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Positive magnetoresistance peaked at ferromagnetic transition in Mn-doped quantum wells GaAs/AlGaAs

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A large positive magnetoresistance peaked at the Curie temperature was observed in quantum well structures GaAs/AlGaAs doped by Mn. We suggest a new mechanism of magnetoresistance within low T_c ferromagnets resulting from a pronounced dependence of spin polarization at the vicinity of T_c on the external magnetic field. As a result, any contribution to resistance dependent on the Zeeman splitting of the spin subbands is amplified with respect to the direct effect of the external field. In our case we believe that the corresponding contribution is related to the upper Hubbard band. We propose that the mechanism considered here can be exploited as the mark of ferromagnetic transition.

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1. Introduction. As it is known, the quantum well structures on the base of GaAs/AlGaAs doped with Mn, representing ferromagnetic behavior, are of undoubted interest for spintronics. Thus, these structures were intensively studied during recent years. Clearly, the natural characteristic of ferromagnetic behavior is the sample magnetization. However, in many occasions the direct measurements of magnetization for such 2D layers, especially for nano- and even microscale devices can be problematic. Another conventional method to prove a presence of ferromagnetism is related to anomalous Hall effect. However, being an indirect evidence and in the same time being based on rather complex physical backgrounds, the latter also does not give a convincing proof.

Then, an additional (although also indirect) evidence can be given by negative magnetoresistance related to scattering of the carriers by spin fluctuations in the vicinity of T_c. The external magnetic field suppresses these fluctuations leading to a peak of magnetoresistance at $T = T_c$ [1]. However this mechanism, first, gives rather small contribution to resistance, second, it is pronounced only for structures with metallic conductivity. At the same time many quantum well structures are at the insulating side of the metal-insulator transition.

Note that the mechanism supporting ferromagnetic ordering in such disordered samples (including the bulk ones) is still a subject of discussion [2, 3]. In particular, there are statements that the indirect exchange between Mn ions is supported by carriers (holes) from the valence band. Another point of view relates these carriers to the impurity band (some discussion is given in [4]. We would like to mention the papers arguing that the exchange can be supported even by strongly localized carriers [5] in hopping regime [6]. Practically all previous studies rely upon an assumption that the group responsible for the indirect exchange is identified by a position of the Fermi level. Here we would like to note, that, to our point of view, this assumption in many occasions can be incorrect. Indeed, as it is known, the standard indirect exchange is contributed by all mobile carriers rather than the ones participating in transport. In our papers [7, 8] we have suggested a model relating the indirect exchange to the delocalized carriers within the impurity band. The presence of these carriers is a result of so-called virtual Anderson transition which can take place in narrow impurity band due to weak scatter of the site energies. At the same time, for weakly compensated samples the Fermi level can be outside of this band of mobile carriers, thus only activated (rather than metallic) transport is possible.

Returning to the magnetoresistance, we note that different structures exhibit different sorts of behavior. In particular, the strong negative magnetoresistance was reported for many occasions [4, 9] which was attributed to an increase of the localization length for hole-Mn complexes in external magnetic field. Thus in this case the transport is expected to be dominated by hopping between strongly localized states.

We would like to note that the anomalously large *positive* magnetoresistance (50 % for $H = 5$ T) was earlier observed in GaAs/AlGaAs structures with Mn delta-layer separated from the quantum well for sam-

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ples clearly exhibiting ferromagnetic transition [10]. Although the mechanism was not discussed in detail, the conclusion was made that "positive magnetoresistance is characteristic for granular and disordered materials and is not related to a ferromagnetic ordering". Nevertheless, we would like to emphasize that transport within the quantum well was – at least on microscopic scale – of metallic nature thus excluding the localization length considerations mentioned above. As for the geometric factors shortly mentioned in [10], the value of the parameter $\Omega\tau$ (Ω being the Larmour frequency, τ being the relaxation time) which we estimated for the sample parameters given in [10], appeared to be ≤ 0.07. Since it is this parameter which characterizes classical effect of magnetic field on electron trajectories, the geometric factor can hardly explain the magnitude of magnetoresistance observed.

In this work we will show – both experimentally and theoretically – that for ferromagnetic samples there exist an additional mechanism of positive magnetoresistance related to ferromagnetic ordering. This mechanism manifests itself provided that carrier transport depends on the degree of spin polarization while the corresponding magnetoresistance is peaked at the transition point. In our case we assume that the effect of spin polarization on the hole transport is due to activation of the holes to the upper Hubbard band (note that an effect of the exchange field on the position of the upper Hubbard band was earlier considered in [9]). In its turn, for the samples where Mn impurities were separated from the quantum well [10], the effect can be related to the fact that the polarization of the mobile carriers affects the energy distribution and, correspondingly, the scattering efficiency.

We would like to note that the effects considered in what follows can be used as an additional mark for ferromagnetic transition.

2. Experiment. The samples were grown on MBE installation EB1203, equipped by effusion sources of Ga, Mn, and of tetramers As. GaAs substrates, prepared for epitaxy (epi-ready) were subjected to thermal cleaning in As flow from a natural oxide. The cleaning process was controlled with a help of reflected fast electrons diffraction (DFER). Then the substrate temperature was decreased by 50 K and then the buffer layer of GaAs was grown during 15 min for the flatting of the sample surfaces. The following growth of the barrier layer of $Al_{0.3}Ga_{0.7}As$ with a width 250 Å was carried out after lowering the temperature down to 550 °C. To prevent segregation of the secondary phases of the type of MnAs in course of the growth of GaMnAs layer the substrate temperature was lowered down to 400 °C. The doping by Mn, which in terms of 3D concentrations would correspond to $1 \cdot 10^{19}\,\mathrm{cm}^{-3}$ (sample F934), was made only in course of the growth of the central region of quantum wells with a width of 50 Å, situated between the undoped GaAs layers with the same thickness.

For the sample to have ferromagnetic properties, it is necessary for Mn to be build into the Ga sublattice (Mn_{Ga}). In such a case the corresponding impurity is of acceptor type. In its turn, the possible formation of defects of the type of As_{Ga} and interstitial Mn (Mn_I), corresponding to double donors, can lead to compensation effects which lower the hole concentration. Thus the hole concentration can be significantly lower than that estimated from the degree of doping. The Mn concentration in the well center (estimated from the deposition rate) was $(1{-}5) \cdot 10^{19}\,\mathrm{cm}^{-3}$ in the epitaxial layer with a width 5 nm (or in terms 2D it corresponds to $(5{-}10) \cdot 10^{12}\,\mathrm{cm}^{-2}$). The rough estimate of the critical concentration, corresponding to the Mott transition ($N_c^{1/3} = 2.78/a_0$), can be obtained, based on the experimental binding energy ($E_A = 0.11$ eV) and on the value of the effective Bohr radius $a_0 = 10$ Å. The latter was obtained from the value of E_A within the effective mass approximation for the valence band holes, $m_h = 0.5m_e$. It corresponds for GaAs:Mn to $N_c = 1\,\% = 2.2 \cdot 10^{20}\,\mathrm{cm}^{-3}$. The radius a_0 becomes equal to the distance between Mn atoms at concentration $N_{\mathrm{Mn}} = 10^{21}\,\mathrm{cm}^{-3}$. Experimentally, the crossover to metallic conductivity was observed at concentrations $(1{-}5) \cdot 10^{20}\,\mathrm{cm}^{-3}$, which is about order of magnitude higher than the dopant concentration in our samples.

The normal Hall effect is observed at 300 K and corresponds to the concentration $3 \cdot 10^{12}\,\mathrm{cm}^{-2}$ (F934). This value is comparable with the doping level $5 \cdot 10^{12}\,\mathrm{cm}^{-2}$ for sample F934 (that is $> 60\,\%$ of the doped Mn appear to be electrically active). However, due to a high Bohr energy E_{B}, the concentration of the carriers activated to the valence band even at room temperature is expected to be less than the active dopant concentration. Thus one can conclude that actually nearly all the dopants are active and compensation degree is small.

The Hall resistance, R_{xy}, was measured at high magnetic fields at temperature region 220–50 K, see Fig. 1.

At high temperatures the resistivity demonstrates activation behavior with activation exponent $\varepsilon_1 \sim$ $\sim$ (35–40) meV (activation to valence band) (Fig. 2). This value is nearly equal to a half of the energy necessary for activation from the Mn acceptor level to the valence band of a quantum well which also evidences a small degree of compensation. With a decrease of temperature conductivity demonstrates some intermediated region with an apparent activation energy

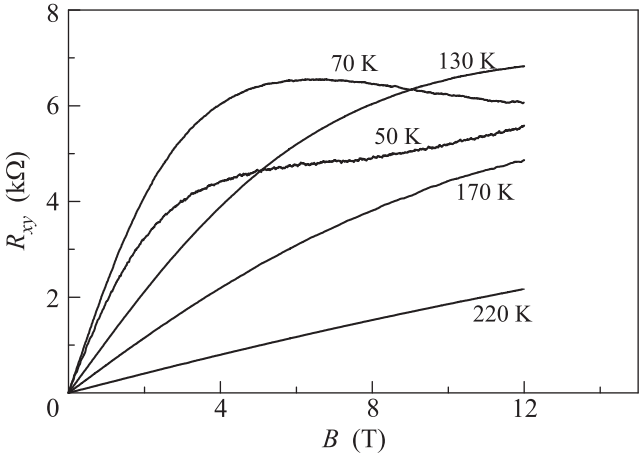

Fig. 1. Magnetic field dependencies of the Hall resistance for sample F934 at different temperatures

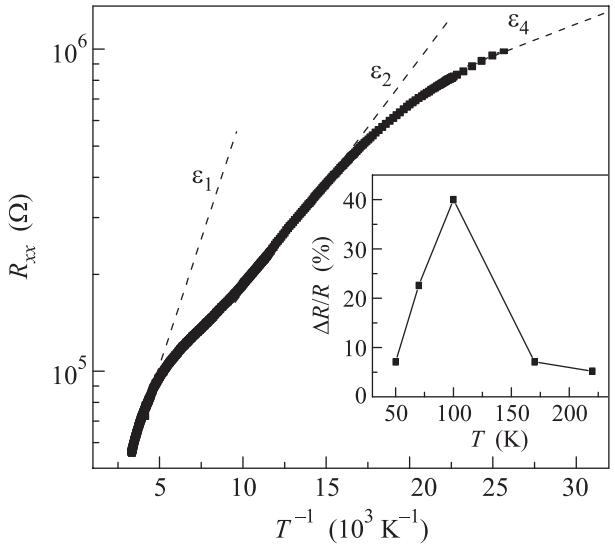

Fig. 2. Temperature dependencies of R_{xx} for samples F934. The insert – temperature behavior of magnetoresistance

$\varepsilon_2 \sim 15$ meV (insert of Fig. 2) and at $T < 20$ K the activation to the mobility edge in the impurity band with $\varepsilon_4 \sim (3-4)$ meV is observed for the sample F934.

On the insert of Fig. 2, a kink in the temperature dependence of R_{xx} is seen at $T = (130-100)$ K for sample F934 (where anomalous Hall effect is observed). It is commonly accepted that such a kink in a temperature dependence of R_{xx} is related to a presence of ferromagnetism and can be used as a measure of the Curie temperature [11].

At high temperatures (~ 200 K) a standard positive quadratic magnetoresistance is observed with relatively small magnitude ($\simeq 5\,\%$ even at $H = 12$ T). At the same time within the temperature region where the ferromagnetic transition takes place the positive magnetoresistance is strongly enhanced (up to 40 %). It is interesting that its magnetic field dependence is different from the one observed for high temperatures. Fig. 3 presents magnetic field dependence of MR in the temperature region 50–220 K. Fig. 4 rescales this temperature behavior in terms of $H^{2/3}$. It is seen that in the vicinity of the transition the magnetoresistance curves tend to be linear in the chosen scale. The positive magnetoresistance increases with temperature decrease from $T \sim 170$ K up to the temperatures $\sim 70-100$ K while a further decrease of temperature the magnetoresistance strongly decreases. At small temperatures the sample demonstrate weak negative magnetoresistance tending to positive magnetoresistance with an increase of magnetic field. Such a behavior is typical for hopping conductivity.

On the Figs. 3a and b magnetoresiastance curves for this sample are shown for temperature interval 220–50 K in the linear and $B^{2/3}$ scales.

3. Discussion. First, let us shortly discuss the temperature dependence of resistance in the absence of external magnetic field. One can easily distinguish the activation behavior at high temperatures characterized by activation energy ~ 44 meV which is nearly a half of the Bohr energy of Mn impurity. Such a behavior is know to be typical for weakly compensated samples. This fact emphasize our conclusion that Mn mostly occupy substitutional position which favor ferromagnetism. At lowest temperatures (less than 50 K) we also observe activation behavior, but with very small activation energy (around 4 meV). To our opinion, this is a signature of so-called $\varepsilon - 4$ conductivity related to activation of carriers from chemical potential to the band of delocalized states [7, 8]. These states result from virtual Anderson transition expected to occur in a narrow impurity band. The fact that the impurity band is narrow is a consequence of the weak compensation leading to a weakness of the disorder potential. In our recent paper we argued that the existence of ferromagnetism in our samples at unusually small Mn concentration is supported by the indirect exchange mediated by delocalized holes near the maximum of the impurity band. Note, however, that at small temperatures the delocalized states are completely occupied by holes, thus the transport is only possible due to activation of "minority" carriers (electrons) from the Fermi level in the tail of the impurity band to the band of delocalized states.

At Fig. 2 one can also see an intermediate region in between of ~ 170 and ~ 50 K with an apparent activation energy ~ 13 meV. Although the possible candidate can be the upper Hubbard band, its states being delocalized, the value of the activation energy seems to be too small. Nevertheless, one cannot exclude the effect of

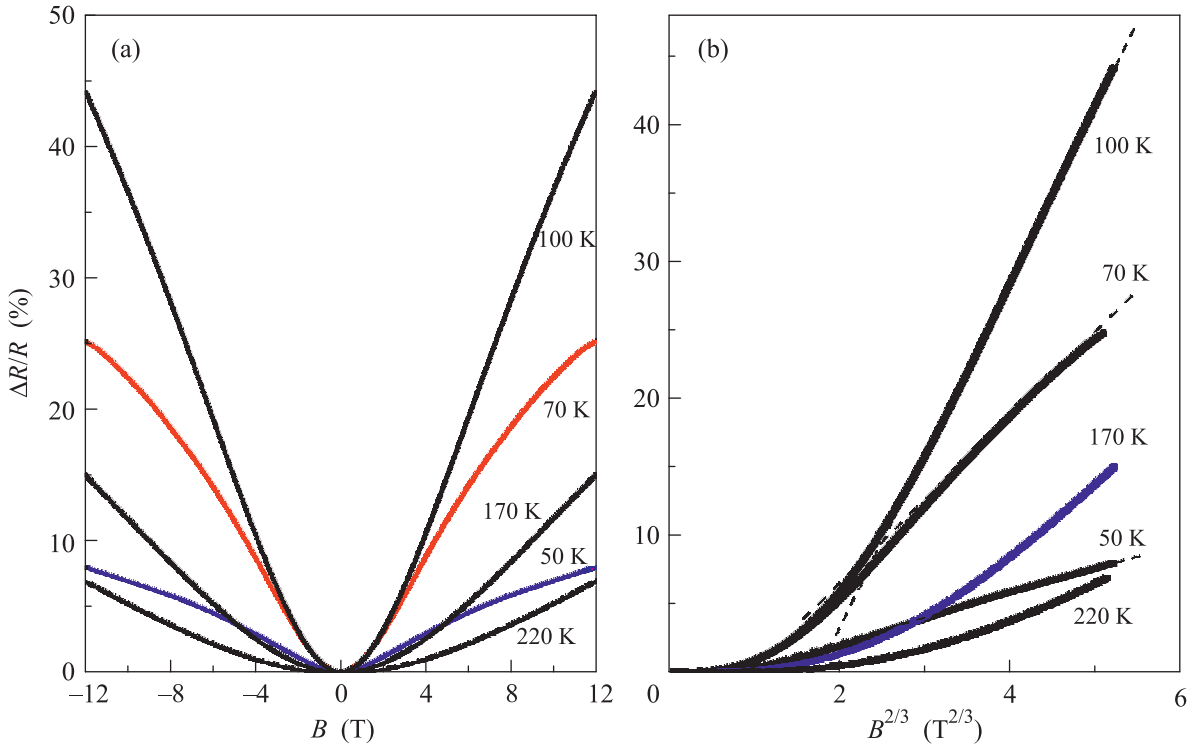

Fig. 3. Magnetic field dependencies of the magnetoresistance for sample F934 at different temperatures

the upper Hubbard band provided its width is not small (typically one expects that the upper Hubbard band is wider than the lower band due to larger overlapping integrals). If we assume the Gaussian density of states of the band centered at energy ε_0 characterized by a width $\Delta\varepsilon$ one expects that the rough estimate of the activation probability is given by

$$\int d\varepsilon' \left\{ \exp\left[-\frac{(\varepsilon_0 - \varepsilon')^2}{\Delta\varepsilon} \right] \right\} \exp\left(-\frac{\varepsilon'}{T} \right). \qquad (1)$$

The simple calculation gives the following temperature-dependent factor:

$$\exp\left\{ -\left[\frac{\varepsilon_0}{T} - \left(\frac{\Delta\varepsilon}{2T} \right)^2 \right] \right\}, \qquad (2)$$

where it is assumed that $\Delta\varepsilon/(2T) < \varepsilon_0/\Delta\varepsilon$. As it is seen, the value of the activation exponent can be in principle twice small with respect to the case of $\Delta\varepsilon = 0$.

Another mechanism which can contribute to the transport in the "intermediate" region is related to electrons activated to the band of delocalized states at temperatures $T > \varepsilon_4$. Although in this case the exponential behavior of resistance is not expected, relatively small interval of temperatures does not allow to discriminate it reliably from power law possible for different scattering mechanism. But, in any case, the existing data do not allow to exclude the contribution of the upper Hubbard band.

Now let us turn to the magnetoresistance behavior. First, we shall give arguments excluding "standard" mechanisms of positive magnetoresistance. The efficiency of purely classical mechanisms is typically described by the parameter $\Omega\tau$ characterizing curvature of electron trajectories in magnetic field. In terms of mobility μ this parameter can be rewritten as $\mu H/c$. As it was noted above, for the best value of μ reported in [10] for the sample, exhibiting pronounced positive magnetoresistance, this parameter appears to be 0.07. For our samples with smaller mobilities we have even smaller value of this geometric factor. We shall also exclude interference effects like antilocalization since for these effects our samples are also too dirty to be pronounced. Having in mind that the maximal value of the positive magnetoresistance is reached in the region centered around the critical temperature of ferromagnetic transition, we are stimulated to ascribe the effect to ferromagnetic properties. To discuss the behavior observed, we shall first discuss the effect of external magnetic field on the ferromagnetic ordering.

It is a conventional approach to consider the magnetization curves of a ferromagnet as resulting mainly from its domain structure. Indeed, for standard ferromagnet the exchange field, even near the Curie temper-

ature, Θ, is typically orders of magnitude larger than any available external magnetic field. Thus at $T > \Theta$ the behavior of magnetization is well described by the Curie–Weiss law while at $T < \Theta$ the magnetization with temperature decrease tends increase to its saturation value according to $(\Theta - T)^{1/2}$ law. The effect of external field on the ferromagnetic transition itself can be thus neglected.

We would like to emphasize that the situation is different for low-Θ materials like magnetic semiconductors where the saturation exchange fields are not as high as for transition metals. Here one can expect that a presence of external field significantly changes the magnetization behavior in the vicinity of transition temperature. In particular, the linear increase of magnetization with external field (Curie–Weiss law) holds only far enough from the Curie temperature, while close to Θ much steeper increase is expected even in reasonable external fields. In its turn, the magnetization value following from $(\Theta - T)^{1/2}$ law can be enhanced in strong enough external fields.

To demonstrate it, let us consider a behavior of magnetization of ferromagnet as a function of external magnetic field near the Curie temperature. For a simplicity we will exploit phenomenological considerations for a simplest model $s = 1/2$. Note that this model preserves most qualitative features relevant for larger spin values except some quantitative factors (see, e.g., [12]). The relative magnetization y in a presence of magnetic field is given by an equation

$$y = \tanh\left(\frac{\Theta}{T}y + \frac{\mu_{\mathrm{B}} g H}{T}\right). \qquad (3)$$

For small y the simple expansion gives

$$y = \frac{\Theta}{T}y + \frac{\mu_{\mathrm{B}} g H}{T} - \frac{1}{3}\left(\frac{\Theta}{T}y + \frac{\mu_{\mathrm{B}} g H}{T}\right)^3. \qquad (4)$$

Let us denote $T - \Theta = \delta$, $H_{ex,0} = \Theta/\mu_{\mathrm{B}} g$. As is easily seen, for $\delta > 0$ we have

$$H_{ex} \equiv H_{ex,0} y = H\frac{\Theta}{T - \Theta}. \qquad (5)$$

However with a decrease of T, $T \to \Theta$, this behavior for a given magnetic field H is saturated at the level

$$y \simeq \left(\frac{3\mu_{\mathrm{B}} g H}{\Theta}\right)^{1/3}; \quad H_{ex} \sim (3 H H_{ex,0}^2)^{1/3} \qquad (6)$$

which corresponds to a neglect of the term linear in y in Eq. (4) for the cubic term taken in the form $y^3/3$ ($T \simeq \Theta$). The saturation takes place at $\delta \simeq$ $\simeq (3^{1/2}\mu_{\mathrm{B}}\, g H \Theta^{1/2})^{2/3}$. A further decrease of δ and change of its sign, $T < \Theta$, does not change the value of y until $|\delta|$ reaches the value $(\mu g H \Theta^{1/2})^{2/3}$, and then y increases with increase of δ according to well-known law $y \simeq (\delta/\Theta)^{1/2}$.

Thus one concludes that: 1) the ferromagnetic transition is "smeared" at finite external magnetic field within the region of temperatures

$$-(3^{1/2}\mu_{\mathrm{B}} g H \Theta^{1/2})^{2/3} < \delta < (3^{1/2}\mu_{\mathrm{B}} g H \Theta^{1/2})^{2/3}, \quad (7)$$

where the exchange filed is strongly sensitive to external magnetic field; 2) at $\delta > 0$ the effective exchange field initially linearly increases with H, then, with a decrease of δ such a behavior is followed by weaker increase $y \propto H^{1/3}$ which still exists for $T < \Theta$. We would like also to note that for a given range of H the increase of $|\delta|$, that is a shift from the critical region finally leads to a suppression of the sensitivity of magnetization to the external magnetic field – except of the effects of domain structure. It is important to note that typically the domain structure effects manifest itself at relatively weak external fields. In contrast, the effects discussed above (pronounced close to the transition point) become important at strong fields (like several T). Now let us estimate the typical temperature interval (δ) around Θ where the effect of external field on ferromagnetic properties is important which is estimated as $\delta \simeq (3^{1/2}\mu_{\mathrm{B}} g H \Theta^{1/2})^{2/3}$. For typical magnetic semiconductor on the base of Mn we have $\Theta \sim 100\,\mathrm{K}$, $g = 5/2$. Thus for $H = 5$ T we have for the half-width of the interval the estimate $\delta \simeq 30$ K. As it is seen, the estimate is by no means a negligible one.

Nevertheless, it seems that the direct evidence for such a behavior cannot be easily extracted from magnetization measurements since the latter are sensitive to a behavior of domain structure. In particular, we can mention recent magnetization measurements in [4]. Although the samples of InMnP demonstrate magnetic field dependence of magnetization in weak fields (which could be attributed to the factor discussed above), this dependence is observed up to very small temperatures. The latter fact excludes the effect of the magnetic field on the exchange field (which is restricted by the vicinity of T_c), but can be easily explained by the effect of the magnetic field on macroscopic distribution of magnetization within the sample. It is important to note that for non-metallic samples the effect of domain structure is not expected to give a noticeable effect in magnetoresistance.

Now let us consider the upper Hubbard band formed by doubly occupied Mn acceptor states, A^+ states, which in the absence of external magnetic field and at $T < \Theta$ is split off the standard impurity band by the

Hubbard energy U. As it is known, the magnetic field increases the value of U by a quantity $\delta U_{A^+} = \mu_B g H$, since the total spin of the corresponding center is equal to 2, while the spin of a single hole – 3/2 [13]. As it is clear, due to the presence of the exchange coupling within the ferromagnet the magnetic field in our estimates should be replaced by H_{ex}. Correspondingly, the effective Hubbard energy is significantly increased by exchange interactions in the ferromagnet.

One has in mind that $E_B > U$, that is the activation of holes to the upper Hubbard band is preferable to activation to the valence band. At the same time we expect that the upper Hubbard band contain delocalized states. Indeed, according to our model, delocalized states exist already within the lower Hubbard band due to virtual Anderson transition. Although supporting exchange between Mn ions, these delocalized states within the lower Hubbard band cannot support electric current since all of them are occupied. In contrast, the states within the upper band have relatively small occupation and thus are current-carrying. Thus we can expect that the activated conductivity at large T is really related to activation into the upper Hubbard band rather than to the valence band. This consideration is supported by the fact that the observed activation energy is smaller than even a half of the Bohr energy E_B for Mn impurity. However, having in mind that the value of U is not expected to be much less than E_B, one still concludes that the observed activation energy still corresponds to a position of chemical potential in the middle between the positions of the upper and the lower Hubbard band, that is compensation is very weak.

Now let us recall our considerations given in [14] concerning positive magnetoresistance resulting from activation to the upper Hubbard band replacing, however, the external magnetic field H by the effective exchange field H_{ex}. According to these estimates the doubly occupied centers for electrons with spin 1/2 have a distribution function

$$n_u = \frac{1}{\exp\left[\dfrac{2\varepsilon + U - 2\tilde{\mu} + 2T\ln 2\cosh(\dfrac{\mu_0 g H_{ex}}{2T})}{T} + 1\right]}, \tag{8}$$

where μ is chemical potential; in the case when the total number of electrons is dominated by single-occupied sites while the total number of D^- sites is small one has

$$\mu = \mu(H=0) - T\ln\left(2\cosh\frac{\mu_0 g H_{ex}}{2T}\right). \tag{9}$$

As it is seen, for $\mu_0 g H_{ex} \gg T$ the Zeeman energy gives an addition to the Hubbard energy equal to $\mu_0 g H_{ex}$ and a contribution of the upper Hubbard band to the conductance is

$$\sigma \propto n_u \propto \exp\left(-\frac{\mu_0 g H_{ex}}{T} - \frac{U}{T}\right). \tag{10}$$

In our case we deal with holes with $s = 3/2$ while the A^+ center has the total spin of the holes equal to 2. For the case of strong fields, $\mu_0 g H \gg T$ the situation is similar to the discussed above: all spins tend to be aligned to magnetic field and the Zeeman energy paid for creation of A^+ center is simply $3\mu_0 g H_{ex} - 2\mu_0 g H_{ex} = \mu_0 g H_{ex}$. It is this energy that gives magnetic field dependent addition to U. Thus in this case the contribution of the A^+ band to conductivity is described by the same equations as for electrons, that is by Eq. (10).

The situation appears to be somewhat different for the case of weak fields, $\mu g H_{ex} < 2T$. As it is seen, for electron case the effect of magnetic field was absorbed by the shift of chemical potential given by Eq. (9). Note that this shift is mainly related to single-occupied states (since, as we believe, $n_u \ll 1$). Let us for a simplicity we will neglect a presence of intermediate spin states, restriction ourselves by two opposite directions of spin. In this case the shift of chemical potential could be described by Eq. (9), if we would replace g by $3g$. However, for the acceptor scenario we should also take into account that the total spin of the two holes at the acceptor is not equal to zero. As it can be easily shown, it leads (in the same approximation as exploited above) to a presence of two configurations of A^+ center described by occupation numbers n_{u+} and n_{u-} corresponding to opposite direction of the total holes spin. Naturally, the conductance is $\propto (n_{u+} + n_{u-})$. One notes that the energies of the two configurations include contributions $2\mu_B g H_{ex}$ and $-2\mu_B g H_{ex}$, correspondingly. Since the chemical potential lays far below the energy positions of A^+ center, one concludes that the magnetic field dependent factor is given as

$$(n_{u+} + n_{u-}) \propto \exp\left(-2\ln 2\cosh\frac{\mu_0 \cdot 3 g H_{ex}}{2T}\right) \times$$
$$\times \left[\exp(2\mu_0 g H_{ex}/T) + \exp(-2\mu_0 g H_{ex}/T)\right]. \tag{11}$$

The simple expansion in terms of $\mu_B g H_{ex}/T \to 0$ gives

$$(n_{u+} + n_{u-}) \propto -(\mu_0 g H_{ex})^2/2T. \tag{12}$$

Thus in the weak fields limit the total magnetoresistance related to the upper Hubbard band is $\propto (\mu_0 g H_{ex})^2/2T$. Having in mind that at a broad temperature region $H_{ex} \propto H^{1/3}$ we expect that within this region around

Θ the magnetoresistance related to the upper Hubbard band behaves as

$$\Delta R(H) \propto H^{2/3}. \tag{13}$$

Note, however, that at $H \to 0$ the width of the temperature region supporting $H^{2/3}$ law appeared to be too narrow. In this case at $T > \Theta$ $H_{ex} \propto H$, and, correspondingly, $\Delta R(H) \propto H^2$. However at $T < \Theta$ the exchange field exists even for $H = 0$, $H_{ex}(H = 0) \simeq \simeq (\Theta - T)^{1/2}\Theta^{1/2}/\mu_{\rm B}g$. We are interested in magnetoresistance resulting from a dependence of H_{ex} on H. As one easily obtains, the corresponding correction to H_{ex} is estimated as

$$\Delta H_{ex} \simeq \frac{\mu_{\rm B}gH}{3T}\frac{H_{ex}^3(T=0,H=0)}{H_{ex}^2(H=0)}. \tag{14}$$

Thus one concludes that in this case $\Delta R(H) \propto |H|$. Here we have an apparent singularity at $H \to 0$. It is a result of our assumption that even at weak external field the exchange field tend to be directed along the external field. However if the magnetization tends to be directed along some anisotropy axis, at weak magnetic fields the collinearity mentioned above does not take place. In particular, for 2D layer the easy axis is expected to be within the plane of the layer. Thus the effect of external magnetic field directed along a normal to the plane is significantly suppressed for weak fields with respect to the estimate of Eq. (14). In particular, for simple model with anisotropy energy $CM\sin^2\theta$ comparison of this energy with the Zeeman energy $-MH\sin\theta$ (taking in mind that in our notations $\cos\angle\mathbf{MH} = \sin\theta$ gives the estimate $\theta \sim H/C$). Then, one notes that actually the product $\mu_{\rm B}gH$ in Eq. (3) should be replaced by $\mu_{\rm B}gH\sin\theta$. Correspondingly, the same replacement should be made in Eq. (14) which gives $\Delta H_{ex} \propto H^2$.

Now let us consider a different situation studied in [10] where the holes were located within GaAs quantum well when the Mn impurities were separated from the well by a spacer. In this case the holes were delocalized within the lateral quantization band. The effect of their exchange interaction with Mn impurities was naturally related to effective exchange field which the holes suffered due to tunneling to the Mn layer. This exchange lead to spin polarization of the holes which, in its turn, affected energy distribution of the holes resulting from Pauli principle and Fermi statistics. To take this effect into account we will exploit the following estimate of the conductance of the spinless carriers:

$$\sigma = \int d\varepsilon\sigma(\varepsilon)\left(-\frac{\partial F_0}{\partial\varepsilon}\right), \tag{15}$$

where $\sigma(\varepsilon)$ is the partial conductance supported by the holes with energy ε while F_0 is the equilibrium Fermi function. The presence of spins coupled to the exchange field naturally affects the positions of partial Fermi levels for the two spin directions. Note that for a simplicity we again will exploit a simple two component model. In this case with an assumption $\mu_{\rm B}gH_{ex} < T$ we obtain for the integrand in Eq. (15)

$$[\sigma(\mu_0 + \mu_{\rm B}gH_{ex}/2 + \xi) + \sigma(\mu_0 - \mu_{\rm B}gH_{ex}/2 + \xi)] \times \times \frac{1}{T[\exp(\xi/2T) + \exp(-\xi/2T)]}. \tag{16}$$

As a result, for the exchange field dependent correction for the conductivity we obtain

$$\delta\sigma = \left(\frac{\mu_{\rm B}gH_{ex}}{2}\right)^2\frac{\partial^2\sigma}{\partial\varepsilon^2}, \tag{17}$$

where the value of ε corresponds to the Fermi level in the absence of the exchange field. Again, we expect that the value of H_{ex} in the vicinity of the ferromagnetic transition depends on the external field as $H_{ex} \propto H^{1/3}$, and thus $\delta\sigma \propto H^{2/3}$. The sign of the effect depends on the sign of the second derivative of the partial conductivity on energy. If this sign is negative, we deal with the positive magnetoresistance. Note that it is expected for conductivity decreasing with energy increase according to any power law:

$$\frac{\partial^2\sigma}{\partial\varepsilon^2} = 2\frac{1}{\rho^3}\left(\frac{\partial\rho}{\partial\varepsilon}\right)^2 - \frac{1}{\rho^2}\frac{\partial^2\rho}{\partial\varepsilon^2}. \tag{18}$$

4. Conclusions. We have studied both experimentally and theoretically magnetoresistance of AlGaAs quantum wells doped by Mn where we have earlier observed ferromagnetic ordering at very small content of Mn. We have shown, that in such structures anomalously large positive magnetoresistance, peaked at the Curie temperature, is observed. To our opinion supported by theoretical estimates such a behavior is a consequence of the two factors. The first one is a dependence of the spin polarization on the external magnetic field which is pronounced in the vicinity of the ferromagnetic transition. The second one is related to a dependence of resistance on the spin polarization. We believe that the positive magnetoresistance is in our case related to activated transport involving the upper Hubbard band. Namely, the position of this band is shifted to higher activation energies due to an increase of the exchange filed stimulated by the external magnetic field.

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