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The neutral ρ meson in a strong magnetic field in SU(2) lattice gauge theory

E. V. Luschevskaya, O. V. Larina

Institute of Theoretical and Experimental Physics, 117218 Moscow, Russia

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The correlators of vector and pseudoscalar currents have been calculated in the external strong magnetic field in SU(2) gluodynamics on the lattice. The masses of the neutral ρ meson with different spin projections $s = 0, \pm 1$ to the axis parallel to the external magnetic field B were calculated. The ρ meson mass with zero spin $s = 0$ decreases with the growth of the magnetic field and the ρ meson masses with $s = \pm 1$ increase with the magnetic field.

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1. Introduction. Magnetic fields of the order of ~ 2 GeV existed in the early Universe during the electroweak transition [1]. The values of the magnetic fields in the non-central heavy-ion collisions can reach the value $15m_\pi^2 \sim 290\,\mathrm{MeV}^2$ [2].

STAR collaboration has detected the chiral magnetic effect at RHIC in the non-central collisions of gold ions [3–6]. Later this effect was also observed in the experiment ALICE at LHC [7]. The strong magnetic field also results to the modification of the phase diagram of QCD. Phenomenological models show that the critical temperature of the transition between the phases of confinement and deconfinement varies with increasing of the external magnetic field B, and the phase transition becomes of the first order [8].

The growth of the phase transition temperature T_c was predicted by the models of Nambu–Jona–Lasinio type: NJL, EPNJL, PNJL [9], and PNJL$_8$ [10], the Gross–Neveu model [11, 12], as well as the first calculations on the lattice QCD with two quarks [13]. However, the lattice calculations in QCD with $N_f = 2+1$ revealed that T_c decreases with increasing of B value [14]. The chiral perturbation theory gives the decrease of T_c with the growth of field value [15].

It has been shown in the framework of the Nambu–Jona–Lasinio model that in the presence of sufficiently strong magnetic fields ($B_c = m_\rho^2/e \simeq 10^{16}$ T) QCD vacuum becomes a superconductor [16] along the direction of the magnetic field. The transition to the superconducting phase is accompanied by a condensation of the charged ρ mesons. Calculations on the lattice [17] also indicate the existence of the superconducting phase. We have investigated the behavior of the masses of the neutral ρ with different spin projection $s = 0$ and ± 1 to the direction of the magnetic field. Quark propagators were calculated with the chiral invariant fermionic operator. In [18] the mass of neutral vector ρ meson was calculated in the relativistic quark-antiquark model, the mass of neutral ρ meson with zero spin does not vanishes with the growth of the magnetic field in the confinement phase in contradiction with the results of [16].

2. Details of the calculations. The improved Symanzik action has been used for the generation of SU(2) gauge field configurations similarly to our previous work [19]. The calculations were performed on symmetric lattices with different lattice volumes 14^4, 16^4, 18^4 and lattice spacings $a = 0.0681$, 0.0998, and 0.138348 fm.

Fermionic spectrum in the background of SU(2) gauge fields were calculted using a chiral-invariant overlap operator, proposed by Neuberger [20]. This operator allows to explore the theory without chiral symmetry breaking.

In a continuous space the analogue of this operator is the Dirac operator $D = \gamma^\mu(\partial_\mu - iA_\mu)$, the corresponding Dirac equation is

$$D\psi_k = i\lambda_k\psi_k. \tag{1}$$

The Neuberger overlap operator allows to calculate the eigenfunctions ψ_k and the eigenvalues λ_k for a test quark in an external gauge field configurations A_μ. A_μ is a sum of SU(2) gauge fields and the external abelian uniform magnetic field. Eigenfunctions of the Dirac operator allow to construct operators and correlators.

Abelian fields interact with quarks, so for the introduction of the external magnetic field it's necessary to perform the following substitution

$$A_{\mu\,ij} \to A_{\mu\,ij} + A_\mu^B \delta_{ij}, \tag{2}$$

$$A_\mu^B(x) = \frac{B}{2}(x_1\delta_{\mu,2} - x_2\delta_{\mu,1}). \tag{3}$$

To match this change with the lattice boundary conditions the twisted boundary conditions for fermions have been used as described in [21].

The value of magnetic field on the lattice is quantized

$$qB = \frac{2\pi k}{(aL)^2}, \quad k \in \mathbb{Z}, \tag{4}$$

where $q = -1/3\, e$ is the charge of d-quark, there is one type of fermions in the theory, a is the lattice spacing in physical units. The quantization condition imposes the limit on the minimum value of the magnetic field. For our calculations it equals to $0.386\,\mathrm{GeV}^2$ for lattice volume 16^4 and lattice spacing 0.1383 fm.

For each value of the quark mass in the interval $m_q a = 0.01{-}0.8$ statistical independent configurations of the gluon field have been used.

3. Calculation of the observables. The following observables were calculated

$$\langle \psi^\dagger(x) O_1 \psi(x) \psi^\dagger(y) O_2 \psi(y)\rangle_A, \tag{5}$$

where $O_1, O_2 = \gamma_5, \gamma_{\mu,\nu}$ are Dirac gamma matrices, $\mu, \nu = 1, \ldots, 4$. In the Euclidean space $\psi^\dagger = \bar\psi$ [22]. The correlators (5) are defined by the Dirac propagators, for their calculation the inverse matrix for the massive Dirac operator $1/(D+m)$ should be found. For M lowest eigenstates Dirac operator it is represented by the sum

$$\frac{1}{D+m}(x,y) = \sum_{k<M} \frac{\psi_k(x)\psi_k^\dagger(y)}{i\lambda_k + m}. \tag{6}$$

In this work $M = 50$ was used. On the lattice the observables (5) have the form

$$\langle \bar\psi O_1 \psi \bar\psi O_2 \psi\rangle_A = \tag{7}$$

$$= \sum_{k,p<M} \frac{\langle k|O_1|k\rangle\langle p|O_2|p\rangle - \langle p|O_1|k\rangle\langle k|O_2|p\rangle}{(i\lambda_k + m)(i\lambda_p + m)}$$

The first term in the numerator represents a disconnected part, and the second one with a minus sign – a connected part. The first term is less than the second one, has large statistical errors, does not affect the result, and so for further calculations only the connected part of the correlator was used.

The mass of a neutral ρ meson was extracted from the correlator of vector currents $\langle j_\mu^V(x) j_\nu^V(y)\rangle_A$, where $j_\mu^V(x) = \psi^\dagger(x)\gamma_\mu\psi(x)$. The mass with a zero spin projection was calculated, which corresponds to the correlator of vector currents along the direction of the magnetic field. In the expression (7) it corresponds to the choice of $O_1, O_2 = \gamma_3$. The correlator $\langle j^{PS}(x) j^{PS}(y)\rangle_A$ gives the mass of π meson, where $j^{PS} = \psi^\dagger(x)\gamma_5\psi(x)$ is the pseudoscalar current.

For the calculation of meson masses we used the method, based on the spectral expansion of the lattice correlation function

$$C(n_t) = \langle \psi^\dagger(\mathbf{0}, n_t) O_1 \psi(\mathbf{0}, n_t) \psi^\dagger(\mathbf{0}, 0) O_2 \psi(\mathbf{0}, 0)\rangle_A =$$

$$= \sum_k \langle 0|O_1|k\rangle\langle k|O_2^\dagger|0\rangle e^{-n_t a E_k}, \tag{8}$$

$$C(n_t) = A_0 e^{-n_t a E_0} + A_1 e^{-n_t a E_1} + \ldots, \tag{9}$$

where A is some constant value, E_0 is the energy of the lowest state, for the particle with average zero momentum $\mathbf{p} = 0$ its energy coincides with its mass $E_0 = m_0$, E_1 is the energy of the first excited state, a is the lattice spacing, n_t is the time coordinate on the lattice. From the expansion (9) one can see that for large values n_t the main contribution comes from the ground energy state.

Due to periodic boundary conditions the contribution of the ground state into the propagator of a meson has the form

$$f(n_t) = A_0 e^{-n_t a E_0} + A_0 e^{-(N_T - n_t) a E_0} =$$

$$= 2A_0 e^{-N_T E_0/2} \cosh[(N_T - n_t) a E_0]. \tag{10}$$

The mass value of the ground state mass can be extracted, fitting the correlator by the function (10), n_t is the lattice site number in the time direction.

4. Results. At first we calculate the mass of neutral π meson on the lattice from the correlators of the pseudoscalar currents $C^{PSPS}(n_t) = \langle j^{PS}(\mathbf{0}, n_t) j^{PS}(\mathbf{0}, 0)\rangle_A$, where $j^{PS}(\mathbf{0}, n_t) = \bar\psi(\mathbf{0}, n_t)\gamma_5\psi(\mathbf{0}, n_t)$. The π mass is shown on the Fig. 1 for the different lattice volumes and spacings, the masses after lattice quark renormalization are also represented.

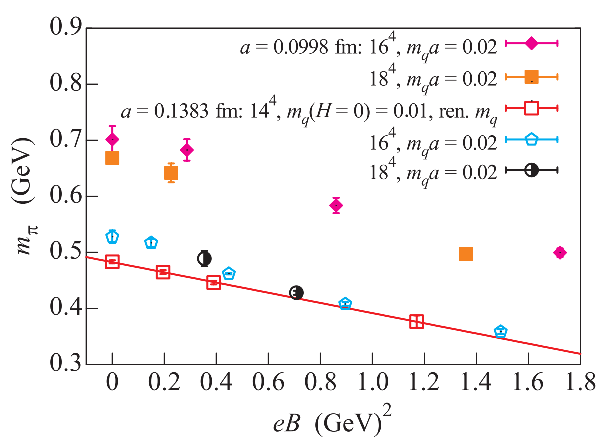

Fig. 1. The mass of the neutral π meson extracted from the $C^{PSPS}(n_t)$ versus the squared value of the magnetic field for the renormalized and nonrenormalized quark mass

The masses of the vector meson were calculated for various magnetic field values. On the lattice the quark masses is renormalized by some quantity $\delta m_{\text{lat}}^{\text{ren}}$, which depends on the lattice parameters – volume and spacing and magnetic field value. The magnitude of this renormalization is easy to estimate, since the squared mass of π meson is a linear function of the quark mass in accordance with the chiral perturbation theory.

To take into account the renormalization of the quark mass we calculate the π meson mass for various values of m_q (the bare quark mass which enter into lattice lagrangian). By extrapolation procedure to small values of m_q we fix the value of the bare quark mass m_{q_0} corresponding to the physical value of the π meson mass at zero magnetic field (135 MeV). We calculate the masses of ρ for several values of m_q in the interval $m_q = 0.01-0.8$, perform the fits and find the coefficients a_i and b_i

$$m_\rho(s=0) = a_0 + a_1 m_q, \tag{11}$$

$$m_A(s=0) = b_0 + b_1 m_q. \tag{12}$$

Then we extrapolate $m_\rho(m_q)$ to the physical values $m_\rho(m_{q_0})$ at $m_q = m_{q_0}$ using the equations (11) and (12).

The components of the correlators of vector currents were calculated, the diagonal components are essentially nonzero, while the nondiagonal ones are zero within the error bars. The external magnetic field is directed along the third coordinate axes. The correlators of vector currents perpendicular to the magnetic field are $C_{11}^{VV}(n_t) = \langle j_1^V(\mathbf{0}, n_t) j_1^V(\mathbf{0}, 0)\rangle_A$ and $C_{22}^{VV}(n_t) = \langle j_2^V(\mathbf{0}, n_t) j_2^V(\mathbf{0}, 0)\rangle_A$, where $j_1^V(\mathbf{0}, n_t) = \bar{\psi}(\mathbf{0}, 0)\gamma_1\psi(\mathbf{0}, n_t)$ and so on. The masses of the neutral mesons with zero spin projection to the magnetic field are extracted from the correlator $C_{33}^{VV}(n_t) = \langle j_3^V(\mathbf{0}, n_t) j_3^V(\mathbf{0}, 0)\rangle_A$. The masses with spin $s = \pm 1$ are found from $C^{VV}(s=1) = (C_{11}^{VV} + iC_{22}^{VV})/\sqrt{2}$ and $C^{VV}(s=-1) = -(C_{11}^{VV} - iC_{22}^{VV})/\sqrt{2}$.

At Fig. 2 the mass of the neutral ρ meson with zero spin is shown versus the value of the magnetic field. For the all lattice volumes 16^4, 18^4 and spacings $a = 0.0998$, 0.11558 fm the mass decreases with the magnetic field. The points are connected by splines to guide the eyes.

Fig. 3 shows the mass of the ρ meson mass with nonzero spin versus the field value. The masses with spin $s = \pm 1$ projections to the magnetic field direction increase with the field.

Unfortunately on the lattice in the presence of the magnetic field the quantum numbers of mesons are not precise. The mixing takes place because of the interaction between photons and the vector quark currents and can occur between neutral pion and the state of ρ meson with zero spin. No severe methods occurs to disentangle these two states in the magnetic field. However we have indications that the masses of vector meson with $s = \pm 1$ definitely increse in our SU(2) theory. The investigations of the mass behavior in QCD with dynamical quarks present the huge interest.

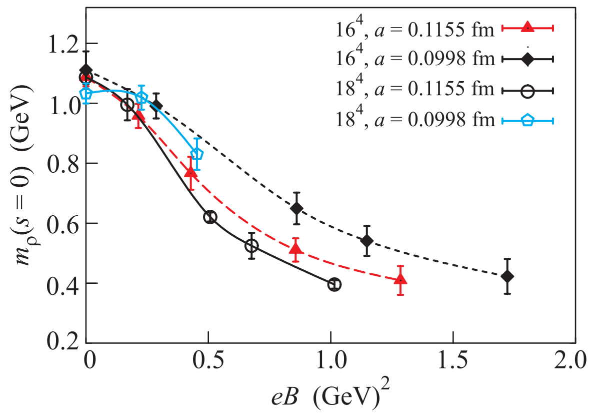

Fig. 2. The mass of the neutral vector ρ meson with zero spin projection $s = 0$ versus the value of the external magnetic field for the lattice volumes 16^4, 18^4 and lattice spacing $a = 0.0998$, 0.1155 fm

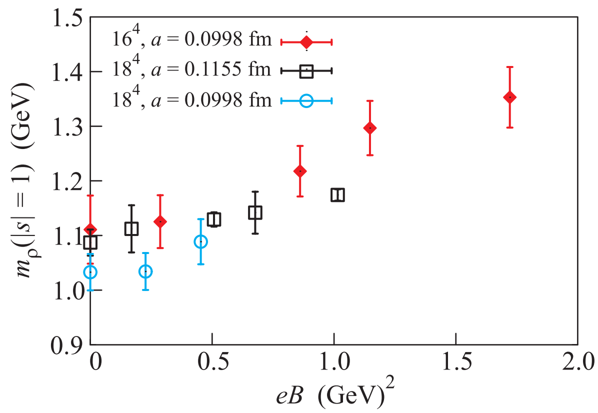

Fig. 3. The mass of the neutral vector ρ meson with nonzero spin projection $s = \pm 1$ versus the value of the external magnetic field for the lattice volumes 16^4, 18^4 and the lattice spacing $a = 0.0998$, 0.1155 fm

5. Conclusions. In this work we explore the masses of the neutral π and ρ mesons in the background of the strong magnetic field of the hadronic scale in the confinement phase. We observe that the masses with zero spin projection to the magnetic field differ from the masses with spin projection $s = \pm 1$. The masses with $s = 0$ decrease with the magnetic field, but the masses with $s = \pm 1$ increase with the field. We consider this phenomena to be the result of the anisotropy, which the strong magnetic field creates. We do not observe any condensation of neutral mesons, so there are no evidences of superfluidity in the confinement phase. However the presence of superconducting phase at high values of the magnetic field B [23] in QCD is a hot topic for discussions. The condensation of charged ρ mesons would be an evidence of the existence of the superconductivity in QCD.

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