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Interplay of total cross sections and ratios of real to imaginary parts of hadron amplitudes

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The impact of different assumptions about high energy behavior of the total cross section of proton–proton interactions on the ratio of the real to imaginary part of the forward elastic scattering amplitude is analyzed. It is shown how experimental data about this ratio at LHC energies can help in the proper choice of the asymptotic dependence of the total cross section.

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The total cross sections and the ratios of real to imaginary parts of forward scattering amplitudes are tightly connected by dispersion relations. Hence, both of them should be analyzed simultaneously. We consider the interrelation of their behaviors as functions of energy discussing how different assumptions about the energy dependence of the total cross section σ_t influence the predictions of the values of the ratio

$$\rho_0 = \rho(s,t=0) = \frac{\mathrm{Re}A(s,t=0)}{\mathrm{Im}A(s,t=0)}, \qquad (1)$$

where $A(s,t)$ is the elastic scattering amplitude at energy $\sqrt{s}$ and transferred momentum t.

In its turn, this ratio measured at LHC can give some hints to the proper choice of the asymptotic behavior of the total cross section.

Experimentally, this ratio is determined from the interference of the Coulomb and nuclear parts of the amplitude at extremely small angles. It is negative at comparatively low energies, becomes positive at energies $\sqrt{s}$ exceeding tens of GeV, and increases at ISR. These features are well reproduced by dispersion relations.

The total cross sections of hadron processes rise with energy increase. This surprising fact was first noticed in kaon-proton interactions in Protvino [1]. Later, it was confirmed at ISR, S$p\bar{p}$S, and Tevatron. Nowadays, with advent of LHC, it has obtained further support.

By the optical theorem, the total cross section is directly related to the imaginary part of the forward elastic scattering amplitude

$$\sigma_t(s) = \frac{\mathrm{Im}A(s,t=0)}{s}. \qquad (2)$$

At the same time, it is well known that due to analyticity properties of the amplitude its imaginary and real parts are interelated by Kramers–Kronig integral dispersion relations in such a way that the real part can be expressed as some integral of the imaginary part, or of the total cross section if the forward direction is considered. This procedure has been widely used for predicting the energy behavior of the real part of the forward scattering amplitude once the total cross section is inserted under the integral sign [2, 3]. The predictions strongly depend on the assumptions about the asymptotic behavior of the total cross section. The bunch of predicted curves becomes very wide at higher energies. Nevertheless, some qualitative statements can be done.

The total cross section of proton–proton scattering has been measured in a wide energy range. So far, the extrapolations of the total cross sections, which have been proposed on the basis of various theoretical arguments, cannot be judged reliable, as can be seen from the fact that many of them have had to be discarded as accelerator energies have moved upward.

At the LHC, the situation simplifies. The energy-decreasing contributions can be neglected. Constant term will be mentioned separately. The phenomenological fits use three main energy-dependent components:

$$\sigma_t(s) = a_1 \ln s + a_2 \ln^2 s + a_3 s^{\Delta} \qquad (3)$$

with variable coefficients a_i and the parameter Δ. The variable s is in GeV^2.

The logarithmic components are related to the so-called Froissart–Martin bound, which states that the total cross section cannot increase asymptotically faster than $\ln^2 s$. Actually, the $\ln^2 s$-dependence is often ascribed to the geometric picture of two hadrons colliding with asymptotically high energies and interacting as Lorentz-contracted disks with logarithmically increasing radii (for example, see [3, 4]).

The third component is considered as preasymptotic one, and related to the hard Pomeron with the intercept $1+\Delta$. We expect that, after unitarization, it will be consumed at extremely high energies by a slower dependence of the $\ln^2 s$-type.

From Fig. 1 in [2], we have learned that linear logarithmic increase of the total cross section leads to very low values of $\rho_0(s)$. The $\ln^2 s$-dependence saturating the Froissart–Martin bound gives rather large values of $\rho_0(s)$. In both cases, it slowly decreases with energy as an inverse logarithm. The power-law dependence provides asymptotically constant values of ρ_0.

These conclusions are supported by the local dispersion relations, which state that, in practice, the value ρ_0 is mainly sensitive to the local derivative of the total cross section. In the first approximation, the result of the dispersion relation can then be written in the form [5–7]

$$\rho_0(s) \approx \frac{1}{\sigma_t}\left[\tan\left(\frac{\pi}{2}\frac{d}{d\ln s}\right)\right]\sigma_t = = \frac{1}{\sigma_t}\left[\frac{\pi}{2}\frac{d}{d\ln s}+\frac{1}{3}\left(\frac{\pi}{2}\right)^3\frac{d^3}{d\ln s^3}+...\right]\sigma_t. \quad (4)$$

It follows that, at high energies, $\rho_0(s)$ is mainly determined by the derivative of the logarithm of the total cross section with respect to the logarithm of energy.

According to Eq. (4), we obtain

$$\rho_0(s) \approx \frac{\pi}{2}\frac{1}{a_1\ln s+a_2\ln^2 s+a_3 s^\Delta}\times \times\left\{a_1+2a_2\ln^2 s+a_3\Delta s^\Delta\left[1+\frac{1}{3}\left(\frac{\pi\Delta}{2}\right)^2\right]\right\}. \quad (5)$$

It is valid at small values of Δ.

First, we consider the three terms of Eq. (3), separately.

1. $a_2=a_3=0$. $\rho_0(s)=\pi/(2\ln s)$.

It gives the values 0.0887 at energy 7 TeV and 0.0823 at 14 TeV.

2. $a_1=a_3=0$. $\rho_0(s)=\pi/\ln s$.

It gives the values 0.1774 at energy 7 TeV and 0.165 at 14 TeV.

We note that the coefficient in front of $1/\ln s$ is twice larger than in the case 1.

3. $a_1=a_2=0$. $\rho_0(s)=\frac{\pi\Delta}{2}\left[1+\frac{1}{3}\left(\frac{\pi\Delta}{2}\right)^2\right]= =\mathrm{const}$.

It gives the values 0.21 at $\Delta=0.13$ [2], 0.131 at $\Delta=0.08$, and 0.094 at $\Delta=0.06$, i.e., they strongly depend on the hard Pomeron intercept accepted at the present stage.

These findings are not at all unexpected and comprise with those of Fig. 1 in [2], but they are somewhat surprising in what concerns the predictions at LHC. According to a general folklore [2, 3, 8–10], we would expect this ratio to be in the interval 0.13–0.14 at LHC energies. The first term in Eq. (3) leads to smaller values, and the second term to larger ones. The simple combination of them with positive weights does not fit either the total cross sections or the expected values of ρ_0. Sometimes, the coefficient a_1 is chosen to be negative (see, e.g., [9, 11]). In these cases, the constant term should be added in the expression for the total cross section.

The old version [11] predicts too high value of 127 mb for the total cross section at 7 TeV. It has been used in extrapolation 4 of [2], and gave rise to a maximum and then to somewhat faster decrease of ρ_0 with energy but its values are still a little bit large. Surely, this is related to larger cross sections predicted at LHC.

The recent fit in [9] predicts slightly smaller value 95.4 ± 1.1 mb compared with 98.3 ± 2.9 mb in experiment but acceptable in view of error estimates. The values of ρ_0 are within the interval 0.13–0.14.

However, up to now we have no reasonable justification for including the terms with negative sign in the expressions for total cross sections. Actually, the total cross section is compiled as a sum of positive contributions of individual channels. Hence, one should develop a model with such negative contributions in particular channels.

The intermediate behavior between variants 1 and 2 was considered in [12, 13]. It is supposed that $\sigma_t \propto \ln^r s$ with $1\le r\le 2$. Then $\rho_0(s)=\pi r/2\ln s$, and fills in the gap between predictions of 1 and 2. However, the value of r is left indefinite. It is claimed that according to some fits of experimental data it can be within the limits $1.3\le r\le 1.52$. It would mean $0.115\le\rho_0\le0.135$ at 7 TeV, and $0.107\le\rho_0\le0.125$ at 14 TeV. The total cross section would increase, correspondingly, by 10–12%.

It looks as if the admission of the third term is necessary not only for fits of the total cross section but is required by the values of ρ_0. Its prediction of constant ρ_0 for a constant Δ is, however, not very appealing intuitively. We would expect that the parameter Δ depends on energy, and, probably, becomes smaller at higher energies to prevent the violation of the Froissart–Martin bound. Theoretically, it would correspond to unitarization of the hard Pomeron. Then, $\rho_0(s)$ becomes smaller than 0.13 as demonstrated above for $\Delta=0.06$.

In general, lower values of ρ_0 at LHC compared to the widely accepted interval 0.13–0.14 would indicate smaller parameter Δ, but show that the way to asymptopia is still very long. Larger values seem less probable, but, if observed, would imply more intriguing situation with competition between pure $\ln^2 s$-law and larger values of the hard Pomeron intercept Δ.

There were attempts to keep the total cross section within $\ln^2 s$-bound suppressing it by small power of s and fit $\rho_0(s)$. For example, such a shape of the total cross section was proposed in [14] with the slight power-like damping

$$\sigma_t \propto cs^{-\alpha} \ln^2 s \tag{6}$$

with $\alpha \approx 0.02$.

It leads to

$$\rho_0(s) = \frac{\pi}{\ln s} \left(1 - \frac{\alpha}{2} \ln s\right). \tag{7}$$

This results in extremely low value at 7 TeV $\rho_0(7\,\text{TeV}) = 3 \cdot 10^{-4}$, and even negative value at 14 TeV $\rho_0(14\,\text{TeV}) = -2.6 \cdot 10^{-2}$, which look unrealistic. Nevertheless, this teaches us that slight variations of $\ln^2 s$-law can drastically diminish $\rho_0(s)$.

The above results can be considered as upper limits of the corresponding estimates for particular models. In most models the energy independent term σ_0 is added in Eq. (3). Therefore it appears in the denominator of Eq. (5) and diminishes the above estimates of ρ_0. It is especially large in the models [3, 11, 12], where it exceeds even the minimum of the cross section at tens of GeV, comparatively small in [10, 14–16], and is absent in [17].

To conclude, we have argued that neither of the three possibilities of energy behavior of the total cross section presented in Eq. (3) seem to satisfy separately our experience with energy dependences of $\sigma_t(s)$ and $\rho_0(s)$. Only their combination with some constant term added seems to be satisfactory. Which one of these contributions wins in the competition for asymptopia, can be guessed from the forthcoming data on ρ_0 at LHC energies.

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