

Dense quark matter conductivity in ultra-intense magnetic field

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Heavy-ion collisions generate a huge magnetic field of the order of 10^{18} G for the duration of about 0.2 fm/c. This time may become an order of magnitude longer if the electrical conductivity of quark matter is large. We calculate the conductivity in the regime of high density and show that contrary to naive expectations it only weakly depends on the MF.

A large body of experimental and theoretical research conducted for over more than ten years of operation of RHIC and the first runs of heavy ion program at LHC has led to a revolutionary change in our view on the nature and properties of the produced strongly interacting matter. A powerful way to investigate the nature of a certain substance is to study its response to the external perturbations. Few years ago it has been comprehended that in heavy ion collisions we have such a tool. The produced quark-gluon plasma is subject to a super-strong magnetic field (MF) generated by colliding ions which have large electric charges and are moving at speed close to the speed of light. At the collision moment and shortly after it ($\tau \leq 0.2$ fm/c) the MF reaches the value $|eB| \geq m_\pi^2 \sim 10^{18}$ G, i.e. it is of a typical QCD scale [1]. In presence of quark matter (QM) the life time of the MF may be several times longer provided the electrical conductivity (EC) of QM is large enough [2]. The properties of QM including its transport coefficients depend on the location of the system in the QCD phase diagram, i.e., on the value of the temperature and chemical potential. At zero chemical potential and high temperature the EC has been calculated by lattice Monte-Carlo method yielding significantly different results [3]. Here, we consider a reverse regime of high density and low or moderate temperature. Qualitatively these conditions will be specified below. Physically such situation may be realized in neutron stars or in future experiments at NICA and FAIR. On the theoretical side use can be made of the ideas and methods developed in condensed matter physics [4].

For a wide class of systems the EC can be decomposed into two contributions. The first one is Boltzmann, or Drude, and it corresponds to semi-classical approximation when the mean free path of a particle is much larger than any other microscopical length scale

and the probabilities of particle interactions are added. Quantum effects enter into this contribution only via Fermi or Bose distribution functions. A variety of quantum effects, such as the interference of trajectories, quantum phase transitions, quantized vortices, fluctuating Cooper pairs, Landau levels in magnetic field, etc., define the quantum contribution to the EC. In solid state physics this contribution is called “quantum correction” [4] since it is inversely proportional to $(k_F l)$, where k_F is the Fermi momentum, l is the mean free path, and $(k_F l) \gg 1$ in solids. This is not the case in dense quark matter where $(k_F l)$ may even approach unity (we remind that according to Ioffe–Regel criterion at $k_F l = 1$ transition to Anderson localization phase occurs). The description of the EC in terms of the above two contributions is legitimate for highly disordered systems [4] in the vicinity of superconducting phase transition [5], and in AdS/CFT correspondence [6].

Our starting point is the general expression for the EC in terms of Matsubara Green’s functions (GF) [4, 5]. The EC momentum and frequency dependent tensor $\sigma_{lm}(\mathbf{q}, \omega_k)$, $\omega_k = 2k\pi T$ is given by:

$$\begin{aligned} \sigma_{lm}(\mathbf{q}, \omega_k) &= \\ &= \frac{e^2 T}{\omega_k} \sum_{\varepsilon_n} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \text{tr} \langle G(\mathbf{p}, \tilde{\varepsilon}_n) \gamma_l G(\mathbf{p} + \mathbf{q}, \tilde{\varepsilon}_n + \omega_k) \gamma_m \rangle. \end{aligned} \quad (1)$$

For $N_c = 3$, $N_f = 2$ we have $e^2 = 3 \times 4\pi\alpha \left[\left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 \right] = 0.15$. The symbol $\langle \dots \rangle$ implies the averaging over the disorder. For Drude EC this procedure is performed independently for each GF and reduces to the substitution of the standard Matsubara frequency $\varepsilon_n = \pi T(2n + 1)$ by $\tilde{\varepsilon}_n = \varepsilon_n + \frac{1}{2\tau} \text{sgn} \varepsilon_n$, where τ is the momentum relaxation time [4, 5]. For the quantum contribution the averaging gives rise to an infinite series of the “fan” diagrams [4, 5, 7] yielding the

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result presented at the end of this paper. The GF has a standard form [8]

$$G(\mathbf{p}, \tilde{\varepsilon}_n) = \frac{1}{\gamma_0(\tilde{\varepsilon}_n + \mu) - \boldsymbol{\gamma}\mathbf{p} - m}, \quad (2)$$

where μ is the quark chemical potential. In the regime of high density and moderate temperature the transport coefficients are dominated by the processes occurring in the vicinity of the Fermi surface. Hence the momentum integration in [1] is performed in the following way

$$\frac{d^3\mathbf{p}}{(2\pi)^3} = \frac{1}{2}\nu\frac{d\Omega}{4\pi}d\xi, \quad (3)$$

$$\nu \simeq \frac{\mu p_F}{\pi^2} \left[1 + \frac{1}{3} \left(\frac{\pi T}{\mu} \right)^2 \right], \quad (4)$$

where $\xi = (\mathbf{p}^2 + m^2)^{1/2} - \mu$. Calculation of the Drude EC reduces to the evaluation of a one-loop diagram defined by (1)–(4). First we perform the tr operation over Dirac induces, then integration in the complex ξ -plane, and finally the ε_n summation. Replacing the Matsubara frequency ω_k by the physical ones $\omega_k = -i\omega$, we write down the resulting expression for the frequency dependent Drude EC

$$\sigma_{II}(\omega) = \frac{2}{3}e^2\nu v_F \frac{\tau}{1 + \omega\tau}, \quad (5)$$

where $v_F = p_F/\mu$. The antiquark contribution is damped near the Fermi surface and dropped in Eq. (5). If in (5) we replace $\mu \rightarrow \tilde{\mu} + m$ and take the limit $|\mathbf{p}| \ll m$, we arrive to the standard Drude formula with $\nu = mp_F/\pi^2$ and $\tilde{\mu}$ bring the non-relativistic chemical potential. For orientation purposes let us estimate $\sigma(\omega \rightarrow 0)$ for the following set of parameters: $\mu = 400$ MeV, $T = 100$ MeV, $v_F = 1$, $\tau = 0.8$ fm. One gets $\sigma \simeq 0.04$ fm $^{-1}$. To our knowledge, the EC has not been calculated before in this domain of the QCD phase diagram. Results obtained at $\mu = 0$ and different values of T [3] differ from each other by an order of magnitude. Our value $\sigma \simeq 0.04$ fm $^{-1}$ lies within the interval of the EC values given in [3].

An important point is that Eq. (5) contains a large parameter $\mu \sim (300\text{--}500)$ MeV. As we shall see this leads to the stabilization of the EC in MF up to $(eB/\mu)\tau \sim 1$. Let the constant MF \mathbf{B} be directed along the z -axis. The Drude EC can be evaluated either from the one-loop diagram with MF entering into the propagators [9], or from the Boltzmann kinetic equation. Diagrammatic calculation is more cumbersome, both methods lead to the same result which is an anticipated generalization of (5). We present the result

leaving the straightforward but lengthy derivation for the forthcoming detailed paper. In line with the symmetry requirements the EC along z -axis remains unchanged while the transverse one is equal to

$$\sigma_{\perp}(\omega = 0, \Omega) = \frac{\sigma_0}{1 + \Omega^2\tau^2}, \quad (6)$$

where $\sigma_0 \equiv \sigma_{II}(\omega = 0)$ given by (5), and $\Omega = eB/\mu$. For the set of parameters considered above, namely for $\mu = 400$ MeV, $\tau = 0.8$ fm we have $\Omega^2\tau^2 < 1$ up to $eB < 5m_{\pi}^2$.

Now we give a cursory glance on the quantum part σ' of the EC. As already mentioned, it includes the interplay of various quantum phenomena. In the regime under consideration the major role is played by the formation of the fluctuation (or precursor) Cooper pairs [5, 10]. Again we refer to the forthcoming paper analyzing diagrams containing propagators of such pairs. The order-of-magnitude estimate of quantum EC is $|\sigma'| \sim \sim e^2/\pi^3 l$, where $l = v_F\tau$ is the mean free path, the sign of σ' may be either positive, or negative. When $\sigma' < 0$ and large by the absolute value, the system approaches the Anderson localization regime. When $B \neq 0$ the above estimate transforms into $|G'| \sim e^2/\pi^3 l_B$, where l_B is the magnetic length, $l_B = (eB)^{-1/2}$. Therefore σ' is not proportional to eB as might be naively expected from the fact that the number of modes degenerating in a unit transverse area is proportional to B .

To summarize, we have for the first time evaluated the EC of dense relativistic quark matter with low and moderate temperature. We have demonstrated the rigidity of the EC on the magnetic field. The details of the calculations will be presented in the forthcoming paper.

The factor $[1 + \frac{1}{3}(\frac{\pi T}{\mu})^2]$ in (4) is not a corollary of the ξ -integration. It arises in more general approaches see [11, 12]. The authors thank E. Megias for this remark. We also note that our conclusion on the rigidity of the EC on the magnetic field was confirmed by a very recent calculation [13] in the instanton liquid model.

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