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The population inversion and novel magnetoconductivity oscillations induced by resonant microwave irradiation in multi-subband two-dimensional electron systems

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The possibility of a population inversion in a multi-subband two-dimensional (2D) electron system exposed to resonant microwave (MW) radiation and a static magnetic field is shown. Theoretical analysis given here indicates that the resonant excitation of the third subband, necessary for the population inversion, is accompanied by novel magnetoconductivity oscillations in the 2D electron system formed on the surface of liquid helium.

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Recently, a body of interesting phenomena have been discovered in 2D electron systems exposed simultaneously to microwave radiation and a static magnetic field oriented normally to the electron layer. Much attention has been attracted to microwave-induced resistivity oscillations and zero-resistance states developed at the minima of these oscillations in high quality GaAs/AlGaAs heterostructures [1–3]. A variety of mechanisms intended to explain 1/B-periodic oscillations and the appearance of zero-resistance states have been proposed [4–6]. The negative linear conductivity effects are assumed to be the origin of the zero-resistance states [7].

Another kind of magneto-oscillations and zero-resistance states were observed in the 2D electron system formed on the free surface of liquid helium [8, 9]. In this electron system, the energy spectrum of surface subbands $\Delta_l \propto l^{-2}$ $(l = 1, 2, ...)$ is similar to the spectrum of a hydrogen atom [10, 11]; the effective Bohr radius $b \sim 100\,\text{Å}$. At low temperatures, only the ground surface subband is occupied. Even though the $1/B$-periodic oscillations of magnetoconductivity σ_{xx} observed in Ref. [8, 9] were similar to that reported for semiconductor systems, they existed only for MW frequencies which are close to the resonance frequency $\omega \to (\Delta_2 - \Delta_1)/\hbar \equiv \omega_{2,1}$. This means that MW excitation of surface subbands is crucial for the observation of these oscillations and zero-resistance states.

Main features of photoconductivity oscillations observed in the electron system formed on liquid helium were explained [12] as the result of quasi-elastic inter-subband electron scattering caused by a nonequilibrium population of the excited subband. Under the resonance condition, the MW field provides a substantial population of the first excited surface subband which sparks off inter-subband electron scattering by capillary wave quanta (ripplons) whose energies $\hbar\omega_q \ll T$. For elastic scattering from $l = 2$ to $l' = 1$, a simple analysis of the energy conservation of an electron in a *dc* driving electric field $E_\parallel$ indicates that the sign of the displacement of the orbit center $X' - X$ changes $1/B$-periodically with the magnetic field:

$$X' - X = \hbar\omega_c\,(\omega_{2,1}/\omega_c - m)\,/eE_\parallel, \qquad (1)$$

where $m = n' - n > 0$ is an integer, $\omega_c = eB/m_e c$ is the cyclotron frequency, and n is the Landau level (LL) number:

$$\varepsilon_{l,n,X} = \Delta_l + \hbar\omega_c\,(n + 1/2) + eE_\parallel X.$$

For example, at $X' - X > 0$ the decay of the excited subband is accompanied by electron scattering against the driving force, which gives a negative contribution into σ_{xx}.

Simple increase in the electron temperature T_e and the corresponding increase in fractional occupancies $\bar{n}_l = N_l/N_e$ of the excited subbands cannot lead to a sign-changing correction to σ_{xx}, because an electron scattering reverse to that considered in (1) compensates completely the negative terms. Thus, (1) indicates that the mechanism of negative corrections to σ_{xx} analyzed is a kind of "displacement" mechanism, which still requires a nonequilibrium population of the excited subband $\bar{n}_2 - \exp(-\hbar\omega_{2,1}/T_e)\bar{n}_1 > 0$. Since electrons on a

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liquid helium surface form a highly correlated system, the electron distribution over in-plane states (LLs) is described by an equilibrium function, which differs from the theory of the "inelastic" mechanism [5] proposed for semiconductor systems.

In the vicinity of level-matching points $B_m^{(2,1)}$ defined by $\omega_{2,1} = m\omega_c(B)$, the difference $\bar{n}_2 - \exp(-\hbar\omega_{2,1}/T_e)\bar{n}_1$ is rather small because of fast decay processes. To increase it substantially we propose to use the MW excitation of the third subband ($l = 3$). According to quantum optics, such an excitation leads to a population inversion ($\bar{n}_2 > \bar{n}_1$) when the decay rate $\nu_{3\to2}$ is much larger than $\nu_{2\to1}$. For a 2D system located near an interface, such a population inversion is seemingly impossible because electrons populating a higher subband has weaker interaction with the interface. The situation changes drastically in the presence of the magnetic field.

Since the electron density of states is squeezed into LLs slightly broadened by scatterers, the elastic decay of excited subbands is strongly suppressed, except vicinities of level-matching points $B_m^{(l,l')}$ where the ratio $\omega_{l,l'}/\omega_c$ is close to an integer m. Generally, $B_m^{(3,2)}$, $B_m^{(2,1)}$, and $B_m^{(3,1)}$ are different, and can be varied continuously by the holding electric field $E_\perp$ oriented normally to the interface. Therefore, important decay processes can be strongly suppressed or enhanced by B and $E_\perp$, giving rise to the population inversion.

In this work we consider theoretically the MW excitation of the third surface subband ($\omega = \omega_{3,1}$) for two extreme relationships between different $B_m^{(l,l')}$ and investigate the influence of the corresponding increase in population of excited subbands on photoconductivity oscillations. The theory introduces two new important frequencies for inter-subband scattering transitions ($\omega_{3,2}$ and $\omega_{2,1}$) which differ strongly from the MW frequency, and there is a sigh-changing correction to σ_{xx} which originates from the decay of the subband which is not excited directly by the MW. We found that, in addition to basic ω/ω_c-periodic oscillations, a variety of new oscillations with different periods can appear in the multi-subband system.

The basic idea of this work is applicable to a quite arbitrary multi-subband 2D electron system provided it is clean enough. Actual evaluations are performed for the electron system formed on a liquid helium surface where the collision broadening of LLs (Γ) is much smaller than temperature.

At low T, surface electrons are predominantly scattered by ripplons representing a sort of 2D phonons whose spectrum $\omega_q = \sqrt{\alpha/\rho}q^{3/2}$ (here α and ρ are the surface tension and mass density of liquid helium, respectively). For one-ripplon scattering, $\hbar\omega_q$ is much smaller than the typical broadening of LLs and T. To describe decay rates of excited subbands $\nu_{l\to l'}$ and the momentum relaxation rate ν_{eff} we shall transform scattering probabilities of the lowest order to the form containing level densities of the initial and final states, which is equivalent to the SCBA theory [13]. Then ν_{eff} is found using an extension of the momentum-balance method described in Refs. [14].

For example, the average probability of scattering from l to l' which is accompanied by the momentum exchange $\hbar\mathbf{q}$ caused by ripplon destruction and creation processes can be found as

$$\bar{\nu}_{l,l'}(\mathbf{q}) = 2u_{l,l'}^2 S_{l,l'}(q, \omega_{l,l'} + q_y V_H), \qquad (2)$$

where $V_H = cE_\parallel/B$ is the Hall velocity,

$$u_{l,l'}^2 = \frac{T}{2\alpha\hbar^2 q^2}\left|(U_q)_{l,l'}\right|^2,$$

and $U_q(z)$ is the electron-ripplon coupling [14]. At $l = l'$, the function $S_{l,l'}(q,\Omega)$ coincides with the dynamic structure factor (DSF) of a nondegenerate 2D electron gas. The $S_{l,l'}(q,\Omega)$ generalizes this correlation function for the multi-subband system by taking into account LL densities of different subbands [12]. The above given equation for scattering probabilities resembles the scattering cross-section of thermal neutrons and X-rays in solids. Here ripplons play the role of a particle flux whereas the electron layer represents a target.

Using (2), one can obtain main relaxation rates. For example, $\nu_{l\to l'}$ and ν_{eff} can be found as

$$\nu_{l\to l'} = \sum_{\mathbf{q}} \bar{\nu}_{l,l'}(\mathbf{q})$$

and

$$\nu_{\text{eff}} = \frac{1}{m_e V_H}\sum_{l,l'}\bar{n}_l \sum_{\mathbf{q}} \hbar q_y \bar{\nu}_{l,l'}(\mathbf{q}), \qquad (3)$$

respectively. To obtain (3) we have calculated the total momentum absorbed by scatterers and represented the frictional force, acting on the whole electron system, as $\mathbf{F}_{\text{fric}} \simeq -N_e m_e \nu_{\text{eff}}\langle\mathbf{v}\rangle$, where $\langle\mathbf{v}\rangle$ is the average velocity. We have assumed also that $\langle v_y\rangle \simeq -V_H$ ($\omega_c \gg \nu_{\text{eff}}$) and neglected edge effects. The conductivity tensor, which is found by balancing $\mathbf{F}_{\text{fric}}$ and the average force of external fields $\langle\mathbf{F}_{\text{ext}}\rangle$, has the Drude form with ν_{eff} standing instead of the classical collision frequency.

For the Gaussian shape of LL densities [15], the $S_{l,l'}(q,\Omega)$ has sharp maxima near Landau excitation energies [12]:

$$S_{l,l'}(q,\Omega) = \frac{2\sqrt{\pi}\hbar}{Z_\parallel}\sum_{n,n'}\frac{J_{n,n'}^2}{\Gamma_{l,l'}}\exp\left[-\frac{\varepsilon_n}{T_e} - D_{m;l,l'}(\Omega)\right], \qquad (4)$$

where $m = n' - n$,

$$J^2_{n,n'}(q) = \left| \left(e^{i\mathbf{q}\cdot\mathbf{r}_e} \right)_{l,X;l',X'} \right|^2,$$

$$D_{m;l,l'}(\Omega) = \frac{\hbar^2 \left(\Omega - m\omega_c - \Gamma_l^2/4\hbar T_e \right)^2}{\Gamma^2_{l,l'}} - \frac{\Gamma_l^2}{8T_e^2}, \quad (5)$$

$Z_{\parallel}$ is the partition function for the Landau spectrum ε_n, and $2\Gamma^2_{l,l'} = \Gamma_l^2 + \Gamma_{l'}^2$. To shorten equations, here we disregard the dependence of the collision broadening of LLs Γ_l on n, though in our numerical evaluations this dependence is taken into account. Generally, surface electrons represent a highly-correlated system, and the Coulomb interaction introduces an additional broadening of the maxima of the DSF [14, 16].

Considering decay rates of excited subbands, one can neglect $q_y V_H$ in (2). Thus, each $\nu_{l\to l'}$ as a function of B has sharp maxima at the vicinity of the level-matching points $B_m^{(l,l')}$, as shown in Fig. 1 for $T_e = T = 0.2$ K,

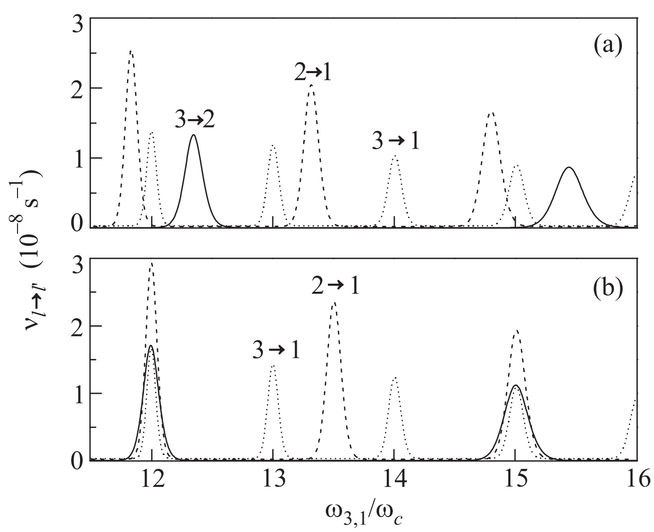

Fig. 1. Decay rates of excited subbands $\nu_{3\to1}$, $\nu_{3\to2}$ and $\nu_{2\to1}$ *vs* $\omega_{3,1}/\omega_c\,(B)$ for the average electron density $n_{\rm av} = 3\cdot10^6\,{\rm cm}^{-2}$, and two different holding fields: $E_\perp \simeq 81.3$ V/cm (a) and $E_\perp \simeq 94.35$ V/cm (b)

and two different holding fields. For the first holding field (Fig. 1a), the peaks of $\nu_{3\to2}$ are located quite distant from peaks of $\nu_{2\to1}$ and $\nu_{3\to1}$. Such a situation is checked to be valid up to $m = 23$. In regions of $\nu_{3\to2}$ peaks the all other decay rates are exponentially small, which causes the population inversion.

Since $\omega_{3,2}$ increases with $E_\perp$ faster than $\omega_{3,1}/3$, a remarkable match can be realized: $\omega_{3,1} \simeq 3\omega_{3,2}$, $\omega_{2,1} \simeq \simeq 2\omega_{3,2}$. Using the variational method, we found that this perfect match occurs at $E_\perp \simeq 94.35$ V/cm. In this case, the position of each peak of $\nu_{3\to2}$ coincides with positions of two other peaks ($\nu_{3\to1}$ and $\nu_{2\to1}$), as shown in Fig. 1b. This remarkable match is preserved at least up to $m = 24$.

Consider $\omega = \omega_{3,1}$. To obtain subband occupancies we use rate equations of dynamic equilibrium

$$r\left(\bar{n}_3 - \bar{n}_1\right) + \nu_{2\to1}\bar{n}_2 + \nu_{3\to1}\bar{n}_3 = 0$$

and

$$-\nu_{2\to1}\bar{n}_2 + \nu_{3\to2}\bar{n}_3 = 0$$

neglecting thermal excitation of surface subbands. Here r is the rate for stimulated absorption and emission which is balanced by decay processes due to the electron-ripplon interaction. Though the solution of the rate equations is found in an analytical form, the complicated dependence of $\nu_{l\to l'}(B)$ requires numerical calculations.

In the regions of B, where one-ripplon decay processes are exponentially small, we took into account two-ripplon decay processes according to Ref. [14]. For a typical r realized in experiments, the dependence of $\bar{n}_l$ on ω/ω_c is illustrated in Figs. 2 and 3.

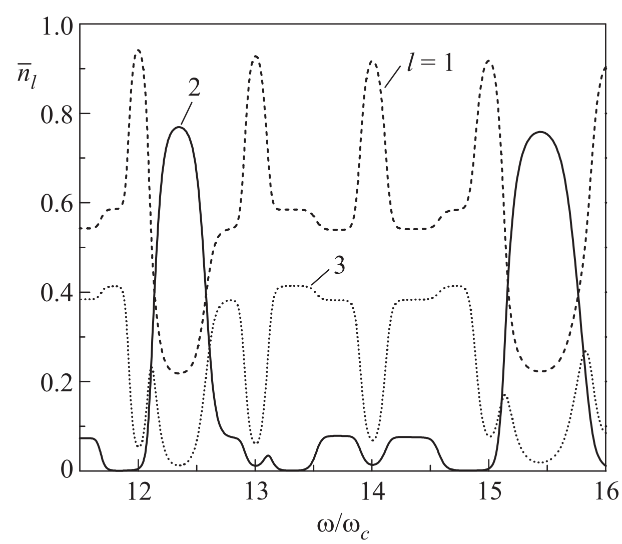

Fig. 2. Subband occupancies $\bar{n}_1$, $\bar{n}_2$, and $\bar{n}_3$ *vs* $\omega/\omega_c\,(B)$ for $\omega = \omega_{3,1}$. Other conditions are the same as in Fig. 1a

One can see that at $E_\perp \simeq 81.3$ V/cm the population inversion ($\bar{n}_2 > \bar{n}_1$) occurs when $12.13 < \omega/\omega_c < 12.57$ and $15.17 < \omega/\omega_c < 15.76$. Under the perfect-match conditions of Fig. 1b the population inversion is small, as follows from Fig. 3. Still, there are sharp maxima of $\bar{n}_2$ (up to 0.46) in the regions of tails of $\nu_{3\to2}$, which should affect magnetoconductivity oscillations.

To obtain magnetoconductivity σ_{xx} under MW irradiation we use (3)–(5). Without the resonant excitation ($\bar{n}_1 \simeq 1$) in the limit $\hbar q_y V_H \ll T_e$, these equations reproduce the result of the conventional SCBA

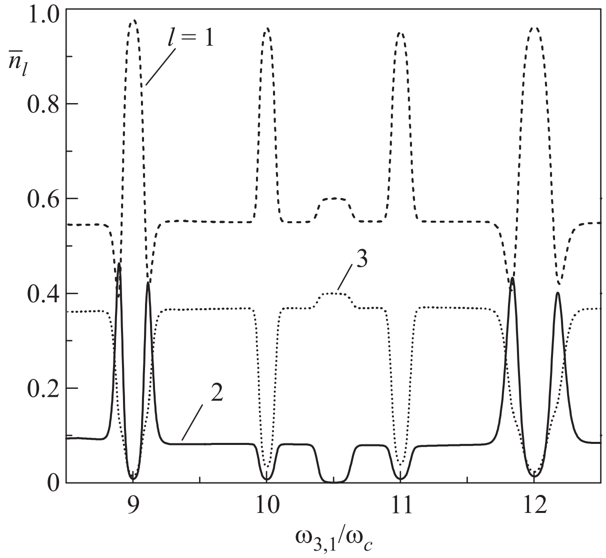

Fig. 3. Subband occupancies $\bar{n}_1$, $\bar{n}_2$, and $\bar{n}_3$ *vs* $\omega/\omega_c\,(B)$ for $\omega = \omega_{3,1}$. Other conditions are the same as in Fig. 1b

theory [13, 15]. The important utility of (3) is that it allows to calculate the momentum relaxation rate caused by inter-subband scattering for arbitrary subband occupancies.

The $S_{l,l'}\,(q,\Omega)$ has an important property

$$S_{l,l'}\,(q,-\Omega) = \exp\left(-\frac{\hbar\Omega}{T_e}\right) S_{l',l}\,(q,\Omega)$$

which is equivalent to $\nu_{l'\to l} = \exp(-\Delta_{l,l'}/T_e)\nu_{l\to l'}$. Using this property, in (3) the terms having the DSF with a negative frequency argument can be transformed to terms having the DSF with a positive frequency argument. This procedure involves also renaming the summation indexes $l \to l'$ and changing the sign of $\mathbf{q}$. Thus, we have

$$\nu_{\mathrm{inter}} = \sum_{(l>l',\mathbf{q})} \frac{\hbar q_y}{m_e V_H} \left[\bar{n}_l - \bar{n}_{l'} e^{-\hbar(\omega_{l,l'}+q_y V_H)/T_e}\right] \bar{\nu}_{l,l'}. \tag{6}$$

This form allows to explore directly the influence of a nonequilibrium distribution $\bar{n}_l$ on the momentum relaxation, and to see the origin of the sign-changing terms. It is notable that the formal replacement of $\sum_{l>l'}$ by $\frac{1}{2}\sum_{l=l'}$ in (6) gives the expression which coincides with the contribution of intra-subband scattering ν_{intra} into the momentum relaxation rate ($\nu_{\mathrm{eff}} = \nu_{\mathrm{intra}} + \nu_{\mathrm{inter}}$).

Without MW irradiation, $\bar{n}_l = \bar{n}_{l'} e^{-\hbar\omega_{l,l'}/T_e}$, it is sufficient to keep the linear expansion term originated from $\exp(-\hbar q_y V_H/T_e)$ in the square brackets, and to set $V_H = 0$ in $\bar{\nu}_{l,l'}\,(\mathbf{q}, V_H)$ defined by (2). In this case, ν_{eff} depends directly on the equilibrium form of $S_{l,l'}\,(q,\omega_{l,l'})$. At $T_e \ll \hbar\omega_{l,l'}$, such a contribution to ν_{inter} is exponentially small, and we shall neglect it.

If $\bar{n}_l > \bar{n}_{l'} e^{-\hbar\omega_{l,l'}/T_e}$, then we have to consider the linear expansion term of $\bar{\nu}_{l,l'}\,(\mathbf{q}, V_H)$ which brings the derivative $S'_{l,l'}\,(q,\Omega) = \partial S_{l,l'}\,(q,\Omega)/\partial\Omega$. In this case the expansion parameter $\hbar q_y V_H/\Gamma_l$ is much larger than the parameter $\hbar q_y V_H/T_e$ because LLs are extremely narrow ($\Gamma_l \ll T$). Therefore, even a small difference $\bar{n}_l - \bar{n}_{l'} e^{-\hbar\omega_{l,l'}/T_e}$ could lead to a strong sign-changing momentum relaxation

$$\frac{2}{m_e} \sum_{(l>l',\mathbf{q})} \left[\bar{n}_l - \bar{n}_{l'} e^{-\hbar\omega_{l,l'}/T_e}\right] \hbar q_y^2 u_{l,l'}^2 S'_{l,l'}\,(q,\omega_{l,l'})\,. \tag{7}$$

From this equation and Eqs. (4) and (5) one can see that ν_{inter} is approximately proportional to $\omega_{l,l'}/\omega_c - m$, as expected from the qualitative analysis given in the beginning of this work.

The results of numerical evaluations based on (7) and on the corresponding equation for ν_{intra} are shown in Figs. 4 and 5 for two extreme cases illustrated above

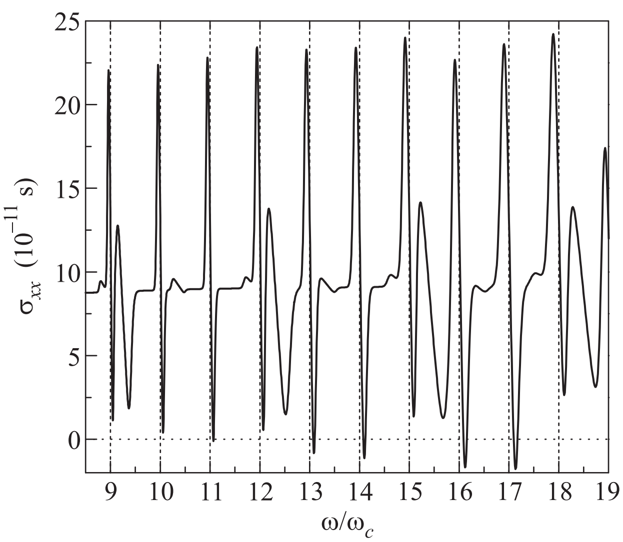

Fig. 4. Magnetoconductivity σ_{xx} *vs* $\omega/\omega_c\,(B)$ for $\omega = \omega_{3,1}$. Conditions are the same as in Fig. 1a

by the two graphs of Fig. 1. In the first case (Fig. 4), corresponding to $E_\perp = 81.3$ V/cm, the main oscillations are located in the vicinity of $\omega/\omega_c = m$. They have a specific shape representing a derivative of a sharp peak. These oscillations originate from ν_{inter} of (7).

There are also additional minima and maxima placed quite distant from the level-matching points between 6 and 7, 9 and 10, 12, and 13, and so on. Though there is a global periodicity in numbers m restricting regions where these new extremes occur, their positions shift slowly to the right if m increases. These σ_{xx} ex-

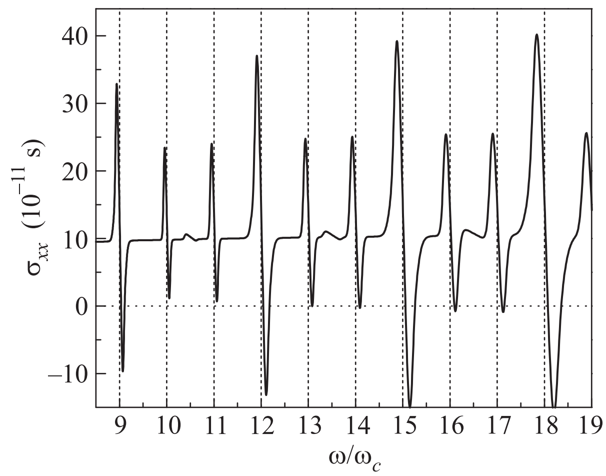

Fig. 5. Magnetoconductivity σ_{xx} *vs* $\omega/\omega_c\,(B)$ for $\omega = \omega_{3,1}$. Conditions are the same as in Fig. 1b

tremes accompany the population inversion discussed above. They originate from minima of intra-subband scattering and from sign-changing variations of $\nu_{\rm inter}$.

Under the condition $E_\perp \simeq 94.35$ V/cm where the perfect level match occurs (see Fig. 1b), the results of magnetoconductivity calculations are shown in Fig. 5. Here the main magneto-oscillations have a periodic amplitude modulation: the amplitude is strongly enhanced at the conditions of the perfect match ($m = 6, 9, 12, 15, ...$). This enhancement is caused by the simultaneous action of the decay processes $3 \to 2$ and $2 \to 1$ and by the strong periodic increase of $\bar{n}_2$ in the vicinity of the perfect level-matching points shown in Fig. 3. Obviously, in these regions the negative conductivity effects are also strongly enhanced. There are also small fractional oscillations whose variations occur in the vicinity of $7{+}1/2$, $10{+}1/2$, $13{+}1/2$, and so on. They are caused by the inter-subband scattering from $l = 2$ to $l = 1$ and described by the corresponding terms of $\nu_{\rm inter}$. In these regions of the magnetic field, $\bar{n}_2 \ll 1$. Nevertheless, the amplification of (7) caused by $T/\Gamma_l \gg 1$ makes oscillations at fractional values of ω/ω_c quite distinct among basic magneto-oscillations.

In summary, we have shown that the *dc* magnetic and electric fields oriented normally to the multi-subband 2D electron system give a remarkable possibility to manipulate the inter-subband scattering probabilities and to realize the population inversion of electron subbands. The analysis of the electron momentum relaxation rate under the resonant MW excitation of the third subband given here indicates that such an excitation induces a variety of new magneto-oscillations of σ_{xx}. Among these, there are oscillations accompanying the population inversion with a period which is incommensurate with the basic period determined by the resonant MW frequency, oscillations with a $1/B$-periodic amplitude modulation, and oscillations located in the vicinity of some fractional values of the ratio of the resonant frequency to the cyclotron frequency.

In experiments on surface electrons [8, 9] and in the theoretical analysis [12] the period of magneto-oscillations is governed by the resonant MW frequency and it actually coincides with that reported for semiconductor systems. Therefore, an experimental observation of new magneto-oscillations with a different period (not related directly to the MW frequency) described here would be crucial for the identification of the origin of magneto-oscillations and zero resistance states in the multi-subband 2D electron system formed on the surface of liquid helium.

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