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Magnetic moments of negative parity baryons from effective hamiltonian approach to QCD

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Magnetic moments of $S_{11}(1535)$ and $S_{11}(1650)$ baryons are studied in the framework of the relativistic three-quark Hamiltonian derived in the Field Correlation Method. The baryon magnetic moments are expressed via the average current quark energies which are defined by the fundamental QCD parameters: the string tension σ, the quark masses, and the strong coupling constant α_s. Resulting magnetic moments for the $J^P = 1/2^-$ nucleons are compared both to model calculations and to those from lattice QCD.

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1. Introduction. The dipole magnetic moment encodes information about the leading-order response of a bound system to a soft external magnetic field. In particular, baryon magnetic moments are dynamical characteristics which provide valuable insights into baryon internal structure in terms of quark and gluon degrees of freedom. In this paper we shall explore the magnetic moments of negative parity resonances employing the QCD dynamics of a baryon in the form of the three-quark Effective Hamiltonian (EH). The EH is derived from the QCD path integral (see, e.g., [1]), and was already used in the studies of baryon spectra without external fields [2–5]. (The extension of the EH to the case of external magnetic field has been done recently in Ref. [6], where the nucleon spectrum as a function of magnetic field was calculated.) Within this method the magnetic moments of the $1/2^+$ octet baryons have been studied analytically in Ref. [7]. The model was shown to agree with experiment within 10% accuracy. The same accuracy was achieved for the baryon magnetic moments in Ref. [8], where the QCD string dynamics was investigated from another point of view.

Negative parity partners of the baryon octet arise from excitation of one unit of orbital angular momentum. Although the magnetic moments of the $1/2^+$ baryon octet are well-known both experimentally and theoretically, little is known about their $1/2^-$ counterparts. Experimentally, magnetic moments of these states can be extracted through bremsstrahlung processes in photo- and electro-production of mesons at intermediate energies. For N(1535) a similar process $\gamma p \to \gamma \eta p$ can be used [9], but to date no such measurements have been made.

There exist limited number of theoretical studies of the magnetic moments of negative parity baryons based on constituent quark model [9], unitarized chiral perturbation theory (UχPT) [10], chiral constituent quark model (χCQM) [11], Bethe–Salpeter approach [12], and on the lattice [13] where magnetic moments of the baryon resonances have been obtained from the mass shifts. Comparison study of magnetic moments for positive- and negative-parity states offers insight into underlying quark-gluon dynamics. Given that the mass spectrum of the $1/2^+$ and $1/2^-$ states has been reasonably well established from the EH, it is instructive to investigate the magnetic moments of these states. In this paper we extend the results of Ref. [7] to calculate the magnetic moments of the negative parity $S_{11}(1535)$ and $S_{11}(1650)$ resonances. The paper builds on the previous work presented in Ref. [4] where the EH contains the three quark string junction confined interaction and the Coulomb potential with the fixed strong coupling constant.

In Section 2 we briefly discuss the theoretical formalism of EH method for baryons, including the techniques required to extract the average quark energies ω_i which are cornerstones of the present calculation. As a result one obtains the resonance magnetic moments without introduction of any fitting parameters. Details of calculation of the magnetic moments for excited $1/2^-$ nucleons are given in Section 4. In this Section, we also report the magnetic moments of the $1/2^+$ and $3/2^+$ octet baryons. Section 5 contains the summary of the obtained results.

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2. Effective Hamiltonian for Baryons. The key ingredient of the EH method is the use of the auxiliary fields (AF) initially introduced in order to get rid of the square roots appearing in the relativistic Hamiltonian [14]. Using the AF formalism allows one to derive a simple local form of the EH for the three-quark system which comprises both confinement and relativistic effects, and contains only universal parameters: the string tension σ, the strong coupling constant α_s, and the bare (current) quark masses m_i. Neglecting the spin-dependent forces responsible for the fine and hyperfine splittings of baryon states the EH has the form

$$H = \sum_{i=1}^{3}\left(\frac{m_i^2}{2\,\omega_i} + \frac{\omega_i}{2}\right) + H_0 + V. \tag{1}$$

In Eq. (1) H_0 is the nonrelativistic kinetic energy operator for the constant AF ω_i, the spin-independent potential V is the sum of the string potential

$$V_Y(\mathbf{r}_1,\, \mathbf{r}_2,\, \mathbf{r}_3) \,=\, \sigma\, r_{\min}, \tag{2}$$

with $r_{\min}$ being the minimal string length corresponding to the Y-shaped configuration, and a Coulomb interaction term

$$V_{\mathrm{C}}(\mathbf{r}_1,\, \mathbf{r}_2,\, \mathbf{r}_3) \,=\, -\, C_F \sum_{i<j} \frac{\alpha_s}{r_{ij}}, \tag{3}$$

arising from the one-gluon exchange. In Eq. (3) $C_F = 2/3$ is the color factor. The constant α_s was treated either as a fixed parameter, $\alpha_s = 0.39$ [4] or as the running coupling constant with the freezing value $\sim$ 0.5 [5]. The results for ω_i coincide with the accuracy better than 1% (compare Tables 1 and 2 of Refs. [4] and [5], respectively). In what follows we use ω_i taken from Ref. [4].

3. The auxiliary field formalism. The EH depends explicitly on both bare quark masses m_i and the constants AF ω_i that finally acquire the meaning of the dynamical quark masses. These quantities with a good accuracy coincide the average kinetic energies of the current quarks $\left\langle\sqrt{\mathbf{p}_i + m_i^2}\right\rangle$ [4]. As the first step the eigenvalue problem is solved for each set of ω_i; then one has to minimize $\langle H\rangle$ with respect to ω_i. Although being formally simpler the EH is equivalent to the relativistic Hamiltonian up to elimination of AF.

The formalism allows for a very transparent interpretation of AF ω_i: starting from bare quark masses m_i, we arrive at the dynamical masses ω_i that appear due to the interaction and can be treated as the dynamical masses of constituent quarks. These have obvious quark model analogs, but are derived directly using the AF formalism. Due to confinement $\omega_i \sim \sqrt{\sigma} \sim 400$ MeV or higher, even for the massless current quarks.

The baryon mass is given by

$$M_B \,=\, M_0 \,+\, C \,+\, \Delta M_{\,\mathrm{string}}, \tag{4}$$

$$M_0 \,=\, \sum_{a=1}^{3}\left(\frac{m_a^2}{2\omega_a} + \frac{\omega_a}{2}\right) \,+\, E_0(\omega_a), \tag{5}$$

where $E_0(\omega_a)$ is an eigenvalue of the Schrödinger operator $H_0 + V$, and the ω_a are defined by minimization condition

$$\frac{\partial\, M_0(m_a, \omega_a)}{\partial\, \omega_a} \,=\, 0. \tag{6}$$

The right-hand side of Eq. (4) contains the perturbative quark self-energy correction C that is created by the color magnetic moment of a quark propagating through the vacuum background field [15]. This correction adds an overall negative constant to the hadron masses. Finally, $\Delta M_{\mathrm{string}}$ in Eq. (4) is the correction to the string junction three-quark potential in a baryon due to the proper moment of inertia of the QCD string [16]. We stress that both corrections, C and $\Delta M_{\mathrm{string}}$ are added perturbatively and do not influence the definition of ω_a.

The confinement Hamiltonian contains three parameters: the current quark masses m_n and m_s and the string tension σ. Let us underline that they are not the fitting parameters. In our calculations we used $\sigma = 0.15\,\mathrm{GeV}^2$ found in the SU(3) QCD lattice simulations [17]. We employed the current light quark masses $m_u = m_d = 9\,\mathrm{MeV}$ and the bare strange quark mass $m_s = 175\,\mathrm{MeV}$.

4. Magnetic moments of $\boldsymbol{S_{11}(1535)}$ and $\boldsymbol{S_{11}(1650)}$ resonances. To calculate the nucleon magnetic moment one introduces a vector potential $\mathbf{A}$ and calculate the energy shift ΔM_B due the Hamiltonian $\mathcal{H} = H(\mathbf{A}) + H_\sigma$ where H is defined by Eq. (1) with the substitution $\mathbf{p}_a \to \mathbf{p}_a - e_a\mathbf{A}_a$ and

$$H_\sigma = -\sum_a \frac{e_a\, \boldsymbol{\sigma}_a}{2\omega_a}\mathbf{B}, \tag{7}$$

where $\mathbf{B}$ is an external magnetic field. The magnetic moment operator consists of contributions from both intrinsic spins of the constituent quarks that make up the bound state $\boldsymbol{\mu}_S$ and angular momentum of the three-quark system $\boldsymbol{\mu}_L$ with the center of mass motion removed. Straightforward calculation using the London gauge $\mathbf{A} = \frac{1}{2}(\mathbf{B}\times\mathbf{r})$ yields

$$\hat{\boldsymbol{\mu}} = \hat{\boldsymbol{\mu}}_S + \hat{\boldsymbol{\mu}}_L. \tag{8}$$

Taking the constituent quarks to be Dirac point particles the spin contribution in Eq. (8) is determined by the effective quark masses ω_a

$$\boldsymbol{\mu}_S = \sum_a \frac{e_a \boldsymbol{\sigma}_a}{2\omega_a}. \tag{9}$$

The orbital contribution in Eq. (8) reads

$$\hat{\boldsymbol{\mu}}_L = \sum_a \frac{e_a}{2\omega_a} \mathbf{r}_a \times \mathbf{p}_a. \tag{10}$$

In what follows instead the usual prescription which is to symmetrize the nucleon wave function between all three quarks we symmetrize only between equal-charge (up or down) quarks. In other words for the proton we use the uud basis in which the d quark is singled out as quark 3 but in which the quarks uu are still antisymmetrized. In the same way, for the neutron we use the basis in which the u quark is singled out as quark 3. The uud basis state diagonalizes the confinement problem with eigenfunctions that correspond to separate excitations of the quark 3 ($\boldsymbol{\rho}$ and $\boldsymbol{\lambda}$ excitations, respectively). In particular, excitation of the $\boldsymbol{\lambda}$ variable unlike excitation in $\boldsymbol{\rho}$ involves the excitation of the "odd" quark (d for uud or u for ddu). The physical P-wave states are not pure ρ or λ excitations but linear combinations of all states with a given total momentum J. Most physical states are, however, close to pure $\boldsymbol{\rho}$ or $\boldsymbol{\lambda}$ states [18]. In terms of the Jacobi variables

$$\boldsymbol{\rho} = \frac{\mathbf{r}_1 - \mathbf{r}_2}{\sqrt{2}} \quad \boldsymbol{\lambda} = \frac{\mathbf{r}_1 + \mathbf{r}_2 - 2\mathbf{r}_3}{\sqrt{6}}. \tag{11}$$

Eq. (10), reads

$$\boldsymbol{\mu}_L = \frac{1}{2}(\mu_1 + \mu_2)\mathbf{l}_\rho + \frac{1}{6}(\mu_1 + \mu_2 + 4\mu_3)\mathbf{l}_\lambda + \\ + \frac{\mu_1 - \mu_2}{2\sqrt{3}}(\boldsymbol{\rho} \times \mathbf{p}_\lambda + \boldsymbol{\lambda} \times \mathbf{p}_\rho), \tag{12}$$

where

$$\mathbf{l}_\rho = \boldsymbol{\rho} \times \mathbf{p}_\rho, \quad \mathbf{l}_\lambda = \boldsymbol{\lambda} \times \mathbf{p}_\lambda \tag{13}$$

and where the quark magnetic moments μ_u, μ_d are expressed in terms of parameters ω_a

$$\mu_a = \frac{e_a}{2\omega_a}. \tag{14}$$

Recall that the quantities ω_a are defined from the eigenvalues $M(\omega_1, \omega_2, \omega_3)$ of the EH H using the stationary point Eqs. (6).

Note also that the magnetic moments in Eq. (14) are in quark natural magnetons. To convert it into nuclear magnetons μ_N, we need to scale the results by the factor $m_N = 0.94$ GeV.

The angular operators in (12) act on spacial wave functions $\psi_{1m}^{\rho,\lambda}$ as follows

$$\begin{aligned} l_{\rho z}\psi_{1m}^{\rho} &= m\psi_{1m}^{\rho}, \quad l_{\rho z}\psi_{1m}^{\lambda} = 0, \\ l_{\lambda z}\psi_{1m}^{\lambda} &= m\psi_{1m}^{\rho}, \quad l_{\lambda z}\psi_{1m}^{\rho} = 0, \\ (\boldsymbol{\rho} \times \mathbf{p}_\lambda)_z\psi_{1m}^{\lambda} &= m\psi_{1m}^{\rho}, \quad (\boldsymbol{\rho} \times \mathbf{p}_\lambda)_z\psi_{1m}^{\rho} = 0, \\ (\boldsymbol{\lambda} \times \mathbf{p}_\rho)_z\psi_{1m}^{\rho} &= m\psi_{1m}^{\lambda}, \quad (\boldsymbol{\lambda} \times \mathbf{p}_\rho)_z\psi_{1m}^{\lambda} = 0. \end{aligned} \tag{15}$$

The contribution of the last term in (12) vanishes for the pure $\boldsymbol{\rho}$- and $\boldsymbol{\lambda}$-excitations.

By definition, the *magnetic moment* μ of the baryon with the spin J is the expectation value of the operator $\hat{\mu}^z$ for the state with $M_z = J$

$$\mu = \langle \hat{\mu}^z \rangle = \langle JJ | \hat{\mu}_S^z + \hat{\mu}_L^z | JJ \rangle. \tag{16}$$

In particular, for baryons with the total orbital momentum $\mathbf{L} = 0$ where $\mathbf{L}$ – the angular momentum of the three-quark system with the correct center of mass motion removed

$$\begin{aligned} \mu = \mu_{\rm spin}^{1/2} &= \left\langle \frac{1}{2}^+ \frac{1}{2} \Big| \sum_a \frac{e_a \sigma_{az}}{2\omega_i} \Big| \frac{1}{2}^+ \frac{1}{2} \right\rangle = \\ &= \left\langle \chi_{1/2\,1/2}^{\lambda}(12;3) \Big| \sum \frac{e_a}{2\omega_a} \Big| \chi_{1/2\,1/2}^{\lambda}(12;3) \right\rangle = \\ &= \frac{1}{3}(2\mu_1 + 2\mu_2) - \frac{1}{3}\mu_3, \end{aligned} \tag{17}$$

where $\chi_{1/2\,1/2}^{\lambda}$ is the doublet spin function symmetric under interchange $1 \leftrightarrows 2$. Eq. (17) is standard result of the additive quark model for the $1/2^+$ baryons [19].

Table 1

The values of ω_a for the $L = 0, 1$ baryons

Baryon	L	Excitation	ω_1	ω_3
nnn	0		408	408
	1	ρ, λ	457	457
nns	0		414	453
	1	ρ	482	459
	1	λ	441	534
ssn	0		458	419
	1	ρ	520	424
	1	λ	483	506

In Table 1 we show the einbein parameters ω_q and ω_s calculated for the different $L = 0$ baryons in Ref. [4] using the constant value of $\alpha_s = 0.39$. The symbol "q" denotes the light quarks u or d. We use the notation $\omega_1 = \omega_2 = \omega_q$, $\omega_3 = \omega_s$ for the qqq and qqs baryons and $\omega_1 = \omega_2 = \omega_s$, $\omega_3 = \omega_q$ for the ssq

Table 2

Magnetic moments of $J^P = (1/2)^+$ baryons. Quark masses are from [4]

Baryon	ω_q	ω_s	μ_u	μ_d	μ_s	μ	Expt.
p	0.408		1.53	−0.77		2.29	2.79
n	0.408		1.53	−0.77		−1.53	−1.91
Λ	0.414	0.453	1.51	−0.76	−0.69	−0.69	−0.61
Σ^+	0.414	0.453	1.51	−0.76	−0.69	2.23	2.46
Σ^0	0.414	0.453	1.51	−0.76	−0.69	0.80	0.83
Σ^-	0.414	0.453	1.51	−0.76	−0.69	-0.91	−1.16
Ξ^0	0.419	0.458	1.485	−0.742	−0.75	−1.40	−1.25
Ξ^-	0.419	0.458	1.689	−0.845	−0.75	−0.50	−0.65
Ω^-		0.463			−0.671	−2.01	−2.02

baryons. These parameters have been calculated for the string tension $\sigma = 0.15\,\mathrm{GeV}$ and the strong coupling constant $\alpha_s = 0.39$ with the values of the current light quark masses, $m_u = m_d = 9\,\mathrm{MeV}$, $m_s = 175\,\mathrm{MeV}$. The very similar values of ω_a have been calculated in Ref. [5] where instead the constant α_s the running coupling constant $\alpha_s(r)$ has been used with $\alpha_s(\infty) \sim 0.5$.

There is no good theoretical reason why ω_a need to be the same in different mesons and baryons. However from the results of Table 1 we conclude that einbeins of the light quarks are increased by $\sim 10\,\mathrm{MeV}$ when going from the nucleon to Ξ. This variation is marginal and is within the accuracy of calculations. For ground states of Λ and Σ hyperons we obtain $\omega_q = 0.411\,\mathrm{MeV}$, $\omega_s = 0.451\,\mathrm{MeV}$ that agrees with the corresponding values for the ground state of K meson [20].

The magnetic moments for the $1/2^+$ baryons with $L = 0$ are presented in Table 2. For the $3/2^+$ baryons one obtains $\mu_{\Delta^{++}} = 3\mu_u = 4.575\mu_N$, other moments are $\mu_{\Delta^+} = 2\mu_u + \mu_d = \frac{3}{2}\,\mu_{\Delta^{++}}$, $\mu_{\Delta^0} = 0$, $\mu_{\Delta^-} = 3\mu_d = -\frac{1}{2}\,\mu_{\Delta^{++}}$. Recall that so far, only the magnetic moment of $\Delta^{++}(1232)$ has been studied in the reaction $\pi^+ p \to \gamma\pi^+ p$ with the result $\mu_{\Delta^{++}} \sim (3.7\text{–}7.5)\,\mu_N$ [21]. The uncertainty in the number arises from the ambiguity in the theoretical analysis of the reaction.

The wave function of an $1/2^-$ resonance is given as a superposition of two spin ($S = 1/2$ and $3/2$) states in the $l = 1$ 70-dimensional representation of SU(6):

$$|S_{11}(1535)\rangle = \cos\vartheta|^2P_{1/2}\rangle - \sin\vartheta|^4P_{1/2}\rangle,$$

$$|S_{11}(1650)\rangle = \sin\vartheta|^2P_{1/2}\rangle + \cos\vartheta|^4P_{1/2}\rangle,$$

where mixing angle ϑ depends on the hyperfine spin interaction between the quarks and the standard spectroscopic notations $|\,^{2S+1}P_{1/2}\rangle$ are used to indicate the total quark spin $S = 1/2, 3/2$, orbital angular momentum $L = 1$, and total angular momentum $J = 1/2$. The corresponding spin-angular functions are given by

$$|\,^2P_{1/2}\rangle = \\ = \frac{1}{\sqrt{2}}\left(\sqrt{\frac{2}{3}}Y_{11}(\boldsymbol{\lambda})\chi^{\lambda}_{1/2-1/2} - \sqrt{\frac{1}{3}}Y_{10}(\boldsymbol{\lambda})\chi^{\lambda}_{1/2\,1/2}\right) + \\ + \frac{1}{\sqrt{2}}\left(\sqrt{\frac{2}{3}}Y_{11}(\boldsymbol{\rho})\chi^{\rho}_{1/2-1/2} - \sqrt{\frac{1}{3}}Y_{10}(\boldsymbol{\rho})\chi^{\rho}_{1/2\,1/2}\right), \quad (18)$$

where $\chi^{\lambda}_{1/2m_s}$ and $\chi^{\rho}_{1/2m_s}$ are the two spin functions symmetric and antisymmetric under interchange $1 \leftrightarrows 2$, and

$$|\,^4P_{1/2}\rangle = \sqrt{\frac{1}{2}}Y_{1-1}(\boldsymbol{\lambda})\chi^{s}_{3/2\,3/2} - \\ - \sqrt{\frac{1}{3}}Y_{10}(\boldsymbol{\lambda})\chi^{s}_{3/2\,1/2} + \sqrt{\frac{1}{6}}Y_{11}(\boldsymbol{\lambda})\chi^{s}_{3/2-1/2}. \quad (19)$$

Note that parameters ω for the $1/2^-$ nucleons depend also on the type of excitation. However, the difference is marginal and does not exceed 2%, see Table 2 of Ref. [5]. In what follows we use the common value $\omega = 0.457\,\mathrm{GeV}$ both for $\boldsymbol{\rho}$ and $\boldsymbol{\lambda}$ excitations.

Straightforward calculation yields (the indexes $+, 0$ refer to the charge of the nucleon $1/2^-$ states)

$$\mu(S^+_{11}(1535)) = \mu(^2P^+_{1/2})\cos^2\vartheta + \mu(^4P^+_{1/2})\sin^2\vartheta - \\ - 2\langle^2P^+_{1/2}|\mu^z_S|^4P^+_{1/2}\rangle\sin\vartheta\cos\vartheta = 1.24\,\mu_N, \quad (20)$$

$$\mu(S^+_{11}(1650)) = \mu(^2P_{1/2})^+\sin^2\vartheta + \mu(^4P_{1/2})\cos^2\vartheta + \\ + 2\langle^2P_{1/2}|\mu^z_S|^4P_{1/2}\rangle\sin\vartheta\cos\vartheta = -0.33\,\mu_N, \quad (21)$$

$$\mu(S^0_{11}(1535)) = \mu(^2P^0_{1/2})\cos^2\vartheta + \mu(^4P^0_{1/2})\sin^2\vartheta - \\ - 2\langle^2P^0_{1/2}|\mu^z_S|^4P^0_{1/2} > \sin\vartheta\cos\vartheta = -0.84\,\mu_N, \quad (22)$$

Table 3

Magnetic moments of $J^P = 1/2^-$ nucleons

State	*CQM* [9]	χPT[10]	χCQM [11]	*BS* [12]	*LQCD* [13]	This work
$S_{11}^{+}(1535)$	1.894	1.1	2.085	0.37	−1.8	1.24
$S_{11}^{0}(1535)$	−1.284	−0.25	−1.57	−0.1	−1.0	−0.84
$S_{11}^{+}(1650)$	0.11		1.85			0.12
$S_{11}^{0}(1650)$	0.951		−0.69			0.74

$$\mu(S_{11}^0(1650)) = \mu(^2P_{1/2}^0)\cos^2\vartheta + \mu(^4P_{1/2}^0)\sin^2\vartheta - \\ - 2\langle ^2P_{1/2}^0|\mu_S^z|^4P_{1/2}^0\rangle \sin\vartheta\cos\vartheta = 0.744\mu_N, \qquad (23)$$

where

$$\mu(^2P_{1/2}^+) = \frac{2}{9}\mu_u + \frac{1}{9}\mu_d = 0.23\,\mu_N, \qquad (24)$$

$$\mu(^4P_{1/2}^+) = \mu_u + \frac{1}{3}\mu_d = 1.14\,\mu_N, \qquad (25)$$

and

$$\langle ^4P_{1/2}^+|\mu_z|^2P_{1/2}^+\rangle = \frac{4}{9}(\mu_u - \mu_d) = 0.91\,\mu_N. \qquad (26)$$

Eqs. (24)–(26) are written for the positive charge resonances. For the neutral resonances one should interchage μ_u and μ_d

$$\mu(^2P_{1/2}^0) = \frac{1}{9}\mu_u + \frac{2}{9}\mu_d = 0, \qquad (27)$$

$$\mu(^4P_{1/2}^0) = \frac{1}{3}\mu_u + \mu_d = -0.232\,\mu_N, \qquad (28)$$

and

$$\langle ^4P_{1/2}^0|\,\mu_z|^2P_{1/2}^0\rangle = \frac{4}{9}(\mu_d - \mu_u) = -\,0.926\,\mu_N. \qquad (29)$$

Assuming a phenomenological value [22]

$$\vartheta \sim -\frac{\pi}{6}$$

we obtain the results summarized in Table 3. In this Table we also quote the magnetic moments obtained using different theoretical models.

5. Conclusions. To summarize, we have carried out a calculation of the magnetic moments of the low-lying negative parity $S_{11}(1535)$ and $S_{11}(1650)$ resonances. In the framework of the quark model these resonances are configuration mixtures of two $SU(6)$ states with excited orbital wavefunctions. Calculating both the quark spin and orbital angular momentum contribution for the magnetic moment, the cross terms due to the configuration mixing, and using the average value of the quark kinetic energy $\omega = 0.457$ GeV obtained from the variational solution for the einbein field in the EH method we obtain the values of magnetic moments of the $S_{11}(1535)$ and $S_{11}(1650)$ listed in Table 3. The results differ from the magnetic moments of the low-lying $J^P = 1/2^-$ nucleon calculated both in hadronic and quark models and from lattice QCD. In particular, the lattice results are different, even by sign. Any future measurement of the magnetic moment would have important implications in understanding the nature of parity partners of the nucleon.

Finally we note that the magnetic moments of the other $1/2^-$ low-lying baryon resonances can similarly be calculated using EH approach. The results will be published elsewhere.

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