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SCATTERING OF ELECTRON EDGE STATES IN MAGNETIC FIELD BY IMPURITIES AND PHONONS

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We calculate the scattering length for transitions between edge states in two-dimensional electron gas due to impurities short-range and long-range potential and acoustical phonons (deformation and piezoelectric interaction), assuming that the shape of the confining potential is arbitrary

Recent experiments [1-4] on two-dimensional electron gas (2DEG) in quantum Hall regime have demonstrated that when nonideal current probes are used the population of electron edge state can be unequal. In this case scattering between edge states (propagating in the same direction) affects four-probe measurments with nonideal voltage probe (the so-called anomalous quantum Hall effect).

Scattering of the edge states by irregularities of the boundary is discussed in ref. [5,6], by impurities and phonons in ref. [7]. The confining potential $U(y)$ in ref. [7] was assumed to be a parabolic one. However the parabolic potential has no flat domain which corresponds to the interior of the sample. This is why there exist no quasibulk states and one cannot reveal the properties of scattering which appear when the Fermi level is close to a bulk Landau level. In this Letter we calculate the scattering length of the edge states due to impurities and phonons for an arbitrary potential $U(y)$. Further, the impurity potential in ref [7] was assumed to be of a short range, while it was shown both theoretically [8,9] and experimentally [10,11] that for GaAs/$\mathrm{Ga}_{1-x}\mathrm{Al}_x$As heterostructures the dominant scattering mechanism is a long-range potential due to the remote ionized donors. This is why in this Letter we calculate the impurity scattering length for the long-range as well as for the short-range potential.

The wave function of the edge state is

$$\psi_{nk}(r) \sim \exp(ikx)\chi_{nk}(y)\varphi(z) \tag{1}$$

The location of the edge state with respect to the boundary of the 2DEG depends on the wave vector k. When $k \rightarrow +\infty$ state (1) transforms into a quasibulk Landau state, and its energy $E_{nk} \rightarrow E_n = \hbar\omega_H(n+1/2)$, where ω_H is the cyclotron frequency. $\varphi(z)$ is the wave function of the spatial quantization of the 2DEG in the direction normal to the heterostructure interface.

Impurity scattering

In the Born approximation one can calculate the elastic scattering length for transition $n \rightarrow n'$

$$1/l_{n \rightarrow n'} = (1/v_n v_{n'}) \int dq_y < UU >_q |P_{nn'}|^2 / 2\pi \tag{2}$$

where $< UU >_q$ is the 2D Fourier component of the scattering potential correlation function taken at the 2DEG plane, $\mathbf{q} = (q_x, q_y)$, v_n is the group velocity of the edge state n,

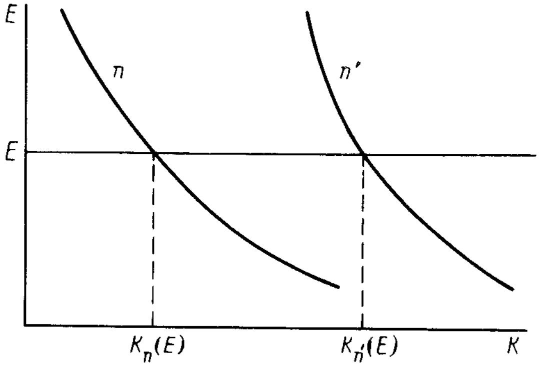
$$P_{nn'}(q_y) = \int dy \chi_{n'}(y) \exp(-iq_y y) \chi_n(y) \tag{3}$$

In (2) one should put $q_x = \delta k_{nn'} = |k_n(E) - k_{n'}(E)|$ with E being the energy of the initial state (see fig 1). The velocities v and functions χ for states n and n' correspond to the energy $E = E_F$. Note that $l_{n \rightarrow n'} = l_{n' \rightarrow n}$.

As in the refs. [8,9], we assume that the ionized donors are situated in a narrow layer separated from the 2DEG plane by the undoped spacer of the thickness z_0. Due to the electroneutrality the density of these donors (per cm^{-2}) equals $N_s + N_d$, where N_s is the density of the 2DEG and N_d is the density of the depletion charge layer on the GaAs side, the latest is assumed to be uniformly doped by the acceptors with the net density (per cm^{-3}) N_{AC}. Then the correlation function $< UU >_q$ can be written in the form [9]

$$< UU >_q = \left(\frac{2\pi e^2}{\epsilon \epsilon_s(q) q} \right)^2 [(N_s + N_d) \exp(-2qz_0) + N_{AC}/2q] \tag{4}$$

where ϵ is the lattice dielectric constant taken to be the same for GaAs and GaAlAs, $\epsilon_s(q) = 1 + q_s/q$ is the dielectric function of the 2DEG, q_s is the screening parameter. The first term in (4) corresponds to the long-range part of the scattering, while the second does to the short-range part.



For the long-range scattering one can proceed further assuming that $\delta k_{nn'} z_0 \gg 1$. In this case only small values of $q_y \simeq (\delta k_{nn'}/z_0)^{1/2}$ contribute to the integral (2). Using this simplification one can calculate the scattering length due to the long-range potential

$$1/l^L_{n \rightarrow n'} = (1/\bar{l}_L) \exp(-2\delta k_{nn'} z_0) A^2_{nn'} \tag{5}$$

where $A_{nn'} = P_{nn'}(0)$ and the nominal scattering length was defined

$$1/\bar{l}_L = 2\pi^{3/2}[2\pi e^2/\hbar\epsilon(v_n v_{n'})^{1/2}]^2(N_s + N_d)(\delta k_{nn'}/z_0)^{1/2}/(\delta k_{nn'} + q_s)^2 \qquad (6)$$

Note that due to the small factor $\exp(-2\delta k_{nn'} z_0)$ in (5) the scattering can be strongly suppressed compared to the case of zero magnetic field even in the case when the spatial separation between edge channels $\delta y_{nn'} = a_H^2 \delta k_{nn'}$, where a_H is the magnetic length, is not large compared to a_H. For the short-range scattering Eq.(2) cannot be reduced to a more simple form without any assumption on the confining potential.

Now we consider the case of the smooth potential $U(y)$,when $U'(y)a_H \ll \hbar\omega_H$. In this case

$$\chi_{nk}(y) = \Phi_n(y - ka_H^2) \qquad (7)$$

$$E_{nk} = E_n + U(ka_H^2)$$

where Φ_n is the harmonic oscillator wave function. In the smooth potential the overlap integral (3) can be calculated explicitly. Using (7) we have

$$A_{nn'}^2 = (2^{n+n'} n! n'!)^{-1} \sigma^{2n+2n'} \exp(-\sigma^2/2) \qquad (8)$$

where $\sigma = (y_{nn'}/a_H)^2 \gg 1$. The short-range scattering length $l_{n\to n'}^S$ becomes

$$1/l_{n\to n'}^S = (2\pi)^{-1/2}[2\pi e^2/\hbar\epsilon(v_n v_{n'})^{1/2}]^2[N_{AC}/a_H \delta k_{nn'}(\delta k_{nn'} + q_s)^2] A_{nn'}^2 \qquad (9)$$

The group velocity in the smooth potential is

$$v_{nk} = a_H^2 U'(ka_H^2) \qquad (10)$$

It follows from (8) that in the smooth potential the dominant transition are $n \to n+1$.

Acoustical phonon scattering

Since the sound velocity $s \ll v_n, v_{n'}$, the scattering is quasielastic, i.e. the energy of the emitted or absorbed phonon $\hbar\omega \ll \hbar\omega_H$. Hence, in the transition $n \to n'$ (see fig.1) the change of k is $\delta k_{nn'}$. The minimal energy of the phonon is $\Delta_{nn'} = \hbar s \delta k_{nn'}$. In what follows we consider low temperetures $T \ll \Delta_{nn'}$. In this case, due to the phonon Bose factor and the Pauli exclusion principle the phonon energy $\hbar\omega$ is close to the threshold $\Delta_{nn'}$. As a result the calculations of the scattering length is greatly simplified. In the Born approximation for deformation potential scattering (DA) [12]

$$\frac{1}{l_{n\to n'}} = \frac{\Xi^2}{2\pi\hbar\rho s^2} A_{nn'}^2 \frac{\delta k_{nn'}^2}{\hbar v_n v_{n'}} T F\left(\frac{\varepsilon}{T}, \frac{\Delta_{nn'}}{T}\right) \qquad (11)$$

where

$$F(\xi, \eta) = (1/2)[\ln(1 + \exp(\xi - \eta) + \exp(\xi)\ln(1 + \exp(-\xi - \eta)] \qquad (12)$$

Here Ξ is the deformation potential constant, ρ is the crystal density, $\varepsilon = E - E_F$. Velocity $v_{n'}$ and function $\chi_{n'}$ of the final state correspond to the energy of this state $E = E'$. Equation (11) is valid if $\exp(\varepsilon - \Delta/T) \ll \exp(\Delta/T)$ or, in other words, if $|\varepsilon| < \Delta_{nn'}$ and if $\varepsilon - \Delta_{nn'} \ll \Delta_{nn'}$. In the first case $\hbar\omega - \Delta_{nn'} \simeq T$,while in the second case $\hbar\omega - \Delta_{nn'} \simeq \varepsilon - \Delta_{nn'}$. It was also assumed that $T \gg ms^2$ and $\Delta_{nn'} \ll \hbar s/d$, where d is the scale of function $\varphi(z)$ .For GaAs/AlGaAs heterostructure $d = 3$nm, $s = 5 \times 10^5$ cm/s , $ms^2 = 0.1$K and $\hbar s/d = 13$K.Taking $\delta k_{nn'} = a_H^{-1}$, we have for $H = 2$T: $a_H = 18$nm ,$\Delta_{nn'} = 2$K and $\hbar\omega_H = 39$K. The inverse scattering length (3) is to be averaged near Fermi energy:

$$\left\langle \frac{1}{l_{n\to n'}} \right\rangle = \int dE \left(-\frac{\partial f_0}{\partial E}\right) \frac{1}{l_{n\to n'}(E)} \qquad (13)$$

Function F grows exponentially with ε for $\varepsilon > 0$ and since the average value is due to hot electrons ($\varepsilon \simeq \Delta_{nn'} \gg T$) rather than thermal electrons ($\varepsilon \simeq T$). Since $\Delta_{nn'} \ll \hbar\omega_H$, one can put $E = E_F$. With the above mentioned assumptions for DA-scattering

$$\left\langle \frac{1}{l_{n\to n'}} \right\rangle_{DA} = \frac{1}{(\bar{\tau}_{DA})_H} A^2_{nn'} (\delta k_{nn'} a_H)^3 \frac{s}{v_n v_{n'}} \exp\left(-\frac{\Delta_{nn'}}{T}\right) \qquad (14)$$

For piezoelectric PA-scattering [12] the calculations are similiar:

$$\left\langle \frac{1}{l_{n\to n'}} \right\rangle_{PA} = \frac{1}{(\bar{\tau}_{PA})_H} A^2_{nn'} (\delta k_{nn'} a_H) \frac{s}{v_n v_{n'}} \exp\left(-\frac{\Delta_{nn'}}{T}\right) \qquad (15)$$

Here we defined nominal scattering times

$$(1/\bar{\tau}_{DA})_H = \Xi^2 / 4\pi\hbar\rho s^2 a_H^3 \qquad (16)$$

$$(1/\bar{\tau}_{PA})_H = (e\beta)^2 / 4\pi\hbar\rho s^2 a_H$$

where β is some piezoelectric modulus. For GaAs at $H = 2$T we have $(\bar{\tau}_{DA})_H =$ 800ps and $(\bar{\tau}_{PA})_H = 40$ps (Ξ^2 and β^2 taken from [12]).The exponential suppression of the scattering rate is because of the deficit of free final states below E_F.

To consider the case of the smooth potential one have to substitute Eq.(7) for $A_{nn'}$ into (14) and (15). Comparing (14) or (15) with Eq.(7) in ref. [7] one can see that the corresponding equations agree only in the exponential factors from the overlap integral and from the deficit of the final states.

Discussion

Let us estimate $l_{1\to 0}$ in the simplest case when the Fermi level is far from Landau levels (i.e. field H corresponds to quantum Hall plateau) and potential $U(y)$ is not smooth, i.e. $\delta k_{10} \simeq a_H^{-1}$ and $v_1, v_0 \simeq a_H\omega_H \simeq v_F$ .To learn what is the dominant mechanism of the impurity scattering it is sufficient to put all of the characteristic lengths entering the pre-exponential factors in (5) and (9) to be the same: $a_H = z_0 = k_F^{-1} = q_s^{-1} \equiv \bar{k}^{-1} =10$ nm and $N_d = N_s = 10^{12}$ cm^{-2}. Comparing (5) and (9) one can see that the long-range scattering dominates if the thickness of the spacer z_0 is less than

$$z_0^* = (1/2\delta k_{nn'}) \ln(\bar{k}^3 / N_{AC}) \qquad (17)$$

At $N_{AC} = 10^{14}$ cm^{-3} we have $z_0^* \approx 60$ nm. To calculate $l_{1\to 0}$ we take $\delta y_{10} \approx 1.5 a_H$, $v_1 \approx v_0 \approx v_F/3$, $N_s = N_d = 3.5 \times 10^{12}$cm^{-2} ($k_F = 1.5 \times 10^6$cm^{-1}),

$v_F = 2.6 \times 10^7$cm/s and $q_s = 2.0 \times 10^6$cm^{-1}. The rate of the long-range impurity scattering depends strongly on the value of the parameter $\delta k_{nn'} z_0$. Taking $z_0 = 40$nm we have $l^L_{1\to 0} \approx 3\mu$m at $H = 2$T and $l^L_{1\to 0} \approx 30\mu$m at $H = 5$T, while the transport scattering length at $H = 0$ corresponding to the typical value of the zero-field mobility $\mu = 5 \times 10^5$ cm^2/V s is $\approx 5\mu$m. For DA-scattering $l_{1\to 0} \approx 1000\mu\mathrm{m}\exp(3\mathrm{K}/T)$ and for PA-scattering $l_{1\to 0} \approx 50\mu\mathrm{m}\exp(3\mathrm{K}/T)$. It follows from these calculations that impurity scattering is only strongly suppressed compared to $H = 0$, if H is high enough and the phonon scattering is always weak. Note that the DA-phonon scattering length estimated in ref. [7] is much shorter. The reason of the difference is mainly attributed to the another choice of the electron- phonon interaction constant. The constant used in [7] is not related to the deformation potential constant Ξ.

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