

Optical bistability through cavity effect in a four-level open atomic medium

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Submitted 1 July 2014

The optical bistability and multistability behavior in a ring cavity for the four-level open atomic system, driven by two coupling fields has been analyzed. It is shown that the presence of exit rate from cavity is having a dominant effect on generating the optical bi(multi) stability on the system. We find that, the effects of the injection rates might be of use to control the threshold intensity. Also, it is found that optical bistability (OB) can convert to optical multistability (OM) or vice versa via the effect of intensity and detuning of the coupling fields.

DOI: 10.7868/S0370274X14170032

1. Introduction. During the last decade, there has been significant research on quantum optical phenomena with quantum coherence and interference [1, 2], such as coherent population trapping and electromagnetically induced transparency (EIT) [3], amplification and lasing without population inversion [4], enhancing Kerr nonlinearity [5], multiwave mixing [6–8], optical solitons [9–13], etc. Meanwhile, there has been a great deal of interest in optical bistability (OB) as an effect which can be used for optical computing. The controlling light by light is one of the most interesting fields of research in quantum nonlinear optics. There can be a large number of potential applications in optical and related sciences, such as all-optical switches in optical communication and signal processing. For instance, finding suitable materials to implement optical transistors in various frequency domains, optical atomic memories to store information and quantum information processing using such memory devices and so on.

Recently optical devices with feedback control are used in information technologies. Generating of laser pulses for transmitting through the optical waveguide and error correction during the information processing are remarkable examples. In-depth information about feedback applications in classical optical processing can be found. Quantum theory of feedback and its applications in designing of quantum memories and implementation of quantum processing with embedded control are under consideration recently. In nonlinear optical systems surrounded by loop of feedback (OB) may appear. Input–output relations are not unique in this case and determined by previous evolution of the system. Existence of several output signal levels corresponded to the same input allows implementation of optical triggers, transistors, and other basic elements of information processing. The main reasons of OB are the saturation of atomic transition and nonlinear relationship between refractive index and electromagnetic field.

Optical bistability behavior is very sensitive to absorption-dispersion and nonlinear properties of the interactivity medium. Therefore, studies of atomic OB in different nonlinear systems, especially those exhibiting EIT are very important from the point of view of implementing devices of practical uses. The concept of optical bistability is a powerful principle that could be explored to implement all-optical transistors, switches, logical gates, and memory. Arrays of bistable optical gates can form the basis of an all-optical digital parallel processor [14, 15]. The name optical bistability comes from the characteristic of such systems that for a single input intensity, two (or more) stable output intensities are often possible, one large and one small. The system is like an electronic flip-flop except that it is all-optical. The research for quantum coherence in atomic systems [16, 17] is an interesting topic in the field of laser physics and quantum optics, which can modify the linear susceptibility and enhance the nonlinear optical processes in multilevel atomic systems [18–20] and has potential applications, such as optical buffers and modulators based on the slow light phenomenon, [21, 9, 10] and all-optical switch based on optical bistability and multistability (OM) [22, 23]. Several atomic systems have been proposed in recent years for studying optical bistability in an optical resonator [24–29]. These studies have shown that the OB threshold intensity could be significantly decreased by the effects of quantum coherence

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and interference. Gong et al. [28] investigate the OB behavior of a nearly equispaced ladder-type three-level atomic system contained in a unidirectional ring cavity. Considering the three-level atoms under conditions that the atomic dipole moments are nonorthogonal so that the effect of spontaneously generated coherence (SGC) is important, they found that the SGC effect significantly affects the OB behavior of such system. Within certain parameter range, OB can be realized due to the SGC effect. The bistable hysteresis cycle becomes wider with the enhancement of SGC effect. Furthermore, such system is much sensitive to the relative phase between the coupling and probe fields. This property makes it possible to switch between bistability and multistability via adjusting the relative phase. We [30] analyzed theoretically OB and OM in a medium consisting of four-level cascade-type cold atoms by means of a unidirectional ring cavity. Due to existence of a radio-frequency (RF) field, upper two-folded levels are coupled and double dark resonances (DDRs) can arise. We showed that by proper tuning of the RF field the threshold and the hysteresis cycle shape of OB and OM can be engineered. Also, the effect of intensity and frequency detuning of continuous-wave (cw) control laser field on bistable behavior of the medium was discussed. In addition, we explored the influence of different parameters on switching between OB and OM in this medium, which is applicable in all-optical switching. However, almost all of these studies are considered with a closed system. The effect of open system on transient and steady state behavior of tripod, V-type and four level atomic systems are investigated [31–34]. Recently, the effect of open system on OB of Ladder and Λ-type three-level atomic system is discussed [35, 36]. In this letter, bistable properties of a four-level open atomic system are investigated.

Based on the study of optical bistability in a closed four-level atomic system [24, 26, 30], in this work, we propose a new scheme in an open four-level atomic system to investigate OB. The present scheme is drastically different from the schemes for controlling the OB and OM behavior in a closed system. (i) For the given parameters optical bistability and multistability do not occur for closed system, while multistable behavior appears for open four-level atomic system. (ii) We demonstrate that the behavior of OB including threshold intensity and the hysteresis loop can be controlled by varying the ratio between atomic injection and exit rates. (iii) Compared with the corresponding closed system, intensity of coupling filed leads to different behavior in this open scheme.

2. Model and equations. Consider an open four-level atomic system coupled by a weak probe field, and two strong coupling fields (Fig. 1). A weak probe field with Rabi frequency of $\Omega_p = \mathfrak{P}_3\mathbf{E}_p/2\hbar$, amplitude E_p,

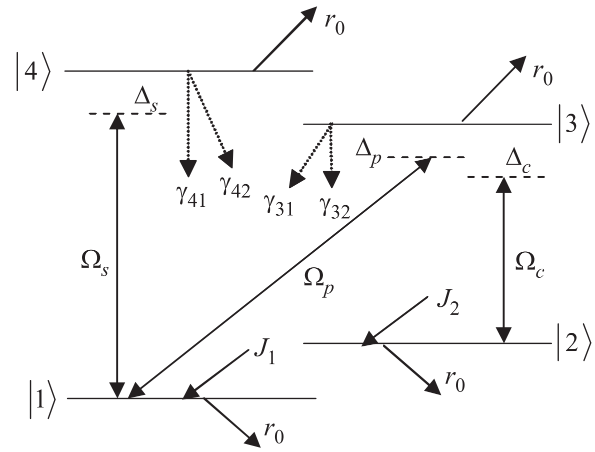

Fig. 1. The schematic of an open four-level atomic system. The system will be a closed system if $r_0 = J_1 = J_2 = 0$

and frequency ω_p, couples levels $|1\rangle$ and $|3\rangle$. An strong driving field with Rabi frequency of $\Omega_c = \mathfrak{P}_{32}\mathbf{E}_c/2\hbar$ (amplitude E_c and frequency ω_c) is applied to transition $|2\rangle \to |3\rangle$. Levels $|1\rangle$ and $|4\rangle$ are coupled via a coherent pump field of Rabi-frequency of $\Omega_s = \mathfrak{P}_{41} \cdot \mathbf{E}_s/2\hbar$ (amplitude E_s and frequency ω_s). The corresponding electric dipole moment are defined by $\mathfrak{P}_{ij}$. The spontaneous decay rates from upper level $|4\rangle$ to levels $|1\rangle$ and $|2\rangle$ are defined as γ_{41}, γ_{42}, while γ_{31}, γ_{32} are the spontaneous decay rates from $|3\rangle$ to levels $|1\rangle$ and $|2\rangle$, respectively. The atomic exit rate from the cavity is defined by r_0. In addition, J_1 and J_2 are the atomic injection rates for levels $|1\rangle$ and $|2\rangle$, respectively. The ratio of the atomic injection rates is $X = J_2/J_1$. We also assume that the number of interacting atoms is constant, which means that $r_0 = J_1 + J_2$. Using the rotating-wave and the electric dipole approximations and in the interaction picture, the density matrix equations of motion of this system can be written as:

$$\dot{\rho}_{12} = i(\Delta_p - \Delta_c)\rho_{12} + i\Omega_p\rho_{32} + i\Omega_s\rho_{42} - i\omega_c\rho_{13},$$

$$\dot{\rho}_{13} = \left[\frac{\gamma_{31}+\gamma_{32}}{2} - i\Delta_p\right]\rho_{13} + i\Omega_s\rho_{43} -$$

$$- i\Omega_c\rho_{12} + i\omega_p(\rho_{33} - \rho_{11}),$$

$$\dot{\rho}_{14} = \left[\frac{\gamma_{41}+\gamma_{42}}{2} - i\Delta_s\right]\rho_{14} + i\Omega_p\rho_{34} + i\Omega_s(\rho_{44} - \rho_{11}),$$

$$\dot{\rho}_{23} = \left[\frac{\gamma_{31}+\gamma_{32}}{2} - i\Delta_c\right]\rho_{23} - i\Omega_p\rho_{21} + i\Omega_c(\rho_{33} - \rho_{22}),$$

$$\dot{\rho}_{24} = \left[\frac{\gamma_{41}+\gamma_{42}}{2} + i(\Delta_p - \Delta_c - \Delta_s)\right]\rho_{44} +$$

$$+ i\Omega_c\rho_{34} - i\Omega_s\rho_{21},$$

$$\dot\rho_{34} = \left[\frac{\gamma_{31}+\gamma_{32}+\gamma_{41}+\gamma_{42}}{2} + i(\Delta_p - \Delta_s)\right]\rho_{34} +$$

$$+ i\Omega_p\rho_{14} + i\Omega_c\rho_{24} - i\Omega_s\rho_{31},$$

$$\dot\rho_{11} = \gamma_{31}\rho_{33} + \gamma_{41}\rho_{44} + i\Omega_p(\rho_{31} - \rho_{13}) +$$

$$+ i\Omega_s(\rho_{41} - \rho_{14}) + J_1 - r_0\rho_{11},$$

$$\dot\rho_{22} = \gamma_{32}\rho_{33} + \gamma_{42}\rho_{44} + i\Omega_c(\rho_{32} - \rho_{23}) + J_2 - r\rho_{22},$$

$$\dot\rho_{33} = -(\gamma_{31}+\gamma_{32}) + i\Omega_p(\rho_{13}-\rho_{31}) + i\Omega_c(\rho_{23}-\rho_{32}) - r_0\rho_{33},$$

$$\rho_{11} + \rho_{22} + \rho_{33} + \rho_{44} = 1. \quad (1)$$

Where $\Delta_p = \omega_{31} - \omega_p$, $\Delta_c = \omega_{32} - \omega_c$, $\Delta_s = \omega_{41} - \omega_s$ are the frequency detuning parameters. In this set of equations, Eq. (1) changes to those for a closed four-level atomic system, if $J_1 = J_2 = r_0 = 0$ [37].

In this paper a standard model is used for studying the OB behavior, i.e., we consider a medium of length L composed of four level open system immersed in the unidirectional ring cavity as shown in Fig. 2. The light

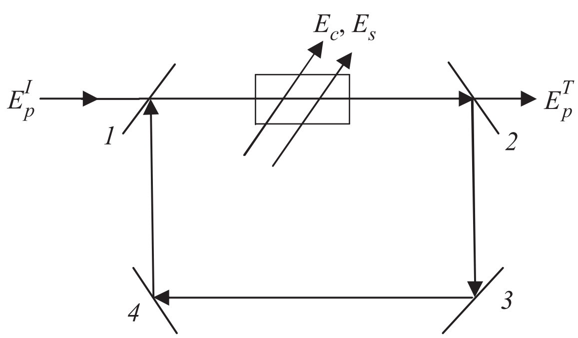

Fig. 2. Unidirectional ring cavity with atomic sample of length L

passes through the nonlinear medium and is redirected back to the entry point by a system of mirrors. The mirrors *3* and *4* are perfect reflectors, while the intensity reflection and transmission coefficients of mirrors *1* and *2* are R and T respectively ($R + T = 1$). By using this geometry, the state of the system can be analyzed by measuring the light transmitted through mirror *2*. Therefore, these two partially transmitting mirrors generate input and output. Here, one partially transmitting mirror is used for input port and another one is used for output port. The total electromagnetic field can be written as:

$$E = E_p e^{i\omega_{31}t} + E_c e^{i\omega_{32}t} + E_s e^{i\omega_{41}t} + \text{c.c.} \quad (2)$$

As we know, the probe field circulates in the ring cavity while the other fields do not. So, the dynamics of the probe field in the optical cavity is governed by Maxwell's equation, which, under slowly varying envelope approximation, is given by

$$\frac{\partial E_p}{\partial t} + c\frac{\partial E_p}{\partial z} = \frac{i\omega_p}{2\varepsilon_0}P(\omega_p), \quad (3)$$

where c and ε_0 are the speed of light and permittivity of free space, respectively, $P(\omega_p)$ is induced polarization in the transitions $|1\rangle \to |2\rangle$ and is given by

$$P(\omega_p) = N\mathfrak{P}_{31}\rho_{31}. \quad (4)$$

Substituting Eq. (4) into Eq. (3), one can obtain the field amplitude relation for the steady state such as

$$\frac{\partial E_p}{\partial z} = i\frac{N\omega_p\mathfrak{P}_{31}}{2c\varepsilon_0}\rho_{31}. \quad (5)$$

For a perfectly tuned ring cavity, in the steady state limit, the boundary conditions impose the following conditions between the incident field E_P^l and the transmitted field E_P^T [38]:

$$E_p(L) = \frac{E_p^T}{\sqrt{T}}, \quad (6a)$$

$$E_p(0) = \sqrt{T}E_p^I + RE_p(L), \quad (6b)$$

where L is the length of the quantum well sample, and the second term on the right-hand side of Eq. (6b) describes a feedback mechanism due to the mirror, which is essential to give rise to OB or OM, that is to say, there is no OB or OM when $R = 0$.

According to the mean-field limit [39, 40], using the boundary condition the steady state behavior of transmitted field is given by

$$Y = 2x - iC\rho_{31}, \quad (7)$$

where the normalized input and output fields are $y = \frac{\mathfrak{P}_{31}E_p^I}{\hbar\sqrt{T}}$ and $X = \frac{\mathfrak{P}_{31}E_p^T}{\hbar\sqrt{T}}$ respectively. The parameter $C = \frac{N\omega_p L\mathfrak{P}_{31}^2}{2\hbar\varepsilon_0 cT}$ is the cooperatively parameter for atoms in a ring cavity. Transmitted field depends on the incident probe field and the coherence term ρ_{31} via Eq. (7). So, the bistable behavior of medium can be determined by atomic variables through ρ_{31}.

3. Results and discussion. Setting $\gamma_{31} = \gamma_{32} = \gamma_{41} = \gamma_{42} = 1\gamma$, $C = 500\gamma$, $\Delta_c = \Delta_s = \Delta_p = 0$, we plot the input-output curves for different values of atomic exit rates r_0, as shown in Fig. 3. It can be found that for $r_0 = 0$ (closed system), no OB and OM occur. When r_0 is increased, the hysteresis cycle appears; OM appears and the threshold of OM will be increased by growth of r_0. Therefore, one can manipulate OM behavior via adjusting the atomic exit rates. In order to

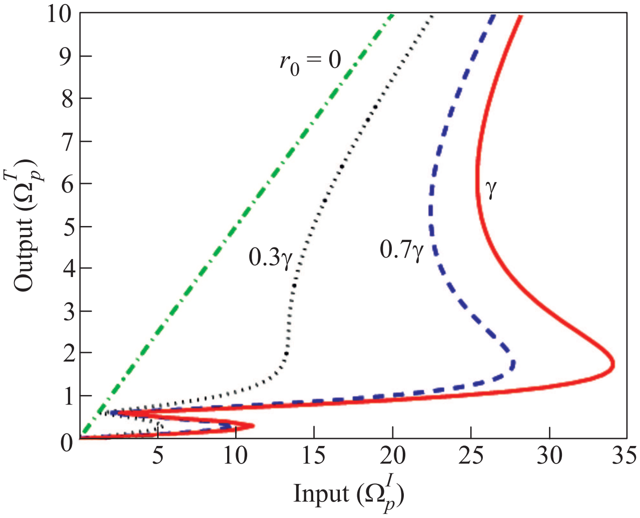

Fig. 3. Plots of the input-output field curves for different values of r_0. The selected parameters are $\gamma_{31} = \gamma_{32} = \gamma_{41} = \gamma_{42} = 1\gamma$, $C = 500\gamma$, $\Delta_c = \Delta_s = \Delta_p = 0$, $\Omega_c = \Omega_s = 0.5\gamma$, $X = J_2/J_1 = 2$

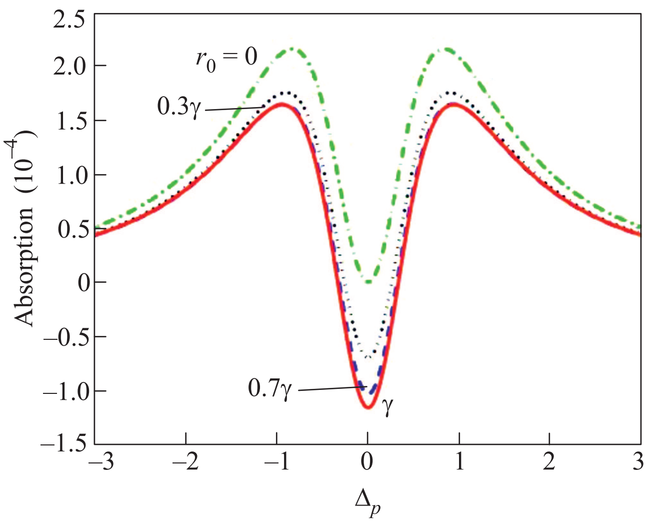

Fig. 4. Plots of absorption versus probe field detuning for different values of r_0. Here, $\Omega_p = 0.001\gamma$ and the other parameters are same as Fig. 3

get the physical origin of such an effect, we plot the absorption spectra versus Δ_p for different values of r_0 in Fig. 4. It is obvious that the atomic sample exhibits EIT when $r_0 = 0$. In addition, for open system ($r_0 \neq 0$), the probe gain enhances with increasing r_0. This makes it harder for the cavity field to reach saturation and thus the multi-stable threshold intensity increases. Thus, the OM threshold increases and the hysteresis cycle changes dramatically. Fig. 5 shows the dependence of the ratio between atomic injections rates ($X = J_2/J_1$) on optical bistability when $r_0 = \gamma$. It is found that for $X \leq 1$,

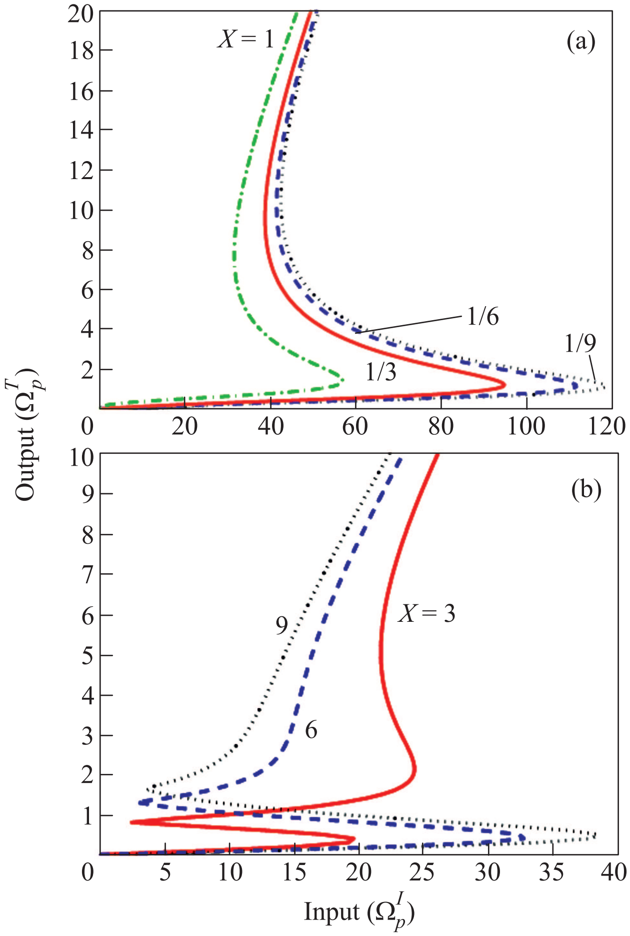

Fig. 5. Plots of the input-output field curves for different values of X. (a) – X from $X = 1/9$ to 1. (b) – X from $X = 3$ to 9. Here, $r_0 = \gamma$ and the other parameters are same as Fig. 3

OB appears (Fig. 5a). In this case, the larger X, the lower the OB threshold intensity and the lower the intensity at which the open system transits from the low branch to the high branch. Furthermore, for X becomes more than one, the procedure is different. In this case, OM emerges and the OM threshold will be increased by increasing parameter X. For physical interpretation, the absorption spectrum of the weak probe field versus probe detuning is plotted in Figs. 5a and b. From Fig. 5a we find out that when $X \leq 1$, increasing X leads to reduction on absorption spectrum at $\Delta_p = 0$. Thus, the cavity field more easily reaches saturation and this enables reduction of the bistable threshold intensity. The absorption spectrum for X values larger than one is displayed in Fig. 6b. It can be seen that by increasing parameter X, the probe gain is increased. This makes it harder for the cavity field to reach saturation and thus the multi-stable threshold intensity increases. In the fol-

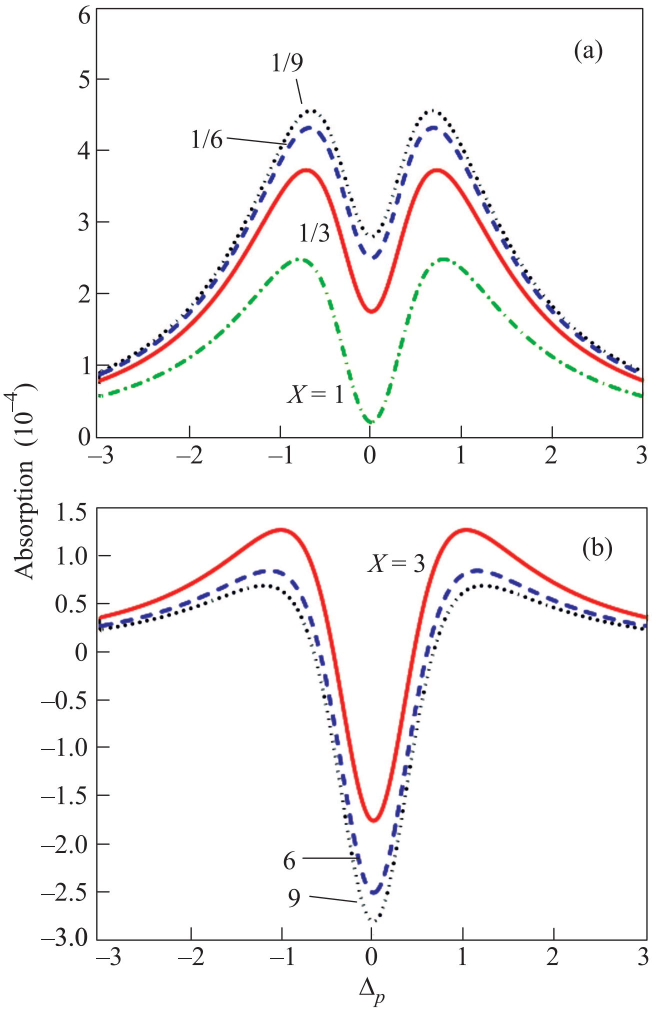

Fig. 6. Plots of absorption versus probe field detuning for different values of X. (a) – X from $X = 1/9$ to 1. (b) – X from $X = 3$ to 9. Here, $r_0 = \gamma$, $\Omega_p = 0.001\gamma$ and the other parameters are same as Fig. 3

lowing, we consider the effect of the Ω_c on OB and OM behavior of the open medium in Fig. 7a. It can easily realized that the OM threshold is reduced with the increase in Ω_c. The impact of Ω_s on optical bistability of the medium is displayed in Fig. 7b. We observe that for $\Omega_s = 0.5\gamma$, OM appears. The more enhancement of Ω_s, OM switches to OB ($\Omega_s = 1.5\gamma$). Finally, OB tends to be disappeared for $\Omega_s = 3\gamma$. Physically, applying an increasingly strong coupling field Ω_c (Ω_s) between states $|2\rangle$ and $|3\rangle$ ($|1\rangle$ and $|4\rangle$), one can modify the absorption and dispersion for the probe field on the $|3\rangle \to |2\rangle$ ($|4\rangle \to |1\rangle$) transition and manipulate the Kerr nonlinearity of the medium, thus obviously changing the threshold and the hysteresis cycle. For better realization, we plot the three-dimensional plot of the steady-state absorption spectra versus Δ_p and Ω_c (Fig. 8a) and

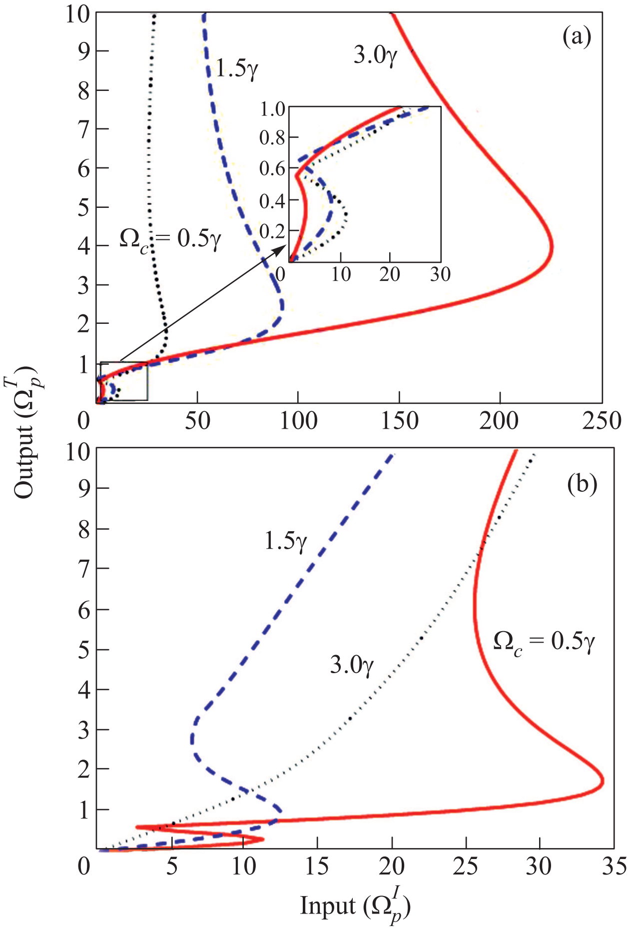

Fig. 7. Plots of the input-output field curves for different values of Ω_c. Here, $r_0 = \gamma$, $X = 2$, $\Omega_s = 0.5\gamma$ and the other parameters are same as Fig. 3

versus Δ_p and Ω_s (Fig. 8b). Investigation on Figs. 8a and b we can discover that the medium experiences a large amplification at $\Delta_p = 0$, but increasing $\Omega_c(\Omega_s)$ leads to significant reduction of the probe gain in such a way that the open system will experience its minimum gain value for $\Omega_c = 3\gamma$ ($\Omega_s = 3\gamma$) around zero probe detuning.

To illustrate the advantage of open four-level system to the corresponding closed four-level system, we plot the input-output curves for the case of $r_0 = 0$ (closed system) and for the same parameters with Fig. 8. It is observed that for $\Omega_c = 0$ no OB and OM occur, which we have seen in Fig. 3. However, increasing Ω_c to the larger values, OB appears. Here we find one of the advantages of our open system to corresponding closed one which is appearance of OM, when more than two output intensity is required. More enhancement of Ω_c to $\Omega_c = 3\gamma$, it can be seen that the OB threshold is increased. In this case the absorption spectrum at $\Delta_p = 0$

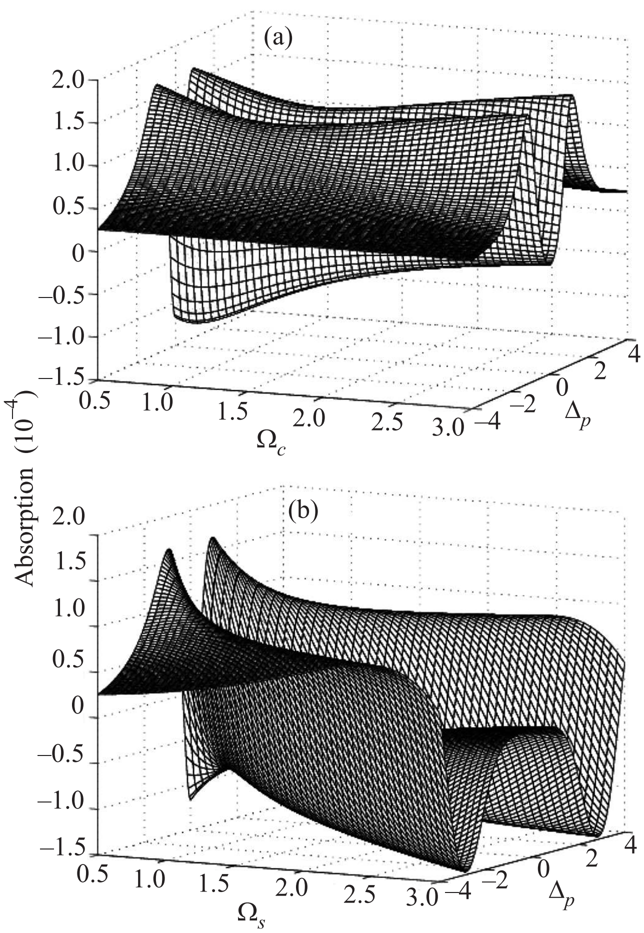

Fig. 8. Plots of the input-output field curves for different values of Ω_s. Here, $r_0 = \gamma$, $X = 2$, $\Omega_c = 0.5\gamma$ and the other parameters are same as Fig. 3

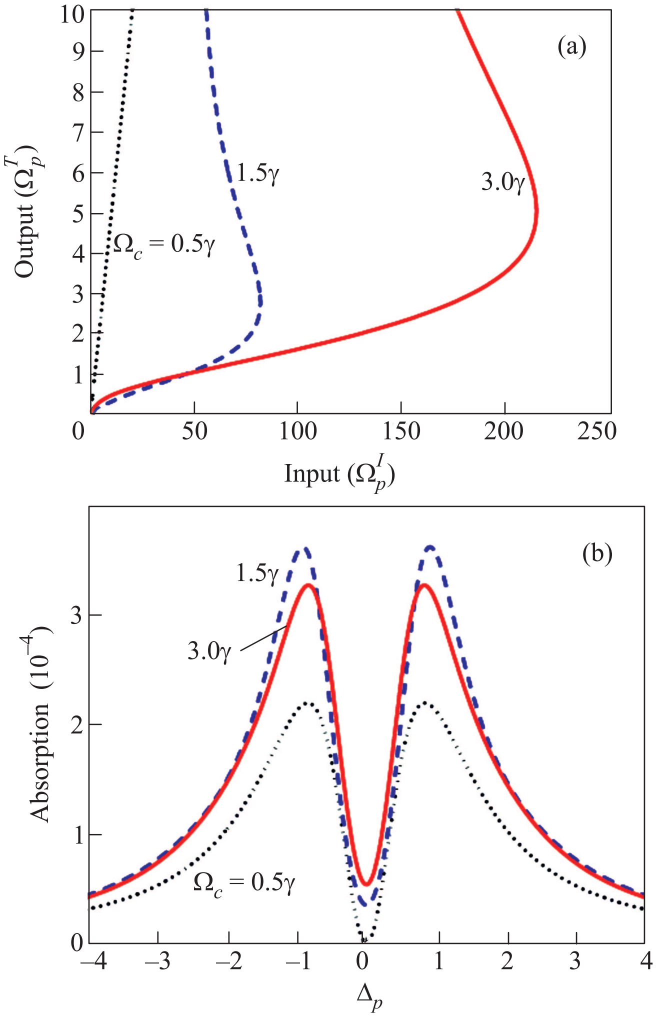

Fig. 9. (a) – Plots of the input-output field curves for different values of Ω_c for the corresponding closed system. (b) – Plot of absorption versus probe field detuning for different values of Ω_c for the corresponding closed system. Here, $r_0 = 0$ and the other parameters are same as Fig. 3

enhances (Fig. 9b), which makes the cavity field harder to reach saturation. This is another advantage of open four-level system rather than the closed one. In other words, in open system the threshold intensity of OM decreases by increasing the intensity of Ω_c (Fig. 7a), while OB threshold is obviously increased in closed system (Fig. 9a). Achieving to smaller threshold intensities is obviously favorable from the viewpoint of practical applications.

At the end, we discuss the influence of coupling field detunings on optical bistability and multi-stability of the four level open atomic system in Fig. 10. It is clear that for $\Delta_c = \Delta_s = 0.2\gamma$ optical multistability appears. However, OM converts to OM for $\Delta_c = \Delta_s = \gamma$, and again OB changes to OM for $\Delta_c = \Delta_s = 2\gamma$. Thus, another way of transition from OB to OM or vice versa is presented. It is important to be pointed out that switching between OB and OM is an interesting result which is very practical in all-optical applications, when two or more output intensity is required.

4. Conclusion. In summary, by using the model of four-level open atomic system inside the unidirectional ring cavity, we have demonstrated that the atomic injection rates and atomic exit rate from the cavity can significantly affect the OB and OM behavior of such system. We found that in closed system OB and OM does not occur. In contrast, in open system, optical bistability and multistability will appear. Increasing the exit rate, the multistable threshold increases which is due to enhancement of probe gain at zero probe detuning. The effect of intensity and frequency detunings of Rabi-frequencies on bistable behavior of the medium is then discussed.

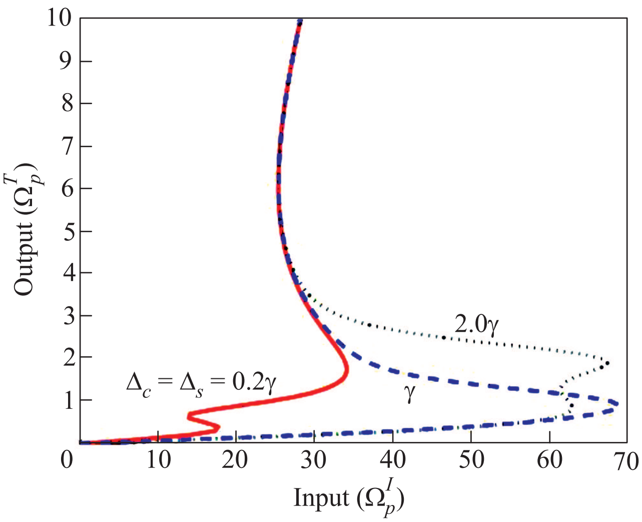

Fig. 10. Plots of the input-output field curves for different values of Δ_c, Δ_s. Here, $r_0 = \gamma$, $X = 2$ and the other parameters are same as Fig. 3

The presented work has been supported by the Lithuanian Research Council (# VP1-3.1-ŠMM-01-V-03-001).

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