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Estimate of the cross section for thermal neutrons

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Cross section of the thermal neutrons in the framework of statistical approach to the complicated nuclei is considered. We calculate probability distribution $\mathcal{P}(z)$ to have given cross section σ (determined by fluctuations of resonance positions and widths). z is the ratio of σ to σ^* where σ^* is the cross section for the model of equidistant resonances with same width. The last quantity can be presented in terms of neutron strength function for given nuclei. Probability distribution $\mathcal{P}(z)$ is universal for all nuclei.

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1. Introduction. To plan experiments with neutrons one must beforehand know their interaction cross sections with nuclei (at least by order of magnitude). In the region of thermal energies the cross sections differ by several orders of magnitude even for nearby nuclei. Therefore they cannot be predicted exactly. The reason for this lies first of all in the different strength functions of different nuclei. However, the main difference comes from fluctuations of the positions and widths of the resonances closest to the zero energy of the incoming neutron. For complicated nuclei number of resonances is very large and their parameters can be considered as random quantities.

A statistical approach to the estimate of thermal cross sections has been first proposed by I.I. Gurevich in 1939 [1]. The random fluctuations of the resonance parameters were taken into account by introducing a universal distribution $\mathcal{P}(z)$ for the cross sections of thermal neutrons. The variable $z = \sigma_r/\sigma_r^*$ is the ratio of the true reaction cross section σ_r to the expected one, σ_r^*, which is calculated individually for each nucleon from its mean resonance parameters. σ_r^* is supposed to be a smooth function from one nucleus to another.

In Ref. [2, 3] the expected cross section of the neutron capture ((n,γ) reaction) was expressed in terms of the strength functions in the following way (for s neutrons, the neutron strength function S_0 and the radiative strength function S_γ):

$$\sigma_\gamma^* = \frac{\pi^3}{k_0^2}\left(\frac{A+1}{A}\right)^2\sqrt{\frac{E_0}{E_{th}}}S_0S_\gamma = \\ = 0.40\cdot 10^8\left(\frac{A+1}{A}\right)^2 S_0S_\gamma,\ \text{barn}. \qquad (1)$$

Here A is the atomic mass of the target nucleus, k_0 is momentum of neutrons with energy $E_0 = 1\,\text{eV}$, $E_{th} = 0.0253\,\text{eV}$ is the thermal energy. In deriving Eq. (1) it was assumed that the resonances are equidistant, their reduced widths are equal, and the zero neutron energy divides the interval between the resonances of positive and negative energy by half (fence model).

Cross section distribution. If the resonance parameters did not fluctuate, the thermal cross section distribution would be described by a delta function $\mathcal{P}_\gamma(z) = \delta(z-1)$ and the capture cross section could be deduced from Eq. (1). As a result of the fluctuations, the delta function is smeared out and forms a wide distribution $\mathcal{P}_\gamma(z)$ [2–6]. The probability $\mathcal{P}_\gamma(z)$ should take into account of the equiprobable position of the zero neutron energy between two resonances, the distribution of neutron widths according to Porter and Thomas, and the distance between neighboring levels according to Wigner distribution.

For large z, the leading term in the probability $\mathcal{P}_\gamma(z)$, with account of all distributions is of the following form (compare with [6]):

$$\mathcal{P}_\gamma(z) = \frac{1}{\pi z^{3/2}}\left\langle\sqrt{\frac{\Gamma_{n0}}{\overline{\Gamma}_{n0}}}\right\rangle_{PT}\left\langle\frac{\overline{D}_0}{D_0}\right\rangle_W = \frac{1}{\sqrt{2\pi z^3}} \qquad (2)$$

($z \gg 1$). We used here $\left\langle\sqrt{\Gamma_{n0}/\overline{\Gamma}_{n0}}\right\rangle_{PT} = \sqrt{2/\pi}$ and $\left\langle\overline{D}_0/D_0\right\rangle_W = \pi/2$.

For small $z \ll 1$, without taking account of the Wigner distribution, the following formula was obtained [2]:

$$\mathcal{P}_\gamma(z) = (2/\pi z)^{3/2}e^{-1/2z}. \qquad (3)$$

The account of the fluctuations of the distances between the resonances changes this behavior strongly (see below, Eq. (30)).

The complete theoretical distribution $\mathcal{P}_\gamma(z)$ was obtained modeling cross sections on the computer by the Monte Carlo method (see Fig. 1). We derived also in-

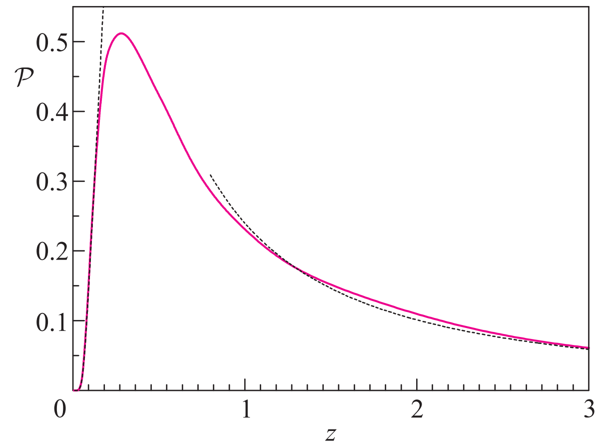

Fig. 1. Monte Carlo evaluation of the $\mathcal{P}_\gamma(z)$ distribution (10^8 hypothetical nuclei; $z = \sigma/\sigma^*$). Dashed and dashed-dotted line are asymptotic at large and small z given by (18) and (30) correspondingly

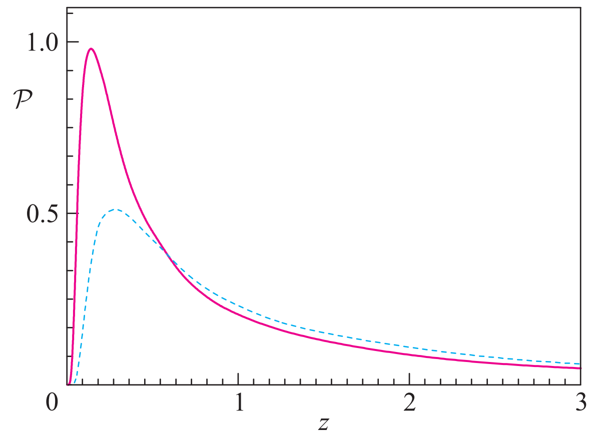

Fig. 2. Distributions $\mathcal{P}(z)$ (dashed curve) and $\mathcal{P}'(z)$ (solid curve) for the cases when one or both widths are fluctuating according to Porter–Thomas (10^8 hypothetical nuclei; 10^3 positive and 10^3 negative resonances)

tegral equation for $\mathcal{P}_\gamma(z)$ which can be used to obtain asymptotic of the distribution.

It can be seen from Fig. 1 that fluctuations of the neutron resonance positions and parameters, predicted by the statistical model, lead to a broad distribution of the neutron capture cross sections $\mathcal{P}_\gamma(z)$. The distribution $\mathcal{P}_\gamma(z)$ together with both asymptotic can be used for a quantitative estimate of the probability for an unknown thermal cross section to lie within given limits.

The similar approach could be used also for other reactions with neutrons. One has to distinguish two cases depending on fluctuations of the exit width [2]. In the statistical approach fluctuations of the reduced widths is described by $\chi^2(\nu)$ where ν is the number of decay channels. If this number is large (capture cross section, total cross section) this distribution reduces to δ-function, i.e. exit width does not fluctuate. In the other limiting case the final state is unique and exit widths are distributed according to Porter–Thomas (i.e. in the same manner as entrance width). Examples are (n,α) and INNA (neutron acceleration [6, 7]) reactions. We consider mainly the first case but the generalization is straightforward and we compare at the end both cases (see Fig. 2).

2. Neutron cross section as the random quantity. It is well-known that in the complicated nuclei the system of levels is essentially random ensemble. The distance between levels is distributed according to Wigner:

$$P_{\mathrm{W}}(\varepsilon) = \frac{\pi}{2}\varepsilon \exp\left(-\frac{\pi}{4}\varepsilon^2\right) \tag{4}$$

while widths of resonances follow so-called Porter–Thomas distribution:

$$P_{\mathrm{PT}}(t) = \frac{1}{\sqrt{2\pi t}}e^{-t/2}. \tag{5}$$

According to the standard picture the thermal neutron interacting with A-nuclei forms a compound $A+1$-nuclei in the excited state with the energy of the excitation equal to the binding energy of the neutron. This energy falls almost randomly in the lattice of energy levels for the compound. Cross section of the process is described by Breit–Wigner formula:

$$z = \sum_i \frac{t_i}{\pi^2(E-E_i)^2}. \tag{6}$$

Here t_i are widths of the resonances and E_i are their energies. We neglect here the width in the denominator as it is usually much smaller than a distance between levels.

The number of resonances in the complex nuclei is very large, it is a good approximation to consider this number to be infinite. Let us divide all resonances into two groups: positive (located to the right to the energy of the neutron) and negative ones. Let us denote by ε the distance between two closest to the neutron energy resonances and choose the scale of the energy in such a way that the energy of the neutron is zero. In this scale the energy of the closest to the neutron energy positive resonance is εx and the energy of the negative one is $-\varepsilon(1-x)$, where x is a random quantity with a flat distribution in the interval $0 < x < 1$.

We denote energies of the positive resonances next to the closest ones as $\varepsilon_1^+ < \varepsilon_2^+ < \ldots$, energies of the negative ones as $-\varepsilon_1^- > -\varepsilon_2^- > \ldots$. In this notation the Breit–Wigner formula Eq. (6) is divided into the contribution of positive and negative resonances:

$$
\begin{aligned}
z &= z_+ + z_-,\\
z_+ &= \frac{t^+}{\pi^2\varepsilon^2 x^2} + \sum_{i=1}^{\infty}\frac{t_i^+}{\pi^2\left(\varepsilon_i^+\right)^2},\\
z_- &= \frac{t^-}{\pi^2\varepsilon^2(1-x)^2} + \sum_{i=1}^{\infty}\frac{t_i^-}{\pi^2(\varepsilon_i^-)^2}.
\end{aligned} \tag{7}
$$

Clearly two contributions can be obtained from each other by $x \leftrightarrow 1-x$ exchange.

The cross section Eq. (7) is a random quantity with $t_i^\pm$ distributed according to Eq. (5), distances between resonances $\varepsilon_i - \varepsilon_{i-1}^\pm$ distributed according to Eq. (4) and flat distribution in x. We would like to find a distribution $\mathcal{P}(z)$ which gives the probability to find the cross-section z in such a system. It is given by infinite-dimensional integral:

$$
\begin{aligned}
\mathcal{P}(z) = \int_0^1 dx \int_0^\infty d\varepsilon\, dt^+\, dt^- P_{\mathrm{PT}}(t^+)P_{\mathrm{PT}}(t^-)P_{\mathrm{W}}(\varepsilon)\times\\
\times\prod_{i=1}^{\infty}\left[\int_0^\infty d\varepsilon_i^+ dt_i^+ P_{\mathrm{W}}(\varepsilon_i^+ - \varepsilon_{i-1}^+)P_{\mathrm{PT}}(t_i^+)\right]\times\\
\times\prod_{i=1}^{\infty}\left[\int_0^\infty d\varepsilon_i^- dt_i^- P_{\mathrm{W}}(\varepsilon_i^- - \varepsilon_{i-1}^-)P_{\mathrm{PT}}(t_i^-)\right]\times\\
\times\,\delta[z - z^+(x,\varepsilon_i^+,t_i^+) - z^-(x,\varepsilon_i^-,t_i^-)]
\end{aligned} \tag{8}
$$

with $z^\pm$ determined by Eq. (7). It is implied in this equation that distribution $P_{\mathrm{W}}(\varepsilon) = 0$ if its argument ε is negative. This corresponds to the ordering of the resonances described above.

The infinite number of integrations in the Eq. (8) makes a problem to be close in spirit to the problems of quantum field theory in the functional integral approach or to the problems of the statistical physics. In fact, the closest analogy is the theory of disordered systems. Fortunately, our problem appears to be rather simple example of this type.

It is convenient to write down expression for the Laplace transform $\widetilde{\mathcal{P}}(\alpha)$ of the distribution $\mathcal{P}(z)$.

$$
\begin{aligned}
\widetilde{\mathcal{P}}(\alpha) = \int_0^\infty dz\, e^{-\alpha z}\mathcal{P}(z) =\\
= \int_0^1 dx \int_0^\infty d\varepsilon dt^+ dt^- P_{\mathrm{W}}(\varepsilon)P_{\mathrm{PT}}(t_+)P_{\mathrm{PT}}(t_-)\times\\
\times\exp\left[-\frac{\alpha t^+}{\pi^2\varepsilon^2 x^2} - \frac{\alpha t^-}{\pi^2\varepsilon^2(1-x)^2}\right]\mathfrak{P}(\alpha,\varepsilon x)\mathfrak{P}[\alpha,\varepsilon(1-x)],
\end{aligned} \tag{9}
$$

where $\mathfrak{P}(\alpha,\varepsilon x)$ and $\mathfrak{P}[\alpha,\varepsilon(1-x)]$ are the contributions of positive and negative resonances:

$$
\mathfrak{P}(\alpha,\varepsilon) = \prod_{i=1}^{\infty}\left[\int dt_i d\varepsilon_i\, P_{\mathrm{W}}(\varepsilon_i - \varepsilon_{i-1})P_{\mathrm{PT}}(t_i)e^{-\frac{\alpha t_i}{\pi^2\varepsilon_i^2}}\right] \tag{10}
$$

(at $i = 0$ the energy $\varepsilon_0 \equiv \varepsilon$).

It is easy to perform averaging in the widths of the resonances in Eq. (10):

$$
S(\alpha,\varepsilon) \equiv \int_0^\infty dt\, P_{\mathrm{PT}}(t)\exp\left(-\frac{\alpha t}{\pi^2\varepsilon^2}\right) = \frac{1}{\sqrt{1+\dfrac{2\alpha}{\pi^2\varepsilon^2}}}. \tag{11}
$$

We introduce further a new function $\Phi(\alpha,\varepsilon) = S(\alpha,\varepsilon)\mathfrak{P}(\alpha,\varepsilon)$ and rewrite Eq. (9) for the Laplace transform of the distribution $\mathcal{P}(z)$ as the average in the distance ε only:

$$
\widetilde{\mathcal{P}}(\alpha) = \int_0^1 dx \int_0^\infty d\varepsilon\, P_{\mathrm{W}}(\varepsilon)\Phi(\alpha,\varepsilon x)\Phi[\alpha,\varepsilon(1-x)]. \tag{12}
$$

Function $\Phi(\alpha,\varepsilon)$ is obtained as a result of the subsequent integrations in the energies of the resonances; it is clear that in the limit of the infinite number of resonances it should obey the integral equation:

$$
\frac{\Phi(\alpha,\varepsilon)}{S(\alpha,\varepsilon)} = \int_\varepsilon^\infty d\varepsilon'\, P_{\mathrm{W}}(\varepsilon'-\varepsilon)\Phi(\alpha,\varepsilon'). \tag{13}
$$

Equations (11)–(13) are sufficient to find the Laplace transform $\widetilde{\mathcal{P}}(\alpha)$ and hence the cross section distribution $\mathcal{P}(z)$. If the exit width of the reaction fluctuates as well and distributed according to Porter–Thomas (see Introduction) one has to modify only function S to be:

$$
\begin{aligned}
S(\alpha,\varepsilon) = \int_0^\infty dt_1 dt_2 P_{\mathrm{PT}}(t_1)P_{\mathrm{PT}}(t_2)e^{-\frac{\alpha t_1 t_2}{\pi^2\varepsilon^2}} =\\
= \frac{y}{2\sqrt{\pi}}e^{y^2/8}K_0\left(\frac{y^2}{8}\right), \quad y = \frac{\pi\varepsilon}{\sqrt{\alpha}}.
\end{aligned} \tag{14}
$$

where K_0 is modified Bessel function. Integral equation (13) as well as representation for distribution $\mathcal{P}(z)$ Eq. (12) are valid also in this case. We will proceed, however, with the situation when only entrance width is fluctuating.

From the practical point of view direct numerical simulations seem to be more simple task than solution of integral Eq. (13), so we will use Monte Carlo method to obtain exact $\mathcal{P}(z)$. However derived equations are convenient to find analytical asymptotic of $\mathcal{P}(z)$ in the regions of large and small z.

Asymptotic of large z. At large $z \gg 1$ we have to calculate $\widetilde{\mathcal{P}}(\alpha)$ at small $\alpha \ll 1$. At $\alpha = 0$ the integral equation (13) has an obvious solution $\Phi(0,\varepsilon) = 1$. This corresponds also to the limit of large $\varepsilon \gg \sqrt{\alpha}$, see Eq. (11) for $S(\alpha,\varepsilon)$. However the main contribution to small z asymptotics comes from the region $\varepsilon \sim \sqrt{\alpha} \ll 1$. In this region the integral in r.h.s of Eq. (13) converges owing to the Wigner dirstribution $P_w(\varepsilon'-\varepsilon)$ and ε' is of order of unity. Therefore we can neglect ε in the argument of Wigner distribution and obtain:

$$\Phi(\alpha,\varepsilon) = S(\alpha,\varepsilon) + O(\alpha,\sqrt{\varepsilon}), \quad \sqrt{\alpha}/\varepsilon \sim 1. \qquad (15)$$

Substituting Eq. (15) into Eq. (12) we obtain for the Laplace transform

$$\widetilde{\mathcal{P}}(\alpha) = \int_0^1 dx \int_0^\infty d\varepsilon\, P_{\mathrm{W}}(\varepsilon) S(\alpha,\varepsilon x) S[\alpha,\varepsilon(1-x)] =$$

$$= \int_0^\infty d\varepsilon\, P_{\mathrm{W}}(\varepsilon) \int_0^1 \frac{dx}{\sqrt{\left(1+\frac{2\alpha}{\pi^2\varepsilon^2 x^2}\right)\left(1+\frac{2\alpha}{\pi^2\varepsilon^2(1-x)^2}\right)}}. \qquad (16)$$

In fact, this is the expression for the distribution $\widetilde{\mathcal{P}}(\alpha)$ in the approximation when only two closest resonances are taken into account (after averaging in widths of resonances). This is, of course, to be expected as the large cross section can appear only if the energy of the neutron is close to the energy of the resonance. However, next terms in the expansion of $\widetilde{\mathcal{P}}(\alpha)$ at small α require to account for other resonances.

There are two regions in Eq. (16) $x \ll 1$ and $1 - x \ll 1$ which contribute at small α. Calculating the integrals we get:

$$\widetilde{\mathcal{P}}(\alpha) = \mathrm{const} - \frac{2\sqrt{2\alpha}}{\pi}\left\langle \frac{1}{\varepsilon}\right\rangle_{\mathrm{W}} + O(\alpha) \qquad (17)$$

(const does not contribute to the back Laplace transformation: at large z only singularities of $\widetilde{\mathcal{P}}(\alpha)$ matter). The average with Wigner distribution is equal:

$$\langle \varepsilon^{-1}\rangle_{\mathrm{W}} = \int \frac{d\varepsilon}{\varepsilon} P_{\mathrm{W}}(\varepsilon) = \frac{\pi}{2}.$$

Making back Laplace transformation we get:

$$\mathcal{P}(z) = \frac{1}{\sqrt{2\pi z^3}} - \frac{1}{2\pi z^2} + O(1/z^{5/2}). \qquad (18)$$

We take into account also next term of expansion which is the last one which can be calculated in the approximation of the closest resonances. Let us note that this expression differs from the one obtained in Ref. [1–3].

Small z asymptotic. In the opposite case of small z the main contribution comes from large $\alpha \gg 1$ and $\varepsilon \sim 1$. Let us look for the solution of the integral equation (13) in the form $\Phi(\alpha,\varepsilon) = e^{-\phi(\alpha,\varepsilon)}$ where:

$$\phi(\alpha,\varepsilon) = \sqrt{\alpha}\phi_0(y) + \phi_1(y) + \frac{1}{\sqrt{\alpha}}\phi_2(y) + \ldots \qquad (19)$$

with $y \equiv \varepsilon\pi/\sqrt{\alpha}$. Thus function $\phi(\alpha,\varepsilon)$ is large in the main region but its derivatives are small. The integral in Eq. (13) is determined by the region $\varepsilon' - \varepsilon \sim 1$. Hence we can expand function $\phi(\alpha,\varepsilon')$ near the point ε:

$$\phi(\alpha,\varepsilon') - \phi(\alpha,\varepsilon) \approx \frac{\partial\phi(\alpha,\varepsilon)}{\partial\varepsilon}\tau + \frac{1}{2}\frac{\partial^2\phi(\alpha,\varepsilon)}{\partial\varepsilon^2}\tau^2 + \ldots =$$

$$= \pi\tau\phi'(y) + \frac{\pi\tau}{\sqrt{\alpha}}\left[\frac{\pi\tau}{2}\phi_0''(y) + \phi_1'(y)\right] + O(1/\alpha) \qquad (20)$$

(where $\tau = \varepsilon' - \varepsilon$). Let us plug this expression into Eq. (13). In the leading order we get

$$\sqrt{1+\frac{2}{y^2}} = \int_0^\infty d\tau\, P_{\mathrm{W}}(\tau) e^{-\tau\pi\phi_0'(y)} + O(1/\sqrt{\alpha}). \qquad (21)$$

Integrating in τ in Eq. (21) we obtain the algebraic equation for the derivative $\phi_0'(y)$. Together with the boundary condition $\phi_0(y) = 0$ at $y \to \infty$ (i.e. $\Phi(\alpha,\varepsilon) \to 1$ at $\varepsilon \to \infty$) this equation allows to determine function $\phi_0(y)$ for all y.

Let us do this calculation for $P_{\mathrm{W}}(\tau) = \delta(\tau - 1)$, i.e. in the model of *equidistant resonances*. The corresponding function ϕ_0^{eqd} is

$$\phi_0^{\mathrm{eqd}}(y) = \frac{1}{2\pi}\int_y^\infty dy' \log\left(1 + \frac{2}{y'^2}\right). \qquad (22)$$

We are interested in the function $\Phi(\alpha,\varepsilon)$ in the region $\alpha \gg 1$ and $\varepsilon \sim 1$. For this purpose we need the function $\phi(y)$ only at $y = 0$; it is equal to $\phi_0^{\mathrm{eqd}}(0) = 1/\sqrt{2}$. Thus:

$$\Phi^{\mathrm{eqd}} = \exp[-\sqrt{\alpha/2} + O(1)] \qquad (23)$$

and the back Laplace transform gives:

$$\mathcal{P}^{\mathrm{eqd}}(z) = \exp\left\{-\frac{1}{2z} + O[\log(z)]\right\}, \quad z \ll 1. \qquad (24)$$

This result was already obtained in Ref. [1].

If one takes into account fluctuations of the distances between resonances according to Wigner distributions the value of $\phi_0(0)$ can be found only by numeric solution of Eq. (21). The answer is: $\phi_0(0) = 0.59617$. We would like to find also the next-to leading correction to $\Phi(\alpha,\varepsilon)$. For this purpose it is sufficient to know the terms which are already written in the expansion Eq. (20). In the next order in $1/\sqrt{\alpha}$ Eq. (13) turns into

$$\frac{1}{\sqrt{\alpha}}\int_0^\infty d\tau\, P_{\mathrm{W}}(\tau)\left[\frac{1}{2}\pi^2\tau^2\phi_0''(y) + \pi\tau\phi_1'(y)\right] = 0. \qquad (25)$$

Hence

$$\phi_1(y) = -\frac{\pi}{2}\frac{\langle\varepsilon^2\rangle_{\mathrm{W}}}{\langle\varepsilon\rangle_{\mathrm{W}}}\phi_0'(y) = -2\phi_0'(y),$$

$$\Phi(\alpha,\varepsilon) \approx \exp\left[-\sqrt{\alpha}\phi_0(y) + 2\phi_0'(y)\right]. \qquad (26)$$

This expression has accuracy $O(1/\sqrt{\alpha})$.

Function $\phi_0(y)$ is determined by Eq. (21) and we are interested by its limit at $y \to 0$. It can be seen that the derivative $\phi_0'(y)$ is singular in this limit. Indeed, if $\phi_0'(y)$ is large the integral in Eq. (21) can be calculated by saddle point method and Eq. (21) takes a form:

$$2\pi\phi_0'(y)\exp\{-\pi[\phi_0'(y)]^2\} = -\sqrt{1+2/y^2},$$

with the solution

$$\phi_0'(y) = -\sqrt{\frac{\log\left(\frac{1}{\pi y^2}\right)}{2\pi}}\left\{1-\frac{1}{2}\frac{\log\left[\log\left(\frac{1}{\pi y^2}\right)\right]}{\log\left(\frac{1}{\pi y^2}\right)}+\ldots\right\}. \tag{27}$$

The further calculations are straightforward. By means of Eq. (27) one can restore the function $\phi_0(y)$ at small y (we remind that $\phi_0(0)$ is already known). Substituting it into Eq. (26) we obtain $\Phi(\alpha,\varepsilon)$ at small α and $\varepsilon \sim 1$ with required accuracy. Next, we have to substitute it into Eq. (12) and integrate in ε and x in order to obtain $\widetilde{\mathcal{P}}(\alpha)$. Integral in ε is calculated by saddle point method, the saddle point being

$$\bar{\varepsilon} = \sqrt{\frac{2}{\pi}\log\alpha}. \tag{28}$$

Integral in x produces some constant. The last step is to make back Laplace transformation. Again this can be done by saddle point method with a saddle point for α equal

$$\bar{\alpha} = \phi_0(0)^2/z^2, \quad z \ll 1. \tag{29}$$

We see now that $y \sim z\sqrt{\log(1/z)}$ is indeed small at small z. We already used this above.

Finally we obtain for the distribution $\mathcal{P}(z)$

$$\mathcal{P}(z) \approx$$

$$\approx \frac{I\phi_0(0)^2}{\sqrt{z^5\log\left(\frac{\phi_0(0)^2}{\pi z^2}\right)}}\exp\left[-\frac{\phi_0(0)^2}{z}-\sqrt{\frac{8}{\pi}\log\frac{\phi_0(0)^2}{\pi z^2}}\right]. \tag{30}$$

Here

$$I = \int_0^1 \frac{dx}{\sqrt{2\pi}}\exp[1-x\log x-(1-x)\log(1-x)] \approx 1.0256. \tag{31}$$

Expression (30) has accuracy only $O[1/\log(1/z)]$.

Thus we see that the probability to find a small cross-section behaves in a rather non-trivial way. This is due to the fact that small cross section can appear either because of the small width or because of the large distance to the resonance. These mechanisms compete one with another. Behavior of the probability at small z depends strongly on the concrete form of Wigner and Porter–Thomas distributions.

Numerical simulations. Distribution $\mathcal{P}(z)$ can be calculated at arbitrary z by means of computer simulations. The simulations are fast and can have high accuracy. To simulate we take 1000 positive resonances and 1000 negative ones and calculate the Breit–Wigner cross section for 10^8 hypothetical nuclei. The resulting curve is plotted in the Fig. 1 together with the asymptotic of large and small z. Distribution represents a broad curve with a peak at $z \approx 0.28$. The curve has a long tail, i.e. probability to have a cross section σ much larger than σ^* decays slowly according to Eq. (18).

At the Fig. 2 we compare the cases when one or two (entrance and exit widths) are fluctuating. The last curve is more concentrated at small z, its peak shifted to $z \approx 0.13$. Still the probability to find a cross section much larger than σ^* remains significant. The asymptotic of this curve at large z is described by

$$\mathcal{P}'(z) = \frac{1}{\pi z^{3/2}} - \frac{1}{2\pi z^2} + O(1/z^{5/2}). \tag{32}$$

The leading term comes here from $\sqrt{\alpha}$ in $\widetilde{\mathcal{P}}(\alpha)$. It differs by the factor $(2/\pi)^{1/2}$ which is the averaged $\langle\sqrt{\Gamma_{\text{exit}}}\rangle_{\text{PT}}$. The next term (it appears from $\alpha\log\alpha$) has the same coefficient as $\langle\Gamma_{\text{exit}}\rangle_{\text{PT}} = 1$. This is in line with Eq. (14).

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