MAGNETIZATION VS EXTERNAL MAGNETIC FIELD IN LAYERED SUPERCONDUCTORS

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For an anisotropic layered superconductor we have calculated the magnetization vs absolute value $\mathcal{H}$ and orientation of the external magnetic field. It displays a cusp at a field $\mathcal{H} = \mathcal{H}_1(\theta)$, where θ is the angle between the field and the layers, and a maximum at some characteristic field $\mathcal{H} = \mathcal{H}_3(\theta) \propto (\sin\theta)^{-1}$ in agreement with the experimental measurments by Zavaritsky N.V. and Zavaritsky V.N.

We predict the existence of an intermediate critical field $\mathcal{H} = \mathcal{H}_2(\theta) \propto (\sin\theta)^{-1}$ at which the perpendicular to the layers component of the magnetic induction first penetrates the sample. The magnetic susceptibility $\chi = \partial M/\partial \mathcal{H}$ has a jump at $\mathcal{H} = \mathcal{H}_2(\theta)$. The jump is very small at $\theta \gg \gamma^{-1}$, where γ is the anisotropy coefficient. In this range of angles one can observe a cusp on a graph of χ vs $\mathcal{H}$ at $\mathcal{H} = \mathcal{H}_2$. This prediction of theory is in good agreement with the experiment [2].

In isotropic superconductors of second type the magnetization vs magnetic field has a maximum at the first critical field $\mathcal{H}_{c1}$ as it was first shown by Abrikosov [1]. At lower fields the field doesn't penetrate the superconductor (the complete Meissner-effect). In a recent experiment by N.V.Zavaritsky and V.N.Zavaritsky [2] the magnetization $M_{\|}$ vs magnetic field has been measured in a strongly anisotropic and layered superconductor Bi 2212. The magnetization has displayed a cusp at lower critical field and an additional maximum at a higher field depending on the angle θ between the external magnetic field and the layers.In this article we explain this phenomenon in the framework of a model of a homogeneous anisotropic superconductor. The layer structure leads to a cusp in the graph of magnetization vs magnetic field at an intermediate value of the magnetic field. However this cusp is very weakly pronounced in the total range of angles except the very small ones. Instead one can observe a well pronounced cusp in a graph of the magnetic susceptibility.

For a homogeneous anisotropic superconductor the magnetization vs magnetic field has been calculated by Buzdin and Simonov [3]. They have found the lower critical field H_{c1} and a maximum of magnetization at a higher field. In contrast with their numerical calculations we account for the layer structure and apply a simplified version of free energy which enables us to make a strightforward analysis. Feinberg and Villard [4] were first to predict the locking of kinks in a tilted magnetic field. Unfortunately their analysis did not incorporate the demagnetizing factors which are of great importance for the problem. Besides that they have not calculated the magnetization explicitly.

We start with an approximate expression for the free energy of an anisotropic layered superconductor [5]:

$$F = \frac{\mathbf{B}^2}{8\pi} + \frac{H_1\sqrt{B_x^2 + \gamma^2 B_z^2}}{4\pi} + \frac{H_2 \mid B_z \mid}{4\pi}, \tag{1}$$

where B_x and B_z are the components of the magnetic induction $\mathbf{B}$ parallel and perpendicular to the layers respectively, H_1 and H_2 are the characteristic magnetic fields logarithmically varying with the magnetic field and the angle, $\gamma^2 = \frac{m_c}{m_{ab}}$ is the ratio of effective masses. The second term at the r.h.s. of equation (1) is due to Campbell et al. [6]. Roughly speaking this is a contribution of a homogeneous anisotropic superconductor. The third term is due to kinks on tilted vortices (Ivlev et al. [7]). The free energy (1) has a logarithmic accuracy $(\ln \frac{\lambda}{\xi})^{-1}$ at low magnetic fields $\propto H_2 \propto \gamma H_1$ and a much higher accuracy $\propto H_2/\mathcal{H}$ in the intermediate range of fields $H_2 \ll \mathcal{H} \ll H_{c2}$.

The inner field $\mathbf{H}$ can be found by differentiating :

$$\mathbf{H} = 4\pi \frac{\partial F}{\partial \mathbf{B}}. \tag{2}$$

For an ellipsoidal shape of a sample the magnetostatic problem can be solved explicitly resulting in the following relationship between the external field $\mathcal{H}$ vector and the magnetic induction $\mathbf{B}$

$$\mathcal{H} = \hat{n}\mathbf{B} + (1 - \hat{n})\mathbf{H} = \mathbf{B} - 4\pi(1 - \hat{n})\mathbf{M} \tag{3}$$

Further we consider a symmetric case when the ellipsoid axes coinside with the crystallographic ones. Then the nondiagonal components of the demagnetizing tensor are equal to zero. Differentiating the free energy (1) and substituting to equation (3) we get

$$\mathcal{H}_x = B_x + (1 - n_{xx})\frac{H_1}{\sqrt{1+p^2}}, \tag{4}$$

$$\mathcal{H}_z = B_z + (1 - n_{zz})(\frac{\gamma H_1 p}{\sqrt{1+p^2}} + H_2 \mathrm{sign} B_z), \tag{5}$$

where $p = \gamma B_z/B_x$.

We find the region of the complete Meissner effect putting $B_x = B_z = 0$. Jet the values $p = \gamma B_z/B_x$ and $\sigma = \mathrm{sign}\, B_z$ remain undefined varying in the intervals: $-1 \le p, \sigma \le 1$. So this region notified as region 1 is defined parametrically by equations

$$\mathcal{H}_x = (1 - n_{xx})\frac{H_1}{\sqrt{1+p^2}}$$

$$\mathcal{H}_z = (1 - n_{zz})(\frac{\gamma H_1 p}{\sqrt{1+p^2}} + H_2\sigma).$$

The boundaries of this region are two straight lines

$$B_x = \pm(1 - n_{xx})H_1, \ \mid B_z \mid \le (1 - n_{zz})H_2$$

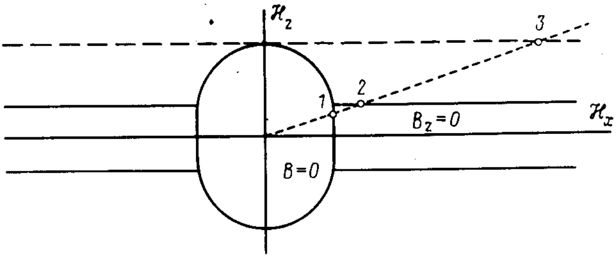

Fig.1. The phase diagram of a layered superconductor in the $\mathcal{H}_z, \mathcal{H}_x$ plane. See explanations in the text

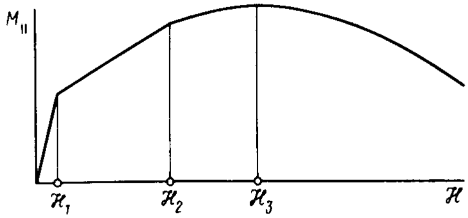

Fig.2. The schematic graph of the parallel magnetization vs magnetic field

and two ellipses

$$\frac{\mathcal{H}_x^2}{(1-n_{xx})^2 H_1^2} + \frac{[\mathcal{H}_z \pm (1-n_{zz})H_2]^2}{(1-n_{zz})^2(\gamma H_1)^2} = 1$$

(see Fig.1). Besides there exists the region of partial Meissner effect, where the parallel component of the induction B_x penetrates while the perpendicular component B_z does not (region 2). This region lays outside region 1 and between two straight lines

$$\mathcal{H}_z = \pm(1 - n_{zz})H_2$$

In the experiment by Zavaritsky and Zavaritsky [2] the angle θ between layers and the external magnetic field has been fixed and the parallel magnetization $M_{\parallel}$ vs the absolute value of magnetic field $\mathcal{H}$ has been measured. We present here the analytical calculations of $M_{\parallel}$ as well as the perpendicular component of magnetization $M_{\perp}$ based on equations (4) and (5).

In the region 1 we get obvious formulae

$$M_{\parallel} = -\frac{\mathcal{H}}{4\pi}\left(\frac{(\cos\theta)^2}{1-n_{xx}} + \frac{(\sin\theta)^2}{1-n_{zz}}\right) \tag{6}$$

$$M_{\perp} = \frac{\mathcal{H}\sin\theta\cos\theta}{4\pi}\left(\frac{1}{1-n_{zz}} - \frac{1}{1-n_{xx}}\right). \tag{7}$$

At a fixed angle θ the complete Meissner effect proceeds till $\mathcal{H} = \mathcal{H}_1 = (1 - n_{xx})H_1/\cos\theta$ provided $\theta < \theta_c$, where

$$\theta_c = \arctan\frac{(1-n_{zz})H_2}{(1-n_{xx})H_1}$$

As it is shown in Fig.1, a straight line corresponding to a fixed value of $\theta < \theta_c$ crosses first the vertical line $\mathcal{H} = \mathcal{H}_1$ and then the horizontal line $\mathcal{H}_z = (1 - n_{zz})H_2$. Between these two crossing points the magnetization obeys the following equations:

$$M_{\parallel} = -\frac{1}{4\pi}\left(H_1 \cos\theta + \mathcal{H}\frac{(\sin\theta)^2}{1-n_{zz}}\right), \tag{8}$$

$$M_{\perp} = \frac{1}{4\pi}\left(-H_1 \sin\theta + \mathcal{H}\frac{\sin\theta\cos\theta}{1-n_{zz}}\right). \tag{9}$$

A simple calculation shows the existence of the parallel magnetic susceptibilityjump $\Delta\chi$ at $\mathcal{H} = \mathcal{H}_2$:

$$\Delta\chi_{\parallel} = \frac{[H_2(1-n_{zz})\cos\theta - H_1(1-n_{zz})\sin\theta]\sin\theta}{4\pi H_1\gamma^2(1-n_{zz})^2}. \tag{10}$$

For $\theta \ll 1$ the ratio $\Delta\chi/\chi \sim (1+\gamma\theta)^{-1}$. Thus the cusp is well pronounced at small $\theta \leq \gamma^{-1}$ and very weakly pronounced at $\theta \gg \gamma^{-1}$. The corresponding value of θ for Bi 2212 is about 1°. Neglecting the jump of χ one finds an approximate expression

$$M_{\parallel} = -\frac{1}{4\pi(1-n_{zz})}\left[\frac{\cos\theta}{\gamma}\sqrt{(\gamma H_1(1-n_{zz}))^2 - (\mathcal{H}\sin\theta - (1-n_{zz})H_2)^2} + \mathcal{H}(\sin\theta)^2\right]. \tag{11}$$

Equation (11) is valid at $\gamma\theta \gg 1$ and in the range of fields $\mathcal{H}_2 \leq \mathcal{H} \leq \mathcal{H}_3 = (\gamma H_1 + H_2)/\sin\theta$. It displays a spurious singularity at $\mathcal{H} = \mathcal{H}_3$. The expression (11) is invalid in a small vicinity $\propto (\gamma\theta)^{-1/3}$ of the point $\mathcal{H} = \mathcal{H}_3$. In this range the parallel magnetization has a maximum and then decreases slowly till $\mathcal{H}_{c2}$ (see Fig.2).

Returning to the point $\mathcal{H} = \mathcal{H}_2$ we find from equation (11) a jump of the parallel susceptibility derivative

$$\Delta\frac{\partial\chi_{\parallel}}{\partial\mathcal{H}} = \frac{\cos\theta(\sin\theta)^2}{4\pi[(1-n_{zz})\gamma H_1]^2}. \tag{12}$$

The relative jump $\Delta\frac{\partial\chi_{\parallel}}{\partial\mathcal{H}}/\frac{\partial\chi_{\parallel}}{\partial\mathcal{H}}$ is of the order of unity. For the completness we write down the expression for the transverse magnetization:

$$M_{\perp} = \frac{\sin\theta}{4\pi(1-n_{zz})}\left[\mathcal{H}\cos\theta - \frac{\sqrt{(\gamma H_1(1-n_{zz}))^2 - (\mathcal{H}\sin\theta - (1-n_{zz})H_2)^2}}{(1-n_{zz})\gamma}\right].$$

We have elaborated the experimental data [2] for magnetization vs field to find the magnetic susceptibility $\chi_{\parallel}$. Cusps in graphs of $\chi_{\parallel}$ vs $\mathcal{H}$ are clearly seen in the tilt angle range from 15 till 80°, though the general calculation is rather poor.

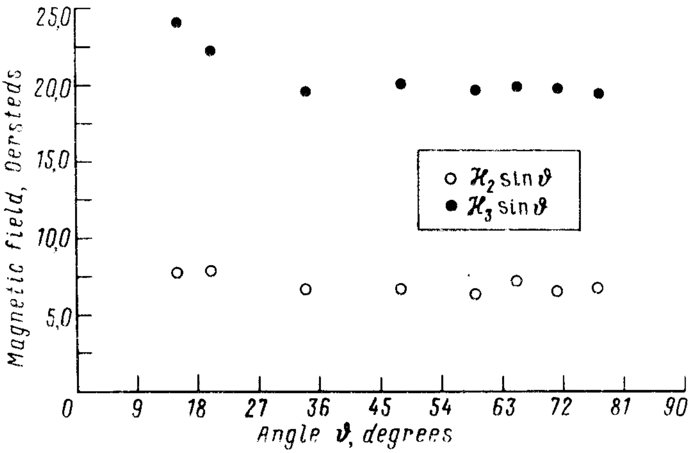

Fig.3. The critical value of $\mathcal{H}_z$ corresponding to a cusp in the magnetic susceptibility (the empty circles) and the value of $\mathcal{H}_z$ corresponding to maximum of $M_{\parallel}$ (the full circles) obtained by elaboration of the experimental data [2]

The external magnetic field corresponding to the cusp at a fixed angle θ has been multiplied by $\sin\theta$. The result is shown in Fig.3. According to the theory it should be a constant equal to $(1 - n_{zz})H_2$. We observe good agreement between theory and experiment. So the experiment [2] can be considered as the first clear evidence of kinks appearance.

For the completeness we show z-component of the external magnetic field at maxima of the parallel magnetization $(1 - n_{zz})(\gamma H_1 + H_2)$ due to [2]. From the experimental data we find $H_1 \approx 3oe$, and $H_2 \approx 75oe$.

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