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Non-stationary effects in the coupled quantum dots influenced by the electron-phonon interaction

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We analyzed time evolution of the localized charge in the system of two interacting single level quantum dots (QDs) coupled with the continuous spectrum states in the presence of electron-phonon interaction. We demonstrated that electron-phonon interaction leads to the increasing of localized charge relaxation rate. We also found that several time scales with different relaxation rates appear in the system in the case of non-resonant tunneling between the dots. We revealed the formation of oscillations in the filling numbers time evolution caused by the emission and adsorption processes of phonons.

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1. Introduction. Recent progress in the engineering and fabrication of well-defined artificial systems – quantum dots (QDs) leads to the possibility of ultra small electronic devices formation with a relatively high control of system parameters (size, shape, and energy spectrum) [1]. In addition to the QDs potential industrial applications, these nanoscale objects provide an ideal test ground for the study of basic physics including, many-body interaction effects, electron transport and time-dependent effects. Moreover, QDs integration in a little quantum circuits deals with careful analysis of non-equilibrium charge distribution, relaxation processes and non-stationary effects. These processes influence strongly on the electron transport through the system of QDs [2–7]. Electron transport in such systems is governed not only by the Coulomb interaction between localized electrons [5–7] but also by the electron-phonon interaction [8–10]. For correct interpretation of non-stationary effects it is necessary to analyze the influence of electron-phonon interaction on the localized charge time evolution because it leads to the appearance of additional inelastic tunneling channels and results in new specific features in the non-stationary electron transport. Detailed analysis of quantum effects in the system of interacting QDs in the presence of electron-phonon interaction gives an opportunity to create high speed electronic and logic devices.

Most of the theoretical works devoted to the problem of electron transport through the coupled QDs in the presence of electron-phonon interaction deal with the tunneling current and current-current correlations (shot noise) investigations [10, 11]. Only a few attempts have been made to analyze phonon assisted localized charge relaxation [12–14]. It was found theoretically that lateral confinement influence on the single-electron relaxation rates in parabolic QDs [15]. Author considered the deformation potential coupling between electrons and longitudinal acoustic phonons, while neglecting piezoelectric coupling on the grounds of it's weaker contribution in two dimensional structures. On the other hand further theoretical and experimental works suggested that electron-acoustic phonon scattering due to the piezoelectric field interaction is relevant for momentum and spin relaxation processes [16, 17] and may even provide charge decoherence in laterally coupled QDs [12, 13]. In [14] authors analyzed phonon induced single electron relaxation rates in the models of weakly confined single and vertically coupled QDs taking into account both mentioned above mechanisms. The regimes where each coupling mechanism prevails were found.

In this paper we use the Keldysh diagram technique [18] to analyze charge relaxation in double QDs due to the coupling with the continuous spectrum states in the presence of electron-phonon interaction. Tunneling to the reservoir is possible only from one of the dots. We have found that electron-phonon interaction results in the increasing of localized charge relaxation rate and also leads to the formation of well resolved oscillations.

2. The suggested model. In the present paper we consider a system of coupled QDs with the single

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particle levels ε_1 and ε_2 connected with the continuous spectrum states. At the initial time two electrons with opposite spins are localized in the first QD on the energy level ε_1 ($n_{1\sigma}(0) = n_0 = 1$). The second QD with the energy level ε_2 is connected with the reservoir (ε_p). Relaxation of the localized charge is governed by the Hamiltonian:

$$\hat{H} = \hat{H}_D + \hat{H}_{\text{tun}} + \hat{H}_{\text{res}}. \tag{1}$$

The Hamiltonian $\hat{H}_D$ of interacting QDs

$$\hat{H}_D = \sum_{i=1,2\sigma} \varepsilon_i c_{i\sigma}^+ c_{i\sigma} + \sum_{\sigma} T(c_{1\sigma}^+ c_{2\sigma} + c_{1\sigma} c_{2\sigma}^+) + \\ + \omega_0 b^+ b + g(c_{1\sigma}^+ c_{2\sigma} + c_{1\sigma} c_{2\sigma}^+)(b^+ + b) \tag{2}$$

contains the spin-degenerate levels ε_i (indexes $i = 1$ and 2 correspond to the first and to the second QD) and electron-phonon interaction. The creation/annihilation of an electron with spin $\sigma = \pm 1$ within the dot is denoted by $c_{i\sigma}^+/c_{i\sigma}$ and n_σ is the corresponding filling number operator. Operators b^+/b describe the creation/annihilation of the phonons. ω_0 is the optical phonon frequency and g is the electron-phonon coupling constant. The interaction between the dots is described by the tunneling transfer amplitude T which is considered to be independent of momentum and spin.

The coupling between the second dot and the continuous spectrum states is described by the Hamiltonian:

$$\hat{H}_{\text{tun}} = \sum_{p\sigma} t(c_{p\sigma}^+ c_{2\sigma} + c_{p\sigma} c_{2\sigma}^+), \tag{3}$$

where t is the tunneling amplitude, which we assume to be independent on momentum and spin. By considering a constant density of states in the reservoir ν_0, the tunnel rate γ is defined as $\gamma = \pi\nu_0 t^2$.

The continuous spectrum states are modeled by the Hamiltonian:

$$\hat{H}_{\text{res}} = \sum_{p\sigma} \varepsilon_p c_{p\sigma}^+ c_{p\sigma}, \tag{4}$$

where $c_{p\sigma}^+/c_{p\sigma}$ creates/annihilates an electron with spin σ and momentum p in the lead. Phonons are always presented in such systems and give significant contribution to the tunneling characteristics [19]. The typical values of optical phonons frequencies are about 50–200 meV. So depending on the size of the QDs and the system geometry it can be smaller or higher than the coupling between dots, levels spacing and the value of relaxation rate to the leads. We shall use Keldysh diagram technique to describe charge density relaxation processes in the considered system. Time evolution of the electron density in the QD is determined by the Keldysh Green function $G_{11}^<$ which is connected with the localized state filling numbers in the following way:

$$G_{11}^<(t,t) = in_1(t). \tag{5}$$

Integro-differential equations for Green function $G_{11}^{<\mathrm{T}}(t,t')$ without electron-phonon interaction has the form:

$$G_{11}^{<\mathrm{T}}(t,t') = G_{11}^{0<} + G_{11}^{0R} T^2 G_{22}^{0R} G_{11}^< + \\ + G_{11}^{0R} T^2 G_{22}^{0<} G_{11}^{AT} + G_{11}^{0<} T^2 G_{22}^{0A} G_{11}^{AT}. \tag{6}$$

The superscript T means that coupling between the QDs and interaction with the reservoir are exactly taken into account in the absence of electron-phonon interaction. In the case when initial charge is localized in the first QD and the second dot is empty, the third term in the Eq. (6) can be neglected [5]. Retarded Green's function $G_{11}^{AT}(t',t) = [G_{11}^{RT}(t,t')]^*$ yields density of states in the first QD and can be found exactly from the integral equation:

$$G_{11}^{RT} = G_{11}^{0R} + G_{11}^{0R} T^2 G_{22}^{0R} G_{11}^R, \tag{7}$$

where Green's functions $G_{11}^{0R}(t-t')$ and $G_{22}^{0R}(t-t')$ in the absence of coupling between the dots are determined by the expressions:

$$G_{11}^{0R}(t-t') = -i\Theta(t-t')e^{-i\varepsilon_1(t-t')}, \\ G_{22}^{0R}(t-t') = -i\Theta(t-t')e^{-i\varepsilon_2(t-t')-\gamma(t-t')}. \tag{8}$$

The eigenfrequencies $E_{1,2}$ of Eq. (7) are determined in the following way

$$(E-\varepsilon_1)(E-\varepsilon_2+i\gamma) - T^2 = 0, \\ E_{1,2} = \frac{1}{2}(\varepsilon_1+\varepsilon_2-i\gamma) \pm \frac{1}{2}\sqrt{(\varepsilon_1-\varepsilon_2+i\gamma)^2+4T^2}. \tag{9}$$

Finally retarded Green's function can be written as:

$$G_{11}^{RT}(t,t') = -i\Theta(t-t')\left[\frac{E_1-\varepsilon_2+i\gamma}{E_1-E_2}e^{-iE_1(t-t')} - \right. \\ \left. - \frac{E_2-\varepsilon_2+i\gamma}{E_1-E_2}e^{-iE_2(t-t')}\right] \tag{10}$$

and interaction with the continuous spectrum states is included in the Green's function $G_{22}^{0R}(t-t')$. Electron-phonon interaction results in the appearance of corrections to the Green's function G_{11}^{RT} in the Eqs. (6) and (7). Consequently the equation for Green function has the following form:

$$G_{11}^R(t,t') = G_{11}^{0R} + G_{11}^{0R} T^2 G_{22}^{0R} G_{11}^R + G_{11}^{0R} \Sigma_{11}^R G_{11}^R + \\ + G_{11}^{0R} \Sigma_{12}^R G_{21}^R + G_{12}^{0R} \Sigma_{21}^R G_{11}^R + G_{12}^{0R} \Sigma_{22}^R G_{21}^R, \tag{11}$$

where self-energies $\Sigma_{11}^R(t,t')$, $\Sigma_{12}^R(t,t')$, $\Sigma_{21}^R(t,t')$, and $\Sigma_{22}^R(t,t')$ can be written as:

$$\begin{aligned}
\Sigma_{11}^R(t,t') &= ig^2[D^> G_{22}^{AT} + D^R G_{22}^{<T}],\\
\Sigma_{12}^R(t,t') &= ig^2[D^> G_{21}^{AT} + D^R G_{21}^{<T}],\\
\Sigma_{21}^R(t,t') &= ig^2[D^> G_{12}^{AT} + D^R G_{12}^{<T}],\\
\Sigma_{22}^R(t,t') &= ig^2[D^> G_{11}^{AT} + D^R G_{11}^{<T}].
\end{aligned} \tag{12}$$

In the Eq. (12) the following relation between Green functions is used $G_{22}^{AT}(t',t) = [G_{22}^{RT}(t,t')]^*$ and expression for Green's function $G_{22}^{RT}(t,t')$ analogous to the equation (10) has the following form:

$$G_{22}^{RT}(t,t') = -i\Theta(t-t')\left[\frac{E_2-\varepsilon_1}{E_1-E_2}e^{-iE_1(t-t')} - \right.$$
$$\left. - \frac{E_1-\varepsilon_1}{E_1-E_2}e^{-iE_2(t-t')}\right]. \tag{13}$$

In our equations Green functions G_{ij}^T (see expressions (10), (13)) obtained when coupling between the QDs and interaction with the reservoir are exactly taken into account in the absence of electron-phonon interaction. These functions also determine Σ_{ij} given by the equations [11,12]. All these Green functions are the result of summation of the infinite diagram rows. But function G_{ij} is of the order of T/γ compared with the diagonal Green functions. The last three terms in Eq. (11) are of the order of T^2/γ^2 compared to the first three terms. Consequently, localized charge relaxation in the presence of electron-phonon interaction is mostly governed by the term $G_{11}^{0R}\Sigma_{11}^R G_{11}^R$. So one can re-write the equation (11) in the following way:

$$(G_{11}^{0R-1} - T^2 G_{22}^{0R} - \Sigma_{11}^R)G_{11}^R(t,t') = \delta(t-t'). \tag{14}$$

The eigenvalues of Eq. (14) can be found from characteristic equation written in the following form:

$$\begin{gathered}
[G_{11}^{0R-1}(\omega)G_{22}^{0R-1}(\omega) - T^2] \times \\
\times [G_{11}^{0R-1}(\omega-\omega_0)G_{22}^{0R-1}(\omega-\omega_0) - T^2] - \\
- g^2(2N_{0\omega}+1)G_{22}^{0R-1}(\omega)G_{11}^{0R-1}(\omega-\omega_0) = 0,
\end{gathered} \tag{15}$$

where $N_{0\omega}$ is the standard equilibrium Bose–Einstein distribution function for phonons and functions G_{ii}^{0R-1} can be determined as

$$G_{ii}^{0R-1} = i\frac{\partial}{\partial t} - \varepsilon_i. \tag{16}$$

Consequently retarded Green's function G_{11}^R can be written in the following form:

$$G_{11}^R(t,t') = \sum_i -i\Theta(t-t')A_i e^{-iE_i(t-t')}, \tag{17}$$

where E_i are eigenvalues of Eq. (15). Coefficients A_i can be found from the system of linear equations obtained in the first order perturbation theory in the parameter g^2:

$$\begin{gathered}
\sum_{i=1}^4 A_i = 1,\\
-\sum_{i=1}^4 A_i \sum_{j\neq i} E_j = -(E_3^0 + E_4^0 + \varepsilon_2 - i\gamma),\\
\sum_{i=1}^4 A_i \sum_{k\neq l\neq i} E_k E_l = E_3^0 E_4^0 + (\varepsilon_2 - i\gamma)(E_3^0 + E_4^0),\\
\sum_{i=1}^4 A_i \prod^{j\neq i} E_j = -(\varepsilon_2 - i\gamma)E_3^0 E_4^0,
\end{gathered} \tag{18}$$

where E_i^0 are the eigenvalues of Eq. (15) with electron-phonon coupling constant $g = 0$,

$$\begin{aligned}
E_{1,2}^0 &= E_{1,2},\\
E_{3,4}^0 &= \omega_0 + E_{1,2}.
\end{aligned} \tag{19}$$

Equation for Keldysh Green function $G_{11}^<(t,t')$, which determines localized charge time evolution $n_1(t)$ than has the form:

$$\begin{gathered}
G_{11}^<(t,t') = G_{11}^{0<} + G_{11}^{0R}T^2 G_{22}^{0R}G_{11}^< + \\
+ G_{11}^{0R}T^2 G_{22}^{0<}G_{11}^A + G_{11}^{0<}T^2 G_{22}^{0A}G_{11}^A + \\
+ G_{11}^{0R}\Sigma_{11}^< G_{11}^A + G_{11}^{0R}\Sigma_{11}^R G_{11}^< + G_{11}^{0<}\Sigma_{11}^A G_{11}^A,
\end{gathered} \tag{20}$$

where self-energy $\Sigma_{11}^<(t,t')$ can be written as:

$$\Sigma_{11}^<(t,t') = ig^2 D^<(t,t')G_{22}^<(t,t') \tag{21}$$

with phonon function $D^<(t_1,t_2)$:

$$\begin{gathered}
D^<(t_1,t_2) = -i(N_{\omega 0}+1)e^{-i\omega_0(t_1-t_2)} - \\
- iN_{-\omega 0}e^{i\omega_0(t_1-t_2)}
\end{gathered} \tag{22}$$

acting with operator G_{11}^{0R-1} Eq. (20) can be re-written in the following form:

$$\begin{gathered}
G_{11}^{0R-1}G_{11}^<(t,t') = \left(i\frac{\partial}{\partial t} - \varepsilon_1\right)G_{11}^<(t,t') = \\
= T^2\int_0^\infty dt_1 G_{22}^{0R}(t,t_1)G_{11}^<(t_1,t') + \\
+ \int dt_1[\Sigma_{11}^<(t,t_1)G_{11}^A(t_1,t') + \Sigma_{11}^R(t,t_1)G_{11}^<(t_1,t')].
\end{gathered} \tag{23}$$

Green's function $G_{11}^<(t,t) = in_1(t)$ is determined by the sum of homogeneous and inhomogeneous solutions:

$$\begin{gathered}
n_1(t) = n_1^h(t) + \widetilde{n}_1(t) = n_1^h(t) + \\
+ \int_0^t G_{11}^R(t,t_1)\Sigma_{11}^<(t,t_2)G_{11}^A(t_2,t)dt_1 dt_2.
\end{gathered} \tag{24}$$

Homogeneous solution of the equation can be written in the following way:

$$n_1^h(t) = n_1^0 \sum_{ij} A_i A_j^* e^{-i(E_i - E_j^*)t}, \tag{25}$$

where coefficients A_i correspond to the Green's function G_{11}^R, which is determined by the Eq. (17). Function $G_{22}^{<\mathrm{T}}(t_1, t_2)$ can be written as:

$$G_{22}^{<\mathrm{T}}(t_1, t_2) = \sum_{i' j'=1,2} a_{i' j'} e^{-iE_{i'}^0 t_1} e^{iE_{j'}^{*0} t_2} \tag{26}$$

and coefficients $a_{i' j'}$ have the following form:

$$\begin{gathered} a_{11} = a_{22} = \tfrac{iT^2}{|E_2^0 - E_1^0|^2}, \\ a_{12} = a_{21}^* = -a_{11}. \end{gathered} \tag{27}$$

Consequently one can find the inhomogeneous solution of the equation

$$\begin{gathered} \widetilde{n}_1(t) = g^2 \sum_{iji' j'=1}^{4} A_i A_j^* a_{i' j'} \times \\ \times \frac{-1}{i(E_j^* - E_{j'}^{0*} - \omega_0)} \frac{1}{i(E_i - E_{i'}^0 - \omega_0)} \times \\ \times [e^{-i(E_{i'}^0 + \omega_0)t} - e^{-iE_i t}][e^{i(E_{j'}^{0*} + \omega_0)t} - e^{iE_j^* t}]. \end{gathered} \tag{28}$$

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Considering only the leading terms in parameters g^2/ω_0^2, T^2/γ^2 in Eq. (28), the following expression for the inhomogeneous solution can be obtained:

$$\begin{gathered} \widetilde{n}_1(t) = \frac{g^2}{\omega_0^2} \frac{T^2}{\gamma^2} \sum_{i' j'=1}^{2} [e^{-i(E_{i'}^0 + \omega_0)t} - e^{-iE_1 t}] \times \\ \times [e^{i(E_{j'}^{0*} + \omega_0)t} - e^{iE_1^* t}]. \end{gathered} \tag{29}$$

For the small values of electron-phonon interaction $g/\omega_0 \ll 1$, relaxation of the localized charge is determined mostly by the homogeneous part of Eq. (25).

3. Results and discussion. The behavior of filling numbers time evolution depends on the parameters of the system: energy levels detuning, the relation between tunneling rates and electron-phonon coupling constant, the value of optical phonon frequency. The general feature of all dependencies is the increasing of localized charge relaxation rate caused by the electron-phonon interaction.

We start by discussing the filling numbers time evolution in the case of the positive initial detuning between energy levels in the coupled QDs ($\Delta\varepsilon = \varepsilon_1 - \varepsilon_2 > 0$). Obtained calculation results are presented on the Fig. 1. It

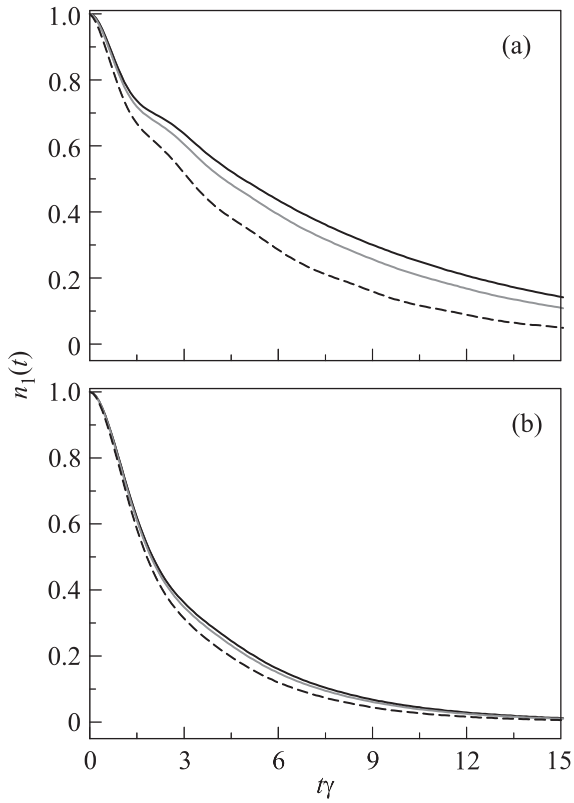

Fig. 1. Filling numbers time evolution in the presence of electron-phonon interaction in the case of positive initial detuning $\Delta\varepsilon$. Black line corresponds to the case when $g = 0$, grey line describes the situation when $g = 0.1$ and black-dashed line – $g = 0.2$. (a) – $\Delta\varepsilon = 2.0$, $\omega_0 = 2.0$. (b) – $\Delta\varepsilon = 1.0$, $\omega_0 = 1.0$. For all the figures values of parameters $T = 0.6$, $\gamma = 1.0$ are the same

is clearly evident that electron-phonon interaction leads to the increasing of localized charge relaxation rate. The growth of the electron-phonon coupling constant g for a given set of system parameters results in the increasing of filling numbers relaxation rate. With the increasing of the initial detuning the influence of electron-phonon interaction on the charge time evolution is clearly pronounced (see Fig. 1a).

A critical value of the detuning exists in the system under investigation for a given set of parameters which corresponds to the relaxation regime changing. For the smaller values of the detuning charge relaxation takes place with the only one relaxation rate both in the presence (see grey and black-dashed lines on the Fig. 1b) and in the absence of electron-phonon interaction (see black line on the Fig. 1b). The typical time scale which determines the localized charge relaxation is close to the value $\gamma_{\mathrm{res}} = 2T^2/\gamma$. For the larger values of detuning than the critical one, localized charge time evolution reveals two typical time intervals with different values of the relaxation rates (see Fig. 1a). The first time interval relaxation rate exceeds the relaxation rate of the second time interval both in the presence and in the absence of electron-phonon coupling. Without electron-phonon interaction the first time interval reveals charge relaxation with the typical rate $\gamma_{\rm res}$. The second time interval demonstrates charge time evolution with relaxation rate close to $\gamma_{\rm nonres} = \gamma_{\rm res}(\gamma^2/\Delta\varepsilon^2)$. When electron phonon-coupling is involved, the filling numbers time evolution in the first time interval reveals charge relaxation with the typical rate very close to $2\gamma_{\rm res}$ and in the second time interval to $-2\gamma_{\rm nonres}$. Consequently, electron-phonon interaction results in the two times increasing of localized charge relaxation rate.

Let us now focus on the charge relaxation processes in the case of negative initial detuning between energy levels in the QDs ($\varepsilon_1 < \varepsilon_2$) (see Fig. 2). One

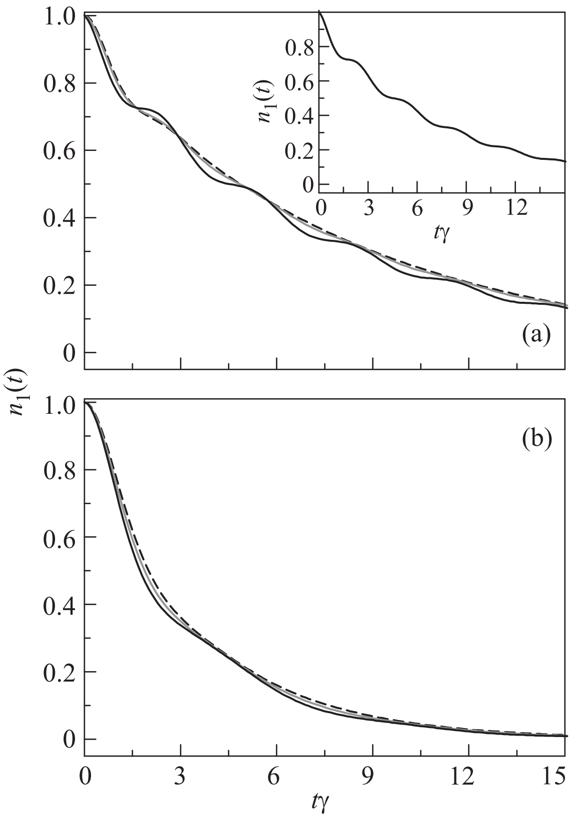

Fig. 2. Filling numbers time evolution in the presence of electron-phonon interaction in the case of negative initial detuning $\Delta\varepsilon$. Black-dashed line corresponds to the case when $g = 0$, grey line describes the situation when $g = 0.1$ and black line – $g = 0.2$. (a) – $\Delta\varepsilon = -2.0$, $\omega_0 = 2.0$. (b) – $\Delta\varepsilon = -1.0$, $\omega_0 = 1.0$. For all the figures values of parameters $T = 0.6$, $\gamma = 1.0$ are the same. The inset demonstrates localized charge relaxation in the case when $g = 0.2$

can clearly see that in the case of negative detuning electron-phonon interaction also results in the increasing of localized charge relaxation rate, but this effect is less pronounced. Charge time evolution changes slightly in comparison with the situation when positive detuning occurs. In the case of negative detuning always exist several time intervals with different values of the relaxation rates (see Fig. 2). For the small value of initial detuning (see Fig. 2b) relaxation rates on the both time intervals are very close to each other and to the value $\gamma_{\rm res}$ both in the presence and in the absence of electron-phonon interaction. For the larger value of initial detuning in the case when electron-phonon coupling is absent the first time interval reveals charge relaxation with the typical rate $\gamma_{\rm res}$ and the second time interval corresponds to the relaxation rate $\gamma_{\rm nonres}$. When electron-phonon coupling is considered, relaxation rates increase slightly and they are continue being very close of the values $\gamma_{\rm res}$ and $\gamma_{\rm nonres}$ for the first and second time intervals correspondingly.

The most interesting effect in this energy levels configuration is the formation of oscillations in the filling numbers time evolution. Oscillations are connected with electron-phonon interaction and they appears due to the formation of new inelastic channel for charge redistribution in the double QDs. This channel is connected with the emission and adsorption of phonons during charge transfer between coupled QDs. This effect is mostly pronounced in the case when detuning is close to the phonon frequency. Oscillations are well pronounced for the large value of initial detuning (see Fig. 2a and the inset). For a given set of system parameters the oscillations amplitude is determined by the value of the electron-phonon coupling constant g (see Fig. 2). The presence of oscillations may even lead to the decreasing of filling numbers relaxation rate in comparison with the case when electron-phonon interaction is absent (see black-dashed and black lines on the Fig. 2a) for a given set of system parameters in the particular time intervals.

4. Conclusion. We investigated filling numbers time evolution in the system of two interacting QDs weakly coupled to the reservoir in the presence of electron-phonon interaction. It was shown that electron-phonon interaction results in the increasing of localized charge relaxation rate. The value of extantion is determined by the system parameters such as energy levels detuning, optical phonon frequency and the ratio between electron-phonon coupling constant and tunneling transfer amplitudes.

We revealed that in the case of positive initial detuning between energy levels in the dots the influence of electron-phonon interaction is mostly pronounced and the relaxation rate increases with the growth of the initial detuning value.

We found that when negative initial detuning is considered, the influence of electron-phonon interaction leads to the formation of oscillations in the filling numbers time evolution. These oscillations are the result of phonons emission and adsorption between the energy levels.

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