Obliquely propagating dust-acoustic solitary waves in magnetized dusty plasmas with two-temperature Maxwellian ions

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The nonlinear propagation of dust-acoustic (DA) waves in an obliquely propagating magnetized dusty plasma, containing Maxwellian distributed ions of distinct temperatures (namely lower and higher temperature Maxwellian ions), negatively charged mobile dust grains, and Maxwellian electrons, is rigorously investigated and analyzed by deriving the Zakharov–Kuznetsov equation. It is investigated that the characteristics of the DA solitary waves (DASWs) are significantly modified by the external magnetic field, relative ion and electron temperature-ratio, and respective number densities of two population of ions. The implications of the results obtained from this analysis in space and laboratory dusty plasmas are briefly discussed.

By now, there has been a rapidly growing interest in understanding different types of collective processes in dusty plasmas. It is noticed that the presence of charged dust grains does not only modify the existing plasma wave spectra [1, 2], but also allows a number of novel eigenmodes. Rao et al. [3] have first theoretically shown the existence of extremely low phase velocity dust-acoustic (DA) waves in an unmagnetized dusty plasma by using the reductive perturbation technique. Five years later, Barkan et al. [4] performed a laboratory experiment on DA waves, and conclusively verified the theoretical prediction of Rao et al.

At present, the properties of the DA solitary waves (DASWs) have attracted a great deal of interest for understanding the fundamental characteristics of localized electrostatic perturbations in laboratory and space dusty plasmas [3, 4–11]. The DASWs have been extensively studied by several authors during last two decades [12–15]. But the effects of two-temperature ions in dusty plasma systems were not discussed in those investigations [12–15]. However, in a recent letter [16], Zhang and Wang considered the effect of nonthermal ions of two distinct temperatures and obtained the *K-dV* equation. On the other hand, Zhou et al. [17] considered the same nonthermal plasma model, and derived the modified *K-dV* (m*K-dV*) equation. Moreover, Masud et al. [18, 19] in 2012 considered two-temperature electrons in unmagnetized dusty plasma systems and analyzed the SWs. Recently, Asaduzzaman et al. [20] analyzed the SWs in a plasma system comprising two-temperature Maxwellian ions. But, these works [16–19] are limited to a finite value of the nonlinear coefficient (A) or considering the models in unmagnetized plasma environments. This indicates that these works are not hold good for magnetized dusty plasma environments as the authors of these papers [16–20] have not considered the effects of magnetic field or obliquness on those SWs. Thus, to obtain a more generalized work on a dusty plasma (consisting of extremely massive, highly negatively charged mobile dust particles, Boltzmann distributed ions of distinct temperatures, and non-inertial electrons), we have derived the Zakharov–Kuznetsov (ZK) equation, and analyzed the SWs both numerically and analytically in this letter.

We consider the nonlinear propagation of collisionless DA waves in a magnetized dusty plasma consisting of negatively charged mobile dust, two-temperature Maxwellian ions of temperatures T_{i1} and T_{i2}, and non-inertial electrons having finite temperature T_e, where $T_e \gg T_{i2} \gg T_{i1}$. Thus, at equilibrium, $n_{i10} + n_{i20} = n_{e0} + Z_d n_{d0}$, where n_{i10} and n_{i20} are the densities of the lower and higher temperature ions respectively, at equilibrium. Z_d is the number of electrons residing onto the dust grain surface, and n_{e0} (n_{d0}) is the equilibrium density of the electron (dust). The nonlinear dynamics of the obliquely propagating DA waves in such a dusty plasma system is governed by

$$\frac{\partial n_d}{\partial t} + \nabla \cdot (n_d \cdot \mathbf{u}_d) = 0, \tag{1}$$

$$\frac{\partial \mathbf{u}_d}{\partial t} + (\mathbf{u}_d \cdot \nabla)\mathbf{u}_d = -\nabla\phi - \alpha(\mathbf{u}_d \times \hat{z}), \tag{2}$$

$$\nabla^2\phi = \mu e^{\sigma_2\phi} - \mu_{i1} e^{-\phi} - \mu_{i2} e^{-\sigma_1\phi} + n_d, \tag{3}$$

where n_d is the dust particle number density normalized by its equilibrium value n_{d0}, $\mathbf{u}_d$ is the dust fluid velocity normalized by $C_d = (Z_d k_B T_{i1}/m_d)^{1/2}$, ϕ is the wave potential normalized by $k_B T_{i1}/e$, the time variable t is normalized by $\omega_{pd}^{-1} = (m_d/4\pi n_{d0} Z_d^2 e^2)^{1/2}$, and the space

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variable is normalized by $\lambda_{\mathrm{D}m} = (k_{\mathrm{B}}T_{i1}/4\pi n_{d0}Z_d e^2)^{1/2}$. Here, $\sigma_1 = T_{i1}/T_{i2}$, $\sigma_2 = T_{i1}/T_e$, $\alpha = \omega_{cd}/\omega_{pd}$, $\mu_{i1} = n_{i10}/Z_d n_{d0}$, $\mu_{i2} = n_{i20}/Z_d n_{d0}$, $\mu = n_{e0}/Z_d n_{d0} = \mu_{i1} + \mu_{i2} - 1$, k_{B} is the Boltzmann constant, and e is the magnitude of the electron charge.

To study small but finite amplitude electrostatic DASWs in the dusty plasma under consideration, one usually constructs a weakly nonlinear theory [21], and uses a scaling of the independent variables through the stretched coordinates [21, 22]

$$X = \epsilon^{1/2}x, \tag{4}$$

$$Y = \epsilon^{1/2}y, \tag{5}$$

$$Z = \epsilon^{1/2}(z - V_p t), \tag{6}$$

$$\tau = \epsilon^{3/2}t, \tag{7}$$

where ϵ is a small parameter measuring the weakness of the dispersion, V_p is the phase speed normalized by the dust-acoustic speed (C_d). It may be noted here that X, Y, and Z are all normalized by the Debye radius ($\lambda_{\mathrm{D}m}$), and τ is normalized by the ion plasma period (ω_{pd}^{-1}).

The perturbed quantities n_d, u_{dx}, u_{dy}, u_{dz}, and ϕ can be expanded along with their equilibrium values as [21, 23–25]

$$n_d = 1 + \epsilon n_d^{(1)} + \epsilon^2 n_d^{(2)} + \cdots, \tag{8}$$

$$u_{dx} = \epsilon^{3/2}u_{dx}^{(1)} + \epsilon^2 u_{dx}^{(2)} + \cdots, \tag{9}$$

$$u_{dy} = \epsilon^{3/2}u_{dy}^{(1)} + \epsilon^2 u_{dy}^{(2)} + \cdots, \tag{10}$$

$$u_{dz} = \epsilon u_{dz}^{(1)} + \epsilon^2 u_{dz}^{(2)} + \cdots, \tag{11}$$

$$\phi = \epsilon\phi^{(1)} + \epsilon^2\phi^{(2)} + \cdots. \tag{12}$$

Now, using (4)–(7) and (8)–(12) into (1)–(3), one can obtain the first order continuity equation, the z-component of the momentum equation, and Poisson's equation which, after simplification, yield

$$n_d^{(1)} = -\frac{\phi^{(1)}}{V_p^2}, \tag{13}$$

$$u_{dz}^{(1)} = -\frac{\phi^{(1)}}{V_p}, \tag{14}$$

$$V_p = \frac{1}{\sqrt{\mu_{i1} + \mu_{i2}\sigma_1 + \mu\sigma_2}}. \tag{15}$$

Equation (15) is the phase speed of the DA waves propagating in the magnetized dusty plasma under consideration.

The first order x- and y-components of the momentum equation can be written as

$$u_{dx}^{(1)} = -\frac{1}{\alpha}\frac{\partial\phi^{(1)}}{\partial Y}, \tag{16}$$

$$u_{dy}^{(1)} = \frac{1}{\alpha}\frac{\partial\phi^{(1)}}{\partial X}. \tag{17}$$

The equations (16) and (17) respectively, represent the x- and y-components of the dust electric field drifts. These equations are also satisfied by the second order continuity equation.

Again, using (4)–(7) and (8)–(12) into (1)–(3), and eliminating $u_{dx,y}^{(1)}$, the next higher order x- and y-components of the momentum equation, and Poisson's equation can be found as

$$u_{dx}^{(2)} = -\frac{V_p}{\alpha^2}\frac{\partial^2\phi^{(1)}}{\partial Z\partial X}, \tag{18}$$

$$u_{dy}^{(2)} = -\frac{V_p}{\alpha^2}\frac{\partial^2\phi^{(1)}}{\partial Z\partial Y}, \tag{19}$$

$$\frac{\partial^2\phi^{(1)}}{\partial X^2} + \frac{\partial^2\phi^{(1)}}{\partial Y^2} + \frac{\partial^2\phi^{(1)}}{\partial Z^2} = \mu\left[p_1 + \frac{\sigma_2^2}{2}(\phi^{(1)})^2\right] + p_2 + n_d^{(2)}, \tag{20}$$

where $p_1 = 1 + \sigma_2\phi^{(2)}$ and $p_2 = \mu_{i1}\phi^{(2)} - \mu_{i1}(\phi^{(1)})^2/2 + \mu_{i2}\sigma_1\phi^{(2)} - \mu_{i2}\sigma_1^2(\phi^{(1)})^2/2$. Equations (18) and (19), respectively, denote the x- and y-components of the dust polarization drifts. Now, following the same procedure one can obtain the next higher order continuity equation, and z-component of the momentum equation. Using these new higher order equations along with (13)–(20), one can eliminate $n_d^{(2)}$, $u_{dz}^{(2)}$, and $\phi^{(2)}$, and can finally obtain

$$\frac{\partial\phi^{(1)}}{\partial\tau} + AB\phi^{(1)}\frac{\partial\phi^{(1)}}{\partial Z} + \frac{1}{2}A\frac{\partial}{\partial Z}\left[\frac{\partial^2}{\partial Z^2} + D\left(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial X^2}\right)\right]\phi^{(1)} = 0, \tag{21}$$

where

$$A = V_p^3, \tag{22}$$

$$B = \frac{1}{2}\left(\mu_{i1} + \mu_{i2}\sigma_1^2 - \mu\sigma_2^2 - \frac{1}{V_p^4}\right), \tag{23}$$

$$D = 1 + \frac{1}{\alpha^2}. \tag{24}$$

Equation (21) is the ZK equation describing the nonlinear propagation of the obliquely propagating DA waves in a magnetized dusty plasma with two-temperature Maxwellian distributed ions. To study the properties of the SWs propagating in a direction making an angle δ with the Z-axis, i.e. with the external magnetic field and lying in the $(Z-X)$ plane, the coordinate axes (X, Z) are rotated through an angle δ, keeping the Y-axis fixed. Thus, we transform our independent variables to

$$\rho = X\cos\delta - Z\sin\delta, \qquad \eta = Y,$$
$$\xi = X\sin\delta + Z\cos\delta, \qquad \tau = t. \tag{25}$$

This transformation of these independent variables allows us to write the ZK equation in the form

$$\frac{\partial\phi^{(1)}}{\partial t}+\delta_1\phi^{(1)}\frac{\partial\phi^{(1)}}{\partial\xi}+\delta_2\frac{\partial^3\phi^{(1)}}{\partial\xi^3}+\delta_3\phi^{(1)}\frac{\partial\phi^{(1)}}{\partial\rho}+ +\delta_4\frac{\partial^3\phi^{(1)}}{\partial\rho^3}+\delta_5\frac{\partial^3\phi^{(1)}}{\partial\xi^2\partial\rho}+\delta_6\frac{\partial^3\phi^{(1)}}{\partial\xi\partial\rho^2}+\delta_7\frac{\partial^3\phi^{(1)}}{\partial\xi\partial\eta^2}+ +\delta_8\frac{\partial^3\phi^{(1)}}{\partial\rho\partial\eta^2}=0, \quad (26)$$

where

$$\begin{aligned}\delta_1&=AB\cos\delta,\\ \delta_2&=\frac{1}{2}A(\cos^3\delta+D\sin^2\delta\cos\delta),\\ \delta_3&=-AB\sin\delta,\\ \delta_4&=-\frac{1}{2}A(\sin^3\delta+D\sin\delta\cos^2\delta),\\ \delta_5&=A\left[D(\sin\delta\cos^2\delta-\frac{1}{2}\sin^3\delta)-\frac{3}{2}\sin\delta\cos^2\delta\right], \quad (27)\\ \delta_6&=-A\left[D(\sin^2\delta\cos\delta-\frac{1}{2}\cos^3\delta)-\frac{3}{2}\sin^2\delta\cos\delta\right],\\ \delta_7&=\frac{1}{2}AD\cos\delta,\\ \delta_8&=-\frac{1}{2}AD\sin\delta.\end{aligned}$$

It is now necessary to look for a steady state solution of this ZK equation in the form

$$\phi^{(1)}=\phi_0(Z), \quad (28)$$

where

$$Z=\xi-U_0t,$$

in which U_0 is a constant speed normalized by the positive DA speed (C_d). Using this transformation the ZK equation can be written in steady state form as

$$-U_0\frac{d\phi_0}{dZ}+\delta_1\phi_0\frac{d\phi_0}{dZ}+\delta_2\frac{d^3\phi_0}{dZ^3}=0. \quad (29)$$

Now, using the appropriate boundary conditions, viz. $\phi^{(1)}\to 0$, $(d\phi^{(1)}/dZ)\to 0$, $(d^2\phi^{(1)}/dZ^2)\to 0$ as $Z\to\pm\infty$, the solitary wave solution of this equation is given by

$$\phi_0(Z)=\psi_m\mathrm{sech}^2(\kappa Z), \quad (30)$$

where $\psi_m=3U_0/\delta_1$ is the amplitude and $\kappa=\sqrt{U_o/4\delta_2}$ is the inverse of the width of the solitary waves. As $U_0>0$, it is clear from (21), (23), and (26) that depending on the sign of B, the SWs will be associated with only positive potential ($\psi_m>0$).

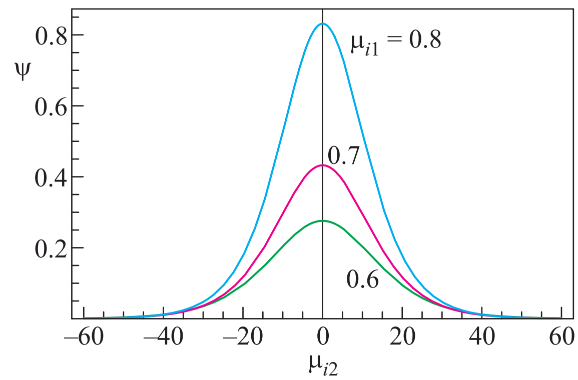

Fig. 1. (Color online) Variation of the solitary profiles with relative ion fluid densities (i.e., μ_{i2}) for different values of μ_{i1}. The upper (blue) curve is for $\mu_{i1}=0.8$, the middle (red) one is for $\mu_{i1}=0.7$, and the lower (green) one is for $\mu_{i1}=0.6$, for $\delta=18^\circ$, $\sigma_1=0.04$, and $\sigma_2=0.06$

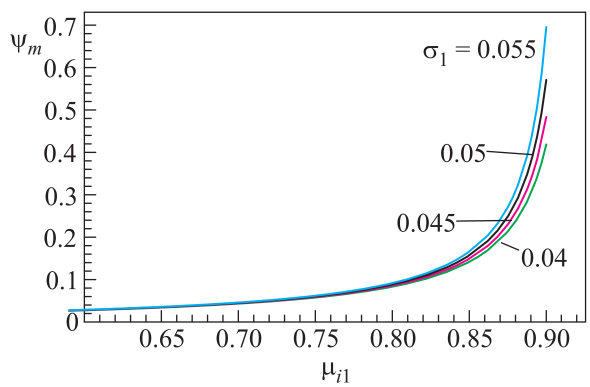

Fig. 2. (Color online) Variation of the amplitudes of solitary waves with μ_{i1} for different values of σ_1. The first (blue) curve is for $\sigma_1=0.055$, second (black) one is for $\sigma_1=0.05$, third (red) one is for $\sigma_1=0.045$, and fourth (green) one is for $\sigma_1=0.04$, for $\delta=18^\circ$, and $\sigma_2=0.06$

Fig. 1 represents the SW profiles for different values of the relative ion-number densities (μ_{i1}), and Fig. 2 indicates the variation of amplitudes of the positive potential DASW for different values of the temperature-ratio (σ_1). Consequently, Fig. 3 describes the variation of widths (Δ) of the DASWs with δ for different values of the frequency-ratio (α).

The effects of obliquness, magnetic field, and two-temperature ions on electrostatic solitary structures, which have been found to exist in a hot magnetized dusty plasma with positive potential only, are investigated rigorously by using the reductive perturbation method which is only valid for small but finite amplitude limit but not valid for large δ which makes the wave amplitude infinitely large. According to our present investigation, we have found that the amplitudes of the DASWs gradually increases with temperature-ratio σ_1.

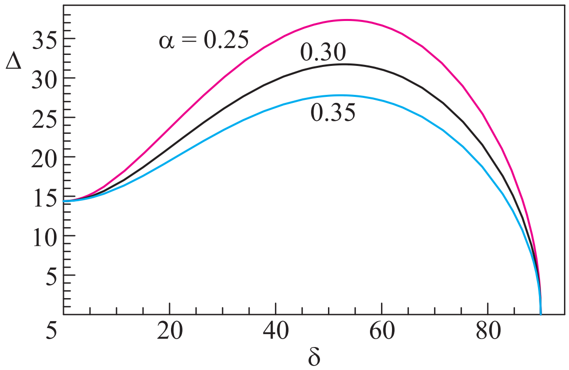

Fig. 3. (Color online) Variation of the width (Δ) of the solitary waves with δ for different values of α. The upper (red) curve is for $\alpha = 0.25$, the middle (black) one is for $\alpha = 0.3$, and the lower (blue) one is for $\alpha = 0.35$, for $\mu_{i1} = 0.6$, $\mu_{i2} = 0.4$, $\sigma_1 = 0.04$, and $\sigma_2 = 0.06$

On the other hand, the height of the DASW profile also varies proportionally with relative ion-number densities (μ_{i1} and μ_{i2}) and with reletive electron number density (μ) for definite values of the temperature ratios σ_1 and σ_2. The effect of variation of the angle δ on the widths (Δ) of the SWs is that the width of these SWs increases with δ for its lower range and decreases for its higher range. It should be pointed out that for large angles the assumptions that the waves are electrostatic is no longer valid, and we should look for fully electromagnetic structures. Moreover, it is also noted that with the increase of frequency-ratio α, the amplitudes become relatively spiky (Fig. 3).

We have analyzed the nature and basic characteristics of the obliquely propagating DASWs in a magnetized dusty plasma system consisting of bi-Maxwellian ions of distinct temperatures, negatively charged mobile dust, and non-inertial electrons. Our present results can be very effective for understanding the localized electrostatic disturbances in space [18, 19, 22, 26, 27] and laboratory magnetized dusty plasmas [16, 17, 22, 27–30], where two population of plasma species (viz. thermal ions or electrons) can dominate the wave dynamics.

To conclude, the time evolution and stability analysis of these solitary structures are also problems of great interest but beyond the scope of the present work.

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