nanofocusing in tapered metal-dielectric-metal (MDM) slot waveguides has been studied [10, 11]. Tapered waveguides suppress Drude losses in metal due to slowing down the 1D plasmon and may lead to strong nonlinear effects because of the field nanofocusing. The Kerr-type nonlinearity of a dielectric filling the slot of the waveguide can be a cause of the spatial plasmon-soliton wave structure formation that provides the 2D focusing of the plasmon beam and also enhancement of the propagation distance without a loss of the field intensity if the taper angle exceeds some critical value [12, 13]. However, nonlinear plasmon nanofocusing requires a sufficiently high initial intensity of a plasmon. Another possibility of 2D plasmon nanofocusing can be realized in a transversely to the propagation direction inhomogeneous MDM slot waveguide with, for example, a parabolic profile of slot thickness [14] that eventually makes up the so-called lens-like medium.

In this Letter, we draw attention to some features of linear 2D plasmon nanolocalization in inhomogeneous

tapered MDM plasmonic slot waveguides. One of the main purposes of this paper is estimation of the plasmon propagation distance in combination with the fundamentally minimum size of the plasmon focal spot.

2. Effective refraction index approximation. Below we use the so-called effective refraction index approximation (ERIA) which allows one to describe the electromagnetic field of surface plasmon modes as a volume wave in some medium with effective refraction index (or dielectric permittivity). Usually, such an approach enables to avoid significant difficulties in the numerical calculations of plasmonic waveguides, especially at the edges of metal and in the places of waveguides joining (see, for example, [15–17]), where ERIA can yield some approximate effective boundary conditions connecting the field of plasmons in different joined waveguides [18, 19].

First of all, using Maxwell's equations in integral form, we rigorously show that propagation of the fundamental even plasmon mode in a MDM slot waveguide with a very thin slot thickness $a < 2\Lambda_p = 2c/\omega_p$, where Λ_p is the field penetration depth into metal at frequencies ω ($\omega_p \gg \omega \gg \nu$), ω_p , ν are the electron plasma frequency and electron scattering rate in a metal, and c is the speed of light, can be described in terms of slot voltage with effective dielectric permittivity. Then we outline the framework of validity of this approach which will be applied to the waveguides with two-dimensional inhomogeneity of the slot.

We shall consider a MDM waveguide with variable thickness of quasi-planar dielectric slot $a(x, y)$ shown in

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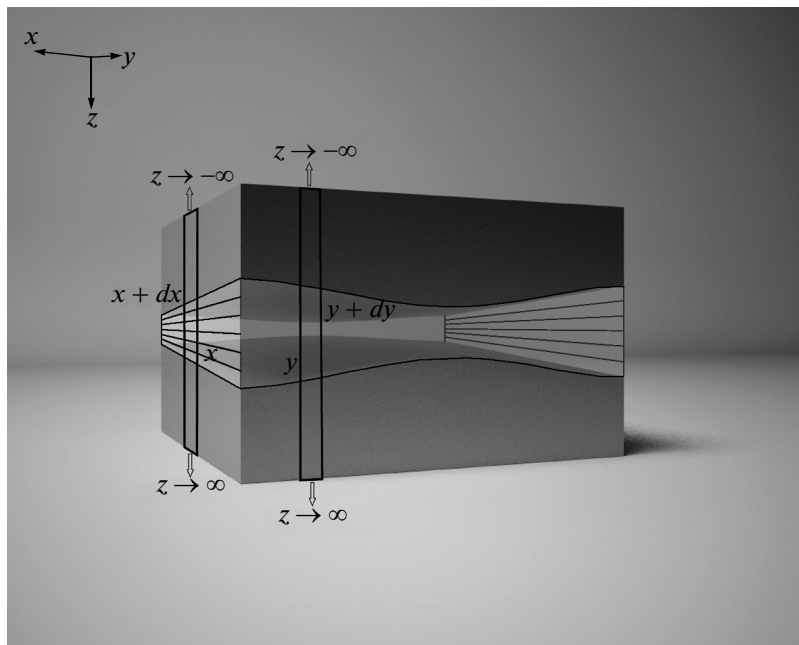


Fig. 1. MDM waveguide with the varying thickness $a(x, y)$ of a quasi-planar dielectric slot

Fig. 1, where x, y are the lateral coordinates and x is the plasmon propagation direction. We also believe that the plasmon transverse structure along the z -axis is kept the same as in a purely planar waveguide with $a = \text{const}$, which, in turn, assumes that the scales of slot inhomogeneity are much greater than the slot thickness a . According to Faraday's law of electromagnetic induction

$$\int_C \mathbf{E} d\mathbf{l} = -ik_0 \oint_S \mathbf{B} d\mathbf{S}, \quad (1)$$

where \mathbf{E} is the electric field tension, \mathbf{B} is the magnetic induction, $d\mathbf{l}$ is the differential element of integration contour, $d\mathbf{S}$ is the differential element of the surface stretched on integration contour directed along the normal to that surface, and $k_0 = \omega/c$. We calculate the electric field circulation along two closed contours (see Fig. 1) placed in two orthogonal planes (x, z) and (y, z) which stretch along the z -axis up to infinity. Taking into account that the plasmon is confined in the vicinity of the slot, the electric field intensity at $z \rightarrow \pm\infty$ is equal to zero. Thus, defining $U = \int_{-\infty}^{+\infty} E_z dz$ as the voltage between plus and minus infinities into plasmon mode we come to the following expressions:

$$U(x + dx) - U(x) = ik_0 \left(\int_{-\infty}^{+\infty} B_y dz \right) dx, \quad (2a)$$

$$U(y + dy) - U(y) = -ik_0 \left(\int_{-\infty}^{+\infty} B_x dz \right) dy. \quad (2b)$$

Hence,

$$\frac{\partial U}{\partial x} = ik_0 \int_{-\infty}^{+\infty} B_y dz, \quad \frac{\partial U}{\partial y} = -ik_0 \int_{-\infty}^{+\infty} B_x dz. \quad (3)$$

One can see that U consists of two parts, namely, $U = U_s + U_m$, where $U_s = \int_{-a/2}^{+a/2} E_z dz$ is the voltage drop inside the slot and $U_m = 2 \int_{a/2}^{\infty} E_z dz$ is the voltage drop inside the metal. Let us consider an even plasmon mode which obeys the following dispersion equation:

$$\tanh \left(\sqrt{h^2 - k_0^2 \varepsilon_d} \frac{a}{2} \right) = -\frac{\varepsilon_d}{\varepsilon_m} \sqrt{\frac{h^2 - k_0^2 \varepsilon_m}{h^2 - k_0^2 \varepsilon_d}}, \quad (4)$$

where h is the propagation constant ($\mathbf{E}, \mathbf{H} \sim \exp(-ihx)$), ε_d and ε_m are the permittivities of a dielectric filling the slot and metal, respectively (which are assumed to be constant). For a thin slot, hyperbolic tangent in (4) may be changed by its argument, namely, $\tanh \left(\sqrt{h^2 - k_0^2 \varepsilon_d} a/2 \right) \approx \sqrt{h^2 - k_0^2 \varepsilon_d} a/2$, and the component of electric field E_z inside the slot can be considered a constant value. It is easy to show that in this case, $|U_s| \gg |U_m|$ if the condition $a \gg a_c = |\varepsilon_d| |\varepsilon_m|^{-1} \Lambda_p$ is fulfilled. For the plasmon frequency ω , which corresponds to the vacuum wavelength $\lambda_0 = 1550$ nm, we have $|\varepsilon_m| \sim 60$ (silver) and $a_c \sim 1$ nm ($\Lambda_p \approx 30$ nm). Using another Maxwell's equation written in differential form, we obtain

$$(\nabla \times \mathbf{B})_z = \frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} = ik_0 \varepsilon(x, y, z) E_z. \quad (5)$$

Integrating Eq. (5) along the z -axis within infinite limits and taking into account expressions (3), we arrive at

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = -k_0^2 \int_{-\infty}^{+\infty} \varepsilon(x, y, z) E_z dz. \quad (6)$$

The integral on the right-hand side of Eq. (6) can be rewritten as

$$\int_{-\infty}^{+\infty} \varepsilon(x, y, z) E_z dz = \varepsilon_d U_s + \frac{2\varepsilon_d}{a\sqrt{h^2 - k_0^2 \varepsilon_m}} U_m. \quad (7)$$

The condition of continuity of the normal component to metal/dielectric interface of electric displacement has been used in expression (7). For a not too large propagation constant $h^2 \ll k_0^2 |\varepsilon_m|$, bearing in mind that $U_s \approx U$, we obtain the wave equation describing an even plasmon in a thin MDM slot waveguide with inhomogeneous slot thickness in terms of effective dielectric permittivity (or effective refraction index) and voltage drop in the waveguide slot:

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + k_0^2 \varepsilon_{\text{eff}}(x, y) U = 0, \quad (8)$$

where

$$\varepsilon_{\text{eff}} = \varepsilon_d \left[1 + \frac{2\Lambda_p}{a(x, y)} \left(1 - i\frac{\nu}{2\omega} \right) \right]. \quad (9)$$

3. Two-dimensional plasmonic eigenmode in an inhomogeneous MDM slot waveguide. In order to study the features of two-dimensional plasmonic eigenmodes, we consider a MDM slot waveguide with slot thickness a dependent on the coordinate y , which is assumed transversal to the propagation direction x . We choose the function $a(y)$ in the following model form (see Fig. 2) which allows one to get an explicit analyti-

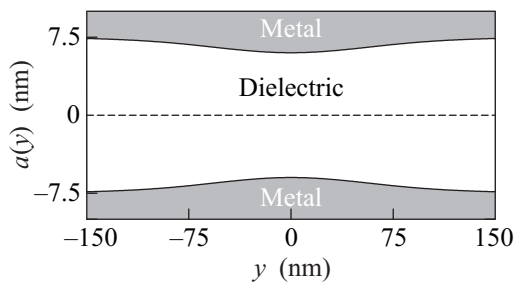


Fig. 2. Transverse profile of the model MDM plasmonic slot waveguide. The parameters are $a_2 = 15$ nm, $a_1 = 12$ nm, and $L = 75$ nm

cal solution to Eq. (8):

$$\frac{2\Lambda_p}{a(y)} = A + \frac{B}{\cosh^2(y/L)}; \quad A = \frac{2\Lambda_p}{a_2};$$

$$B = 2\Lambda_p \left(\frac{1}{a_1} - \frac{1}{a_2} \right); \quad a_2 > a_1. \quad (10)$$

Here, a_2 is the slot thickness at $|y| \rightarrow \infty$ and a_1 is that in the bottleneck at $y = 0$. Such a profile of $a(y)$ makes up the two-dimensional plasmonic waveguide. Indeed, effective dielectric permittivity increases towards the central part of the waveguide $y = 0$, which thus provides the lateral plasmon confinement in the plane of the slot, i.e. in the y direction. It is quite obvious that the fundamental mode of this waveguide is the most localized. Taking into account Eqs. (9) and (10), the solution of Eq. (8) can be expressed in explicit form

$$U_0 = V_0(y) \exp(-ihx);$$

$$V_0(y) = C \frac{1}{\cosh^p(y/L)}, \quad (11)$$

where C is an arbitrary constant value and h is the propagation constant;

$$p = -\frac{1}{2} + \frac{1}{2} \sqrt{1 + 4(k_0 L)^2 \varepsilon_d B}, \quad (12)$$

$$(hL)^2 = p^2 + (k_0 L)^2 \varepsilon_d (1 + A). \quad (13)$$

Formulae (12), (13) were obtained for the lossless case. In order to involve dissipation under consideration, one should replace A and B by $A(1 - i\nu/2\omega)$ and $B(1 - i\nu/2\omega)$, respectively. Dispersion equation (13) is shown in Fig. 3 for different a_1 . The plasmon lateral

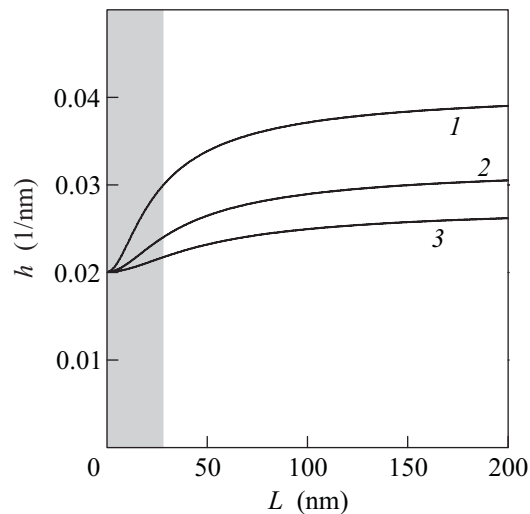


Fig. 3. Dispersion curves of the fundamental even mode of the MDM slot waveguide with $a_2 = 15$ nm, $\Lambda_p = 30$ nm, $\nu = 0.1\omega$, $\lambda_0 = 1550$ nm, $\varepsilon_d = 4.9$ at different a_1 : 3 (1), 5 (2), and 7 nm (3). Shaded area denotes the zone of inapplicability of the accepted ERIA approach

localization scale strongly depends on the value of the parameter p given by expression (12) which is determined, in turn, by the inhomogeneity scale L as well

as by the minimum slot width. By means of Eqs. (11) and (12), one can make sure that the minimum lateral width (at up to the 0.5 intensity level)

$$\Delta_p \approx \frac{2}{k_0 \sqrt{\varepsilon_d B}} \quad (14)$$

of a plasmon is reached at $p \approx 0.5$, and it actually depends only on the relative narrowing of the slot, i.e., on B . It is quite evident that the accepted ERIA is valid up to the lateral scales of the plasmon mode comparable with the field penetration depth into a metal. Hence, we have the right to consider the minimum slot thickness $a_1 \ll a_2$ which provides $\Delta_p \sim \Lambda_p$ or, in other words, $a_1 \sim 0.5 k_0^2 \Lambda_p^3 \varepsilon_d \approx 1$ nm for the frequency corresponding to the vacuum wavelength $\lambda_0 = 1550$ nm, $\Lambda_p = 30$ nm, and $\varepsilon_d = 4.9$ (BiMnO₃). The dependences of the plasmon eigenmode lateral size on L are depicted in Fig. 4 for different values a_1 . Of course, narrowing

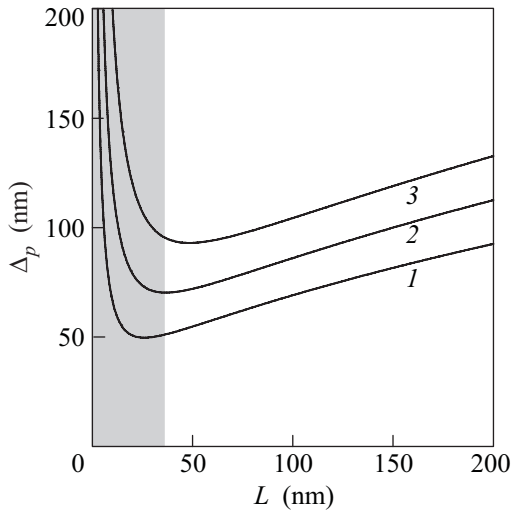


Fig. 4. The dependences of the plasmon eigenmode lateral size Δ_p on L in the case of the MDM slot waveguide with $a_2 = 15$ nm, $\Lambda_p = 30$ nm, $\nu = 0.1\omega$, $\lambda_0 = 1550$ nm, $\varepsilon_d = 4.9$ at different a_1 : 3 (1), 5 (2), and 7 nm (3). Shaded area denotes the zone of inapplicability of the accepted ERIA approach

the slot or, which is the same, increasing B , leads to the loss enhancement and, therefore, decreases the plasmon propagation distance. However, the results obtained in papers [9–12] give grounds to expect an extension of the propagation distance despite the energy dissipation in a tapered slot, which, in our case, means the simultaneous adiabatically slow changing of all slot parameters, which provides the maximum field localization, and, the minimum loss of intensity.

4. The features of plasmon propagation in MDM slot waveguides with slowly varying pa-

rameters. In order to obtain an equation for the slowly varying amplitude of the plasmon in a MDM slot waveguide with slowly varying parameters, we start from initial equation (8) in which the dependence ε_{eff} on x is assumed to be slow compared with that in the transverse direction y : $x \rightarrow \mu x$, where μ is the formally introduced small parameter characterizing slowness. We also rewrite effective dielectric permittivity (9) in the form $\varepsilon_{\text{eff}}(\mu x, y) = \varepsilon_d \cdot 2\Lambda_p [a(\mu x, y)]^{-1} (1 - i\nu/2\omega)$, thereby explicitly emphasizing its imaginary part under the condition $a(\nu x, y) \ll 2\Lambda_p$; the ratio $\nu/2\omega$ is assumed to be of the same order of smallness as μ . Then, the solution of Eq. (11) is sought in the form of asymptotic series:

$$U(x, y) = [Q(\mu x) V_0(\mu x, y, \nu = 0) + \mu U_1(\mu x, y) + \dots] \exp \left[-i \int h(\mu x) dx \right]. \quad (15)$$

In zero order of μ , we come to an equation for the unperturbed lateral structure of the plasmon mode, namely,

$$\frac{\partial^2 V_0}{\partial y^2} + \left[k_0^2 \varepsilon_d \frac{2\Lambda_p}{a(\mu x, y)} - h^2(\mu x, y) \right] V_0 = 0, \quad (16)$$

where $V_0(\mu x, y)$ is the purely real solution perfectly localized along y , which is identical to expression (11) with $\nu = 0$. Let $V_0(\mu x, y)$ be normalized as

$$\langle V_0^2(\mu x, y) \rangle = \int_{-\infty}^{\infty} V_0^2(\mu x, y) dy = 1. \quad (17)$$

Eq. (17) just defines a constant C in expression (11). Then $Q(\nu x)$ can be called the plasmon amplitude. In the first order of μ , we have an equation for the correction $U_1(\mu x, y)$:

$$\begin{aligned} \frac{\partial^2 U_1}{\partial y^2} + \left[k_0^2 \varepsilon_d \frac{2\Lambda_p}{a(\mu x, y)} - h^2(\mu x, y) \right] U_1 = \\ = i \left[2h \frac{\partial Q}{\partial x} V_0 + \frac{\partial h}{\partial x} Q V_0 + 2hQ \frac{\partial V_0}{\partial x} + \right. \\ \left. + \frac{\nu}{2\omega} k_0^2 \varepsilon_d \frac{2\Lambda_p}{a(\mu x, y)} Q V_0 \right] \equiv F. \end{aligned} \quad (18)$$

According to Fredholm theorem of alternative, the condition of solvability of Eq. (18) (nondiverging of the correction U_1) is orthogonality of the eigenmode of homogeneous Eq. (18) for U_1 (which obviously coincides with V_0) on the right-hand side of Eq. (18) $\langle F(\mu x, y) U_0(\mu x, y) \rangle = 0$, which eventually leads to the desired equation for slowly varying amplitude Q

$$2h \frac{\partial Q}{\partial x} + \frac{\partial h}{\partial x} Q + \frac{\nu}{2\omega} Q k_0^2 \varepsilon_d \left\langle \frac{2\Lambda_p}{a(\mu x, y)} V_0^2 \right\rangle = 0. \quad (19)$$

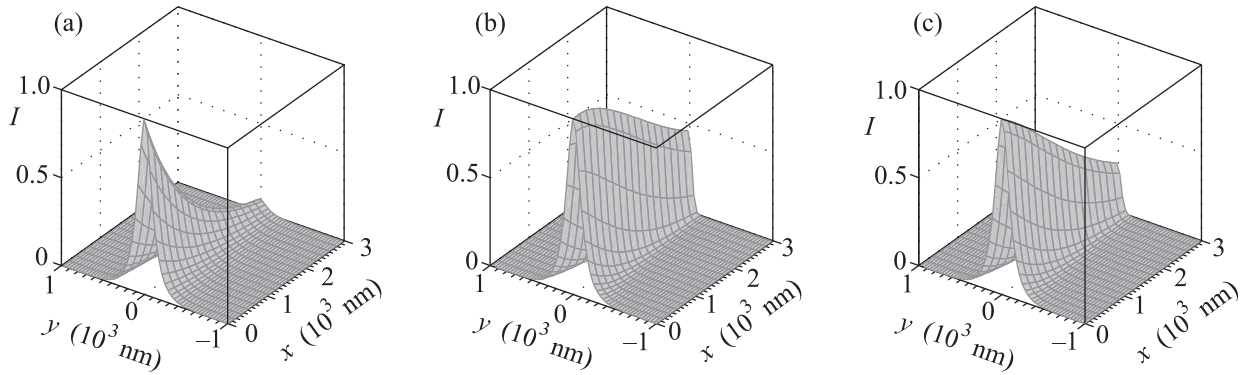


Fig. 5. Spatial distribution of the normalized plasmon intensity in the slot MDM waveguide with $a_2 = 15$ nm, $a_1(0) = 12$ nm, $\Lambda_p = 30$ nm, $\nu = 0.1\omega$, $\lambda_0 = 1550$ nm, $L = 100$ nm, $\epsilon_d = 4.9$, and $a_1(x) = a_1(0)(1 - x/l_a)$ for three different values of the parameter l_a : $l_a \rightarrow \infty$ (a), $l_a = 4500$ nm (b), $l_a = 6000$ nm (c)

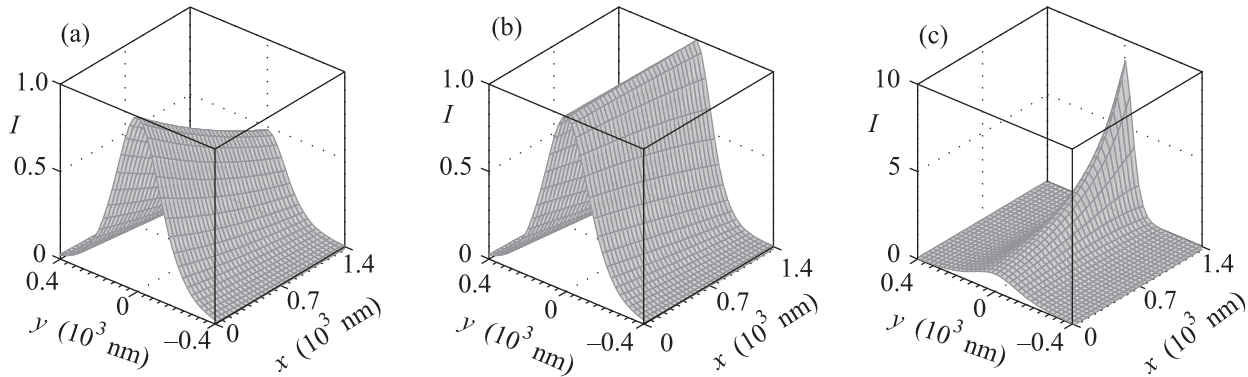


Fig. 6. Spatial distribution of the normalized plasmon intensity in the slot MDM waveguide with $a_1(0) = 12$ nm, $\Lambda_p = 30$ nm, $\nu = 0.1\omega$, $\lambda_0 = 1550$ nm, $\epsilon_d = 4.9$, $p = 1$, and $b = 1.2$ for the different dependences $a_1(x)$: $a_1(x) = a_1(0)(1 - 0.5x/l_p)^2$ (a), $a_1(x) = a_1(0)(1 - x/l_p)^2$ (b), $a_1(x) = a_1(0)(1 - 2x/l_p)^2$ (c)

Equation (19) can easily be integrated and its solution has the following form (now we can assume that the formal auxiliary parameter μ is equal to unity, i.e. $\mu = 1$):

$$\Gamma(x) = \frac{\nu}{4\omega} k_0^2 \epsilon_d \int_0^x \frac{1}{h(x')} \left\langle \frac{2\Lambda_p}{a(\mu x, y)} V_0^2 \right\rangle dx',$$

$$Q(x) = Q_0 \sqrt{\frac{h_0}{h(x)}} \exp[-\Gamma(x)]. \quad (20)$$

Here, h_0 and Q_0 are the values of the propagation constant and plasmon amplitude in the initial section of the waveguide with $x = 0$. The local value $h(x)$ of the propagation constant in the waveguide section x is given by Eq. (13). This equation can be rewritten for the electric field amplitude in the waveguide

$$E(x, y) = Q_0 \sqrt{\frac{h_0}{h(x)}} \frac{V_0(x, y)}{a(x, y)} \exp[-\Gamma(x)]. \quad (21)$$

Then we consider some typical behaviors of a plasmon for different kinds of the longitudinal (along x)

waveguide inhomogeneity which are described by solution (21). All of those scenarios are shown in Figs. 5 and 6. Figure 5a demonstrates the plasmon propagating in a MDM slot waveguide with $a_2 = 15$ nm, $a_1 = 12$ nm, $\Lambda_p = 30$ nm, $L = 100$ nm, $\nu/2\omega = 0.05$, $\lambda_0 = 1550$ nm, and $\epsilon_d = 4.9$ (BiMnO₃), which is accompanied by an usual exponential decay. One can see that the characteristic path length of a plasmon does not exceed one micrometer in this case. Figs. 5b and c show the plasmon intensity behavior $I(x, y) = |E(x, y)|^2$ in a tapered waveguide with $a_1(x) = a_1(0)(1 - x/l_a)$, where $a_1(0) = 12$ nm, $a_2 = 15$ nm and the same parameters indicated above for two different scales of taper: (b) $l_a = 4500$ nm and (c) $l_a = 6000$ nm. As can be seen in Fig. 5, tapering leads to increasing plasmon propagation distance.

The final Fig. 6 demonstrates some particular situation where $a_{1,2}(x)$ and $L(x)$ are changing simultaneously in such a manner that the parameter p is a constant value, $p = 1$. Then, three scenarios are possible, which is shown in Fig. 6 (see panels (a), (b), (c), and respective figure captions). Furthermore, the combination

of parameters assuring conservation of $p = 1$ and $b = hL = \text{const} > 1$ allows one to calculate explicitly the plasmon propagation distance under condition $I(x) = \text{const}$ (Fig. 6b). One can easily show that in order to provide a constant plasmon intensity along the propagation direction, the function $a_1(x)$ must obey the special law

$$a_1(x) = a_1(0) (1 - x/l_p)^2, \quad (22)$$

where

$$l_p = \frac{4\omega \lambda_0}{\nu \pi} \sqrt{\frac{a_1(0) 3b\sqrt{1+b^2}}{2\Lambda_p \varepsilon_d (3b^2 + 1)}}. \quad (23)$$

The zero value of the expression in the parentheses in Eq. (22) just defines the plasmon path length $x_c \equiv l_p$. According to Eq. (23), $l_p \approx 4.2 \mu\text{m}$ for $a_1(0) = 12 \text{ nm}$, $\Lambda_p = 30 \text{ nm}$, $\nu/2\omega = 0.05$, $\lambda_0 = 1550 \text{ nm}$, $\varepsilon_d = 4.9$ (*BiMnO*₃), and $b = 1.2$. In all the cases considered, the transverse scale of a plasmon may achieve values comparable with the skin layer depth Λ_p in a metal. Thus, the special transversal and longitudinal profiling of MDM plasmonic slot waveguide offers an opportunity for light nanolocalization as well as for enhancement (in several times) of the plasmon propagation distance in comparison with homogeneous waveguides.

5. Conclusions. In conclusion, the equation for studying the plasmon propagation in a MDM slot waveguide has been derived directly from integral Maxwell's equations and its domain of applicability has been stipulated. The possibilities of plasmon nanolocalization along with the enhancement of its propagation distance in specially profiled waveguides are shown.

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