

# How learn the branching ratio $X(3872) \rightarrow D^{*0}\bar{D}^0 + c.c.$

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Infant terrible of charmonium spectroscopy, the resonance  $X(3872)$ , generated a stream of interpretations and ushered in a new exotic  $XYZ$  spectroscopy. In the meantime, many (if not all) characteristics of  $X(3872)$  are rather ambiguous. We construct spectra of decays of the resonance  $X(3872)$  with good analytical and unitary properties which allows to define the branching ratio of the  $X(3872) \rightarrow D^{*0}\bar{D}^0 + c.c.$  decay studying only one more decay, for example, the  $X(3872) \rightarrow \pi^+\pi^-J/\psi(1S)$  decay. We next define the range of values of the coupling constant of the  $X(3872)$  resonance with the  $D^{*0}\bar{D}^0$  system. Finally, we show that our spectra are effective means of selection of models for the resonance  $X(3872)$ .

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**1. Introduction.** Discovery of the  $X(3872)$  resonance became the first in discovery of the resonant structures  $XYZ$  ( $X(3872)$ ,  $Y(4260)$ ,  $Z_b^+(10610)$ ,  $Z_b^+(10650)$ ,  $Z_c^+(3900)$ ), the resonant interpretations of which assumes existence in them at least pair of heavy and pair of light quarks in this or that form. Thousand articles on this subject already were published in spite of the fact that many properties of new resonant structures are not defined yet and not all possible mechanisms of dynamic generation of these structures are studied, in particular, the role of the anomalous Landau thresholds is not studied.

Below we suggest an approach which allows to define the branching ratio of the  $X(3872) \rightarrow D^{*0}\bar{D}^0 + c.c.$  decay studying only one more decay of  $X(3872)$  into a non- $D^{*0}\bar{D}^0$  channel and to select models the  $X(3872)$  resonance.

The paper is organized as follows. In Sec. 2 we construct spectra of decays of the resonance  $X(3872)$  with good analytical and unitary properties and define the range of values of the coupling constant of the  $X(3872)$  resonance with the  $D^{*0}\bar{D}^0$  system. In Sec. 3 we show that the constructed spectra can effectively select the model proposed for the  $X(3872)$  resonance.

**2. Spectra and coupling constant.** *2.1. The  $X(3872) \rightarrow D^{*0}\bar{D}^0 + c.c.$  and others spectra.* The mass spectrum  $\pi^+\pi^-J/\psi(1S)$  in the  $X(3872) \rightarrow \pi^+\pi^-J/\psi(1S)$  decay [1] looks as the ideal Breit-Wigner one, see Fig. 1a.

The mass spectrum  $\pi^+\pi^-\pi^0J/\psi(1S)$  in the  $X(3872) \rightarrow \pi^+\pi^-\pi^0J/\psi(1S)$  decay looks in a similar way [2, 3].

The mass spectrum  $D^{*0}\bar{D}^0 + c.c.$  in the  $X(3872) \rightarrow D^{*0}\bar{D}^0 + c.c.$  decay [4] looks as the typical resonance threshold enhancement, see Fig. 2<sup>2)</sup>.

If structures in the above channels are manifestation of the same resonance, it is possible to define the branching ratio  $X(3872) \rightarrow D^{*0}\bar{D}^0 + c.c.$ ,  $BR(X(3872) \rightarrow D^{*0}\bar{D}^0 + c.c.)$  treating data only these (two) decay channels.

We believe that the  $X(3872)$  is the axial vector,  $1^{++}$  [5, 6]. In this case the  $s$ -wave dominates in the  $X(3872) \rightarrow D^{*0}\bar{D}^0 + c.c.$  decay and hence is described by the effective Lagrangian

$$L_{XD^{*0}\bar{D}^0}(x) = g_A X^\mu \left( D_\mu^0(x) \bar{D}^0(x) + \bar{D}_\mu^0(x) D^0(x) \right). \quad (1)$$

The width of the  $X \rightarrow D^{*0}\bar{D}^0 + c.c.$  decay

$$\Gamma(X \rightarrow D^{*0}\bar{D}^0 + c.c., m) = \frac{g_A^2}{8\pi} \frac{\rho(m)}{m} \left( 1 + \frac{\mathbf{k}^2}{3m_{D^{*0}}^2} \right), \quad (2)$$

<sup>2)</sup>An interference between the  $D^0\bar{D}^{*0}$  and  $\bar{D}^0D^{*0}$  channels is negligible for the narrowness of the  $D^{*0}$  and  $\bar{D}^{*0}$  states. Using the isotopical invariance of the  $DD^*\pi$  interaction and the experimental information about the  $D^{*+} \rightarrow D^0\pi^+$  and  $D^{*+} \rightarrow D^+\pi^0$  decays, one can find  $\Gamma_{D^{*0}} = 70$  keV.

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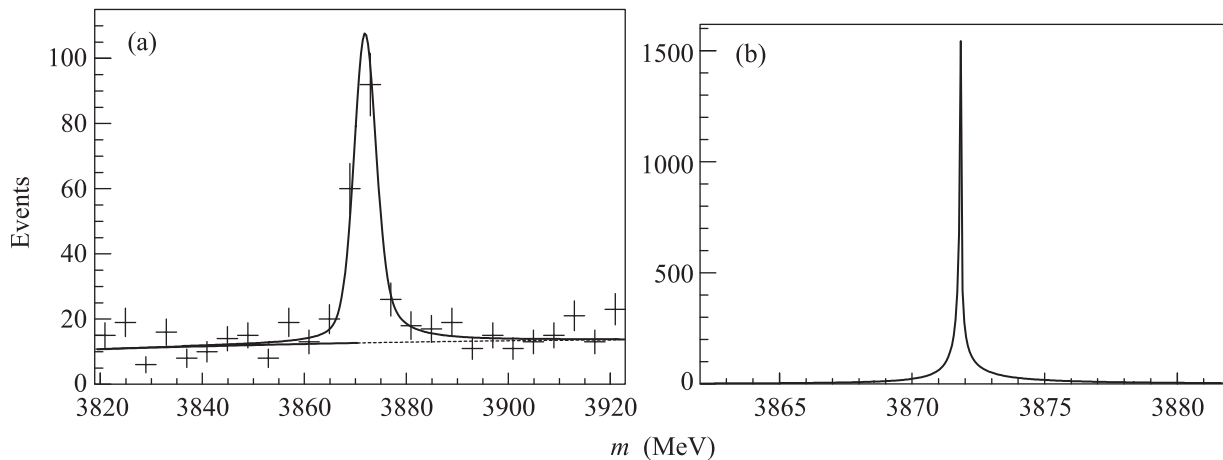


Fig. 1. (a) – The Belle data [1] on the invariant  $\pi^+\pi^- J/\psi(1S)$  mass ( $m$ ) distribution. The solid line is our theoretical one with taking into account the Belle energy resolution. The dotted line is second-order polynomial for the incoherent background. (b) – Our undressed theoretical line

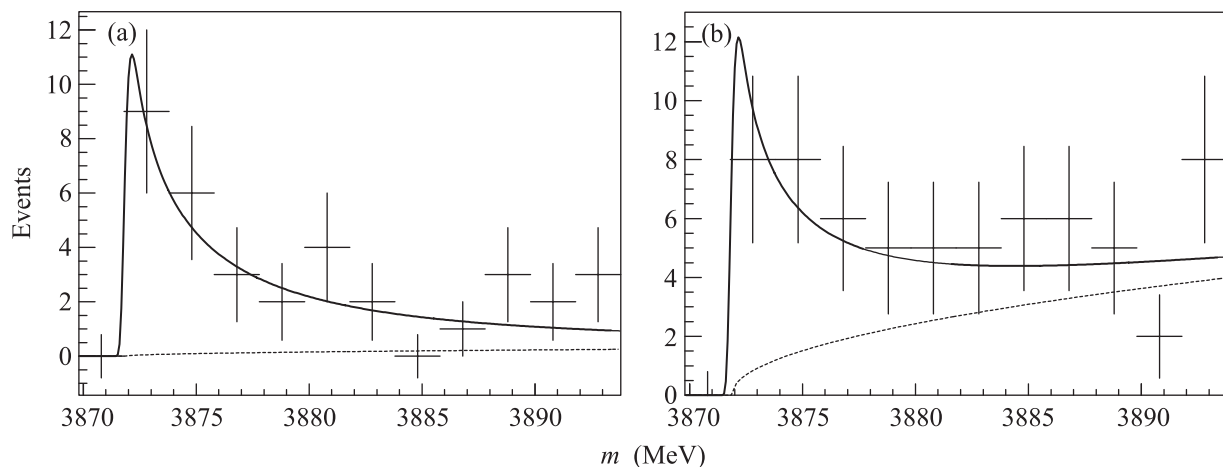


Fig. 2. The Belle data [4] on the invariant  $D^{*0}\bar{D}^0 + c.c.$  mass ( $m$ ) distribution. The solid line is our theoretical one with taking into account the Belle energy resolution. The dotted line is a square root function for the incoherent background. (a) –  $D^{*0} \rightarrow D^0\pi^0$ . (b) –  $D^{*0} \rightarrow D^0\gamma$

where  $\mathbf{k}$  is momenta of  $D^{*0}$  (or  $\bar{D}^0$ ) in the  $D^{*0}\bar{D}^0$  center mass system,  $m$  is the invariant mass of the  $D^{*0}\bar{D}^0$  pair,

$$\rho(m) = \frac{2|\mathbf{k}|}{m} = \frac{\sqrt{(m^2 - m_+^2)(m^2 - m_-^2)}}{m^2}, \quad (3)$$

$$m_{\pm} = m_{D^{*0}} \pm m_{D^0}.$$

The second term in the right side of Eq. (2) is very small in our energy region and can be neglected. This gives us the opportunity to construct the mass spectra for the  $X(3872)$  decays with the good analytical and unitary properties as in the scalar meson case [7, 8].

The mass spectrum in the  $D^{*0}\bar{D}^0 + c.c.$  channel

$$\frac{dBR(X \rightarrow D^{*0}\bar{D}^0 + c.c., m)}{dm} = 4 \frac{1}{\pi} \frac{m^2 \Gamma(X \rightarrow D^{*0}\bar{D}^0, m)}{|D_X(m)|^2}. \quad (4)$$

The branching ratio of  $X(3872) \rightarrow D^{*0}\bar{D}^0 + c.c.$

$$BR(X \rightarrow D^{*0}\bar{D}^0 + c.c.) = 4 \frac{1}{\pi} \int_{m_+}^{\infty} \frac{m^2 \Gamma(X \rightarrow D^{*0}\bar{D}^0, m)}{|D_X(m)|^2} dm. \quad (5)$$

In others  $\{i\}$  (non- $D^{*0}\bar{D}^0$ ) channels the  $X(3872)$  state is seen as a narrow resonance that is why we write the mass spectrum in the  $i$  channel in the form

$$\frac{dBR(X \rightarrow i, m)}{dm} = 2 \frac{1}{\pi} \frac{m_X^2 \Gamma_i}{|D_X(m)|^2}, \quad (6)$$

where  $\Gamma_i$  is the width of the  $X(3872) \rightarrow i$  decay.

The branching ratio of  $X(3872) \rightarrow i$

$$BR(X \rightarrow i) = 2 \frac{1}{\pi} \int_{m_0}^{\infty} \frac{m_X^2 \Gamma_i}{|D_X(m)|^2} dm, \quad (7)$$

where  $m_0$  is the threshold of the  $i$  state;

$$D_X(m) = m_X^2 - m^2 + \text{Re}[\Pi_X^{D^*0\bar{D}^0}(m_X)] - \Pi_X^{D^*0\bar{D}^0}(m) - im_X\Gamma, \quad (8)$$

where  $\Gamma = \Sigma\Gamma_i$  is the total width of the  $X(3872)$  decay into all non- $D^*0\bar{D}^0$  channels.

When  $m_+ \leq m$ ,

$$\begin{aligned} \Pi_X^{D^*0\bar{D}^0}(m) = & \frac{g_A^2}{8\pi^2} \left[ \frac{(m^2 - m_+^2) m_-}{m^2} \ln \frac{m_{D^*0}}{m_{D^0}} + \right. \\ & \left. + \rho(m) \left( i\pi + \ln \frac{\sqrt{m^2 - m_-^2} - \sqrt{m^2 - m_+^2}}{\sqrt{m^2 - m_-^2} + \sqrt{m^2 - m_+^2}} \right) \right]. \quad (9) \end{aligned}$$

When  $m_- \leq m \leq m_+$ ,

$$\begin{aligned} \Pi_X^{D^*0\bar{D}^0}(m) = & \frac{g_A^2}{8\pi^2} \left[ \frac{(m^2 - m_+^2) m_-}{m^2} \ln \frac{m_{D^*0}}{m_{D^0}} - \right. \\ & \left. - 2|\rho(m)| \arctan \frac{\sqrt{m^2 - m_-^2}}{\sqrt{m_+^2 - m^2}} \right], \quad (10) \end{aligned}$$

where  $|\rho(m)| = \sqrt{(m_+^2 - m^2)(m^2 - m_-^2)}/m^2$ .

When  $m \leq m_-$  and  $m^2 \leq 0$ ,

$$\begin{aligned} \Pi_X^{D^*0\bar{D}^0}(m) = & \frac{g_A^2}{8\pi^2} \left[ \frac{(m^2 - m_+^2) m_-}{m^2} \ln \frac{m_{D^*0}}{m_{D^0}} - \right. \\ & \left. - \rho(m) \ln \frac{\sqrt{m_+^2 - m^2} - \sqrt{m_-^2 - m^2}}{\sqrt{m_+^2 - m^2} + \sqrt{m_-^2 - m^2}} \right]. \quad (11) \end{aligned}$$

Our branching ratios satisfy unitarity

$$1 = BR(X \rightarrow D^*0\bar{D}^0 + \text{c.c.}) + \sum_i BR(X \rightarrow i). \quad (12)$$

Fitting the Belle data [1, 4], we take into account the Belle [1] results that  $m_X = 3871.84 \text{ MeV} = m_{D^*0} + m_{D^0} = m_+$  and  $\Gamma_{X(3872)} < 1.2 \text{ MeV}$  90%CL that corresponds to  $\Gamma < 1.2 \text{ MeV}$ , which controls the width of the  $X(3872)$  signal in the  $\pi^+\pi^-J/\psi(1S)$  channel and in every non- $D^*0\bar{D}^0$  channel, see Fig. 1b.

The results of our fit are in the Table 1. The current statistics is not sufficient for serious conclusions.

Nevertheless, one can state that our results are consist with experiment. Really, in view of  $BR(B \rightarrow X(3872)K) \times BR(X(3872) \rightarrow D^*0\bar{D}^0) = (0.80 \pm 0.20 \pm 0.1) \cdot 10^{-4}$  [4],  $BR(B^+ \rightarrow X(3872)K^+) \times BR(X(3872) \rightarrow \pi^+\pi^-J/\psi(1S)) = (8.61 \pm 0.82 \pm 0.52) \cdot 10^{-6}$  [1],  $BR(B^+ \rightarrow X(3872)K^+) \times BR(X(3872) \rightarrow \pi^+\pi^-\pi^0J/\psi(1S)) = (0.6 \pm 0.2 \pm 0.1) \cdot 10^{-5}$  [3], and  $BR(B^+ \rightarrow X(3872)K^+) \times BR(X(3872) \rightarrow \gamma J/\psi(1S)) = (1.78_{-0.44}^{+0.48} \pm 0.12) \cdot 10^{-6}$  [9] it follows that  $BR(X \rightarrow D^*0\bar{D}^0 + \text{c.c.}; m \leq 3892 \text{ MeV})$  is a few times as large as the sum of all non- $D^*0\bar{D}^0$  known branching ratios.

So, when fitting the  $X(3872) \rightarrow D^*0\bar{D}^0$  data and data for any  $X(3872)$  decay into non- $D^*0\bar{D}^0$  state,  $X(3872) \rightarrow i$ , we find  $\Gamma$  and  $g_A^2/4\pi$ , which define  $BR(X(3872) \rightarrow D^*0\bar{D}^0 + \text{c.c.})$ . Generally speaking, we don't need to know  $BR(X(3872) \rightarrow i)$ .

**2.2. Influence of the  $X(3872) \rightarrow D^{*+}\bar{D}^- + \text{c.c.}$  channel.** As seen from Table 1 the sizeable part (near 40%) of  $BR(X \rightarrow D^*0\bar{D}^0 + \text{c.c.})$  accounts for the tail of the  $X(3872)$  resonance ( $m \geq 3891.84 \text{ MeV}$ ). This gives an idea to take into account the  $X(3872) \rightarrow D^{*+}D^- + \text{c.c.}$  decays <sup>3)</sup> on the  $X(3872)$  tail. Since  $X(3872)$  is an isoscalar, the effective Lagrangian has the form

$$\begin{aligned} L(x) = & g_A X^\mu \left( D_\mu^0(x)\bar{D}^0(x) + \bar{D}_\mu^0(x)D^0(x) + \right. \\ & \left. + D_\mu^+(x)D^-(x) + D_\mu^-(x)D^+(x) \right). \quad (13) \end{aligned}$$

Eq. (8) is replaced by

$$D_X(m) = m_X^2 - m^2 + \text{Re}[\Pi_X(m_X)] - \Pi_X(m) - im_X\Gamma, \quad (14)$$

where

$$\Pi_X(m) = \Pi_X^{D^*0\bar{D}^0}(m) + \Pi_X^{D^{*+}D^-}(m). \quad (15)$$

$\Pi_X^{D^{*+}D^-}(m)$  is obtained from  $\Pi_X^{D^*0\bar{D}^0}(m)$ , see Eqs. (9)–(11), by replacement of  $m_{D^*0}$  and  $m_{D^0}$  by  $m_{D^{*+}}$  and  $m_{D^-}$ , respectively.

The unitarity condition, Eq. (12), takes the form

$$\begin{aligned} 1 = & BR(X \rightarrow D^*0\bar{D}^0 + \text{c.c.}) + \\ & + BR(X \rightarrow D^{*+}\bar{D}^- + \text{c.c.}) + \sum_i BR(X \rightarrow i). \quad (16) \end{aligned}$$

The results of our fit are in the Table 2.

The results in Tables 1 and 2 are compatible within the errors. The corresponding curves are similar to ones in Figs. 1 and 2. Of course, one should take into account the  $X(3872) \rightarrow D^{*+}D^- + \text{c.c.}$  channel in the case of the good statistics.

<sup>3)</sup>An interference between the  $D^+D^{*-}$  and  $D^-D^{*+}$  channels is negligible for the narrowness of the  $D^{*\pm}$  states,  $\Gamma_{D^{*\pm}} = 96 \text{ keV}$ .

Table 1

**Results of the analysis of the Belle data [1, 4]\*)**

$\Gamma$	$g_A^2/8\pi$	$\chi^2/Ndf$	$BR_{\text{seen}}$	$BR$	$BR(\text{Oth})_{\text{seen}}$
$1.2_{-0.467}$	$0.857_{-0.481}^{+3.614}$	43.74/42	$0.486_{-0.29}^{+0.061}$	$0.795_{-0.224}^{+0.19}$	$0.191_{-0.179}^{+0.223}$

\*)  $BR_{\text{seen}} = BR(X \rightarrow D^{*0}\bar{D}^0 + c.c.; m \leq 3891.84 \text{ MeV})$ ,  $BR = BR(X \rightarrow D^{*0}\bar{D}^0 + c.c.)$ ,  $BR(\text{Oth})_{\text{seen}} = \sum_i BR(X \rightarrow i; 3851.84 \leq m \leq 3891.84 \text{ MeV})$ .  $\Gamma$  in MeV,  $g_A$  in GeV.

Table 2

$\Gamma^*$	1.2-0.42	Mode	$X \rightarrow D^{*0}\bar{D}^0 + c.c.$	$X \rightarrow D^{*+}D^- + c.c.$	$X \rightarrow \text{Others}$
$g_A^2/8\pi^*$	$1.36_{-0.95}^{+4.85}$	$BR$	$0.586_{-0.101}^{+0.025}$	$0.315_{-0.16}^{+0.132}$	$0.098_{-0.096}^{+0.261}$
$\chi^2/Ndf$	45.49/42	$BR_{\text{seen}}$	$0.285_{-0.188}^{+0.121}$	$0.028_{-0.019}^{+0.004}$	$0.091_{-0.084}^{+0.255}$

\*)  $\Gamma$  in MeV,  $g_A$  in GeV.

**3. Conclusion.** Our approach can serve as the guide in selection of theoretical models for the  $X(3872)$  resonance. Indeed, if  $3871.68 \text{ MeV} < m_X < 3871.95 \text{ MeV}$  [6] and  $\Gamma_{X(3872)} = \Gamma < 1.2 \text{ MeV}$  [6] then for  $g_A^2/4\pi < 0.2 \text{ GeV}^2$  (that does not contradict current experiment, see Tables 1 and 2)  $BR(X \rightarrow D^{*0}\bar{D}^0 + c.c.) < 0.3$ . That is, unknown decays of  $X(3872)$  into non- $D^{*0}\bar{D}^0$  states are considerable or dominant.

For example, in Ref. [10] the authors considered  $m_X = 3871.68 \text{ MeV}$ ,  $\Gamma = 1.2 \text{ MeV}$ , and  $g_{XDD^*} = g_A\sqrt{2} = 2.5 \text{ GeV}$ , that is,  $g_A^2/8\pi = 0.1 \text{ GeV}^2$ . In this case  $BR(X \rightarrow D^{*0}\bar{D}^0 + c.c.; m \leq 3891.84 \text{ MeV}) = 0.15$ , that is, unknown decays  $X(3872)$  into non- $D^{*0}\bar{D}^0$  states are dominant. For details see Table 3.

Table 3

**Branching ratios for the model from Ref. [10] without  $D^{*+}D^- + c.c.$  channel\*)**

$m_X$	$\Gamma$	$g_A^2/8\pi$	$BR_{\text{seen}}$	$BR$	$BR(\text{Oth})_{\text{seen}}$
3871.68	1.2	0.1	0.152	0.189	0.792

\*)  $m_X$  in MeV,  $\Gamma$  in MeV,  $g_A$  in GeV.

Table 4

**Branching ratios for the model from Ref. [10]\*)**

$m_X$	3871.68	Mode	$X \rightarrow D^{*0}\bar{D}^0 + c.c.$	$X \rightarrow D^{*+}D^- + c.c.$	$X \rightarrow \text{Others}$
$\Gamma$	1.2	$BR$	0.176	0.045	0.779
$g_A^2/8\pi$	0.1	$BR_{\text{seen}}$	0.14	0.011	0.761

\*)  $m_X$  in MeV,  $\Gamma$  in MeV,  $g_A$  in GeV.

Account of influence of the  $X(3872) \rightarrow D^{*+}\bar{D}^- + c.c.$  channel has little effect on the results for the model from Ref. [10] since  $g_A^2/8\pi$  is small, see Table 4.

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