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Feedback-enhanced self-organization of atoms in an optical cavity

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We considered an application of a feedback loop to enhance the self-organization of atoms in a cavity. Differently to the original setup, we assumed the light leaking from the cavity was photo-detected and the signal was used to appropriately adjust the atomic potential. It was shown that no additional feedback-induced quantum noise was introduced into the system. Numerical simulations performed in classical approximation showed that the application of feedback weakened the requirement for the atom-field coupling needed to observe the self-organization.

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The search of novel cooling techniques applicable to a wide range of microparticles is still an important problem (see, for example, [1]), as laser cooling is known to be hardly used for complex particles without simple cyclic transitions.

The promising approach is the cavity cooling based on the coupled dynamics of the atoms and the cavity field [2, 3]. It has been shown that in setups with the pumping transverse to the cavity axis the cooling can be accompanied with the formation of the self-organized Bragg grating [4]. The reason of the self-organization is the collective scattering of photons from the transverse pump into the cavity field. The experimental evidence of this effect has been reported [5–9].

The core ingredient of an original self-organization setup is a high-finesse optical cavity capable to provide strong atom-field coupling. Although modern technology allows to build up such a setup, the observation of the self-organization remains technically challenging. In view of this, we propose to supply the original setup with an additional positive feedback loop known to be a standard tool to improve sensitivity of classical electronic devices [10]. We will show below that the feedback helps to reduce the atom-field coupling required to observe the self-organization. One can even think of an alternative setup with no cavity at all, where the self-organization is obtained with the electronic feedback alone.

The application of feedback to control the motion of a single atom has been theoretically discussed [11, 12] and experimentally demonstrated [13, 14]. There are also proposals to control atomic ensembles [15–19]. In all these schemes the feedback-induced back-action noise possesses certain limits to the efficiency of the control [20, 21]. The destructive effect of this noise can be expected when the feedback is applied to an otherwise closed system. The system we are dealing with is essentially open system that includes the cavity field, which is coupled to the vacuum modes of the electro-magnetic field outside the cavity. Thus, the system we would like to improve by the feedback loop is affected by the quantum noise even without the application of the feedback. The feedback is supposed to use the results of the measurements of the field outside the cavity. This information in the standard cavity-induced self-organization setup is simply lost. Thus, the application of feedback even in the quantum regime should not introduce a noise source that would be otherwise absent. These arguments additionally motivate the forthcoming analysis.

Apart from the newly suggested feedback loop the model we are going to investigate is similar to that of the Ref. [23]. An ensemble of cold degenerate bosonic atoms (BEC) is trapped in a quasi-1D potential and placed in an optical cavity, see Fig. 1. Transverse pumping field

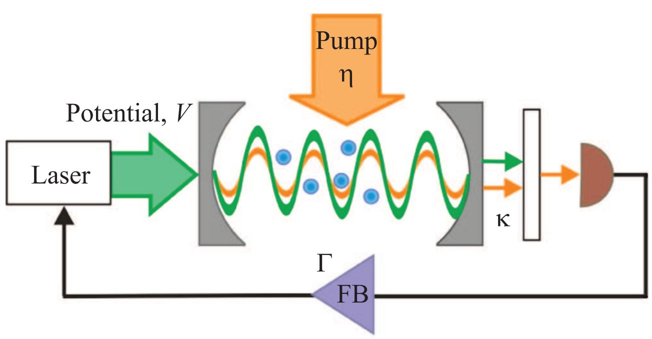

Fig. 1. The conceptual scheme of the self-organization experiment with Feedback

η is applied to the atoms in the cavity. The scattering of the photons from the transverse pump to the cavity

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mode depends on the location of the atoms inside the cavity [4]. On the other hand, the cavity field produces a standing-wave potential for the atoms. Thus, there is a cavity-mediated self-action of the atoms that at certain circumstances results in the self-organization. The Hamiltonian that describes the system is given by

$$H_0 = \hbar\omega_0 a^\dagger a + \int dx \Psi^\dagger(x)\Big[-\frac{\hbar^2}{2m}\partial_x^2 + \frac{\hbar g_0^2}{\Delta}U_0^2(x)a^\dagger a + \\ + \frac{\hbar g_0^2}{\Delta}U_0(x)(\eta^* a + \eta a^\dagger)\Big]\Psi(x). \quad (1)$$

Here, ω_0 is the cavity mode frequency; a is the cavity annihilation operator that obeys commutation relations $\left[a, a^\dagger\right] = 1$; $\Psi(x)$ is the atomic field operator that for bosons has the standard commutator $\left[\Psi(x'), \Psi^\dagger(x)\right] = \delta(x - x')$; m is the atomic mass; g_0 is the atom-field interaction constant; Δ is the atom-cavity detuning; $U_0(x) = \cos(\omega_0 x/c)$ is the cavity mode function; η is the classical amplitude of the transverse pump field. Apart from the Hamiltonian evolution governed by Eq. (1) the system demonstrates irreversible dynamics due to the coupling of the cavity mode to the vacuum modes outside the cavity.

The atom-field coupling g_0 is under normal conditions small compared to other characteristic frequencies of the system. Thus, an experimental observation of the self-organization is non-trivial. To reduce the requirements for the strong atom-field coupling while keeping the intensities of the used lasers within reasonable limits we propose to supply the system with an additional positive feedback loop. The simplest scheme of such a loop would be to measure the cavity photon number and apply to the atoms an additional potential proportional to this result and having the same position dependence $U_0^2(x)$ as the original cavity potential. The example of the setup is shown in Fig. 1. Here, the feedback laser power is controlled by the measurement of the photons inside the cavity. To simplify the treatment of the feedback loop we assume that the light scattered from the transverse pump has slightly different frequency or polarization from the light that produces dipole potential for the atoms. In addition, we assume some trapping potential that ensures transverse confinement and quasi 1D motion.

The feedback is designed to increase the optical potential for the atoms as they start to scatter more pump photons into the cavity mode. This makes the atomic distribution that fulfills the Bragg condition and corresponds to efficient scattering more favorable compared to, for example, uniform distribution. Thus the atoms will gradually self-organize into the Bragg grating. In this paper we consider only linear feedback which means the potential for the atoms is proportional to the number of cavity photons. The dynamics in this case resembles that of the original self-organization setup [4]. In our scheme it is however possible to perform a more general non-linear feedback that might be more efficient. A detailed analysis of this general case we hope to present elsewhere.

The described feedback loop is based on the detection of photons leaking from the cavity. The quantum description of this feedback scheme has been developed in Refs. [24, 25]. According to these references the unconditioned evolution of the quantum state of the atoms-cavity system is described by the following master equation

$$\dot{\varrho} = -\frac{i}{\hbar}\left[H_0, \varrho\right] - \frac{\kappa}{2}\left(a^\dagger a\varrho + \varrho a^\dagger a\right) + \kappa e^{\mathcal{L}\tau} a\varrho a^\dagger. \quad (2)$$

This equation is written in the Markovian limit with κ being the measurement strength, $\mathcal{L}$ being a feedback super-operator acting on the system, and τ being the feedback interaction time. To provide the feedback that changes the atomic potential proportionally to the cavity photon number we assume that the super-operator $\mathcal{L}$ is implicitly given by

$$e^{\mathcal{L}\tau}\varrho = i\frac{g_0^2}{\Delta\kappa}\Gamma\left[V, \varrho\right] - \varrho. \quad (3)$$

The operator V in this equation is the additional feedback-induced potential for the atoms. It is assumed to be of the form of

$$V = \hbar\int dx \Psi^\dagger(x) U_0^2(x)\Psi(x). \quad (4)$$

The parameter Γ in (3) is the gain coefficient of the feedback loop. It describes the depth of the feedback-induced potential for the atoms. Experimentally the feedback-induced potential is realized with an additional laser field that cannot have arbitrary large intensity. Due to this restriction the gain Γ cannot be made very large at all stages of the system evolution. As the atoms become self-organized the feedback signal, though limited by the transverse pump intensity, can become so large that the required potential for the atoms exceeds the capabilities of the available laser. Here we assume that this regime is not reached and the feedback gain Γ can be considered as a constant parameter.

Instead of dealing with the master equation (2) it is reasonable to use the positive P-representation. This approach at least in principal allows for the reformulation of a quantum problem in terms of classical stochastic differential equations (SDE) [22, 26].

For the sake of simplicity we perform a discrete mode expansion of the atom-field operator $\Psi(x)$ as

$$\Psi(x) = \sum_m \phi_m w_m(x). \quad (5)$$

The explicit form of the mode functions $w_m(x)$ will be defined latter. Since the derivation is standard we skip the details as well as omit writing down a lengthy Fokker–Planck equation. Using Einstein summation convention the Ito-type SDE read as

$$d\varphi_i = \left[-\frac{i}{\hbar}K_{ik} - \frac{i}{\hbar}\Pi_{ik}\right]\varphi_k dt -$$
$$- i\sqrt{\frac{g_0^2}{\Delta}}\left(\alpha[U^2]_{ik} + \eta U_{ik}\right)\varphi_k(dW_1 + idW_2),$$
$$d\psi_i = \left[\frac{i}{\hbar}K_{ik} + \frac{i}{\hbar}\Pi_{ik}\right]\psi_k dt +$$
$$+ i\sqrt{\frac{g_0^2}{\Delta}}\left(\beta[U^2]_{ik} + \eta^* U_{ik}\right)\psi_k(dW_3 - idW_4),$$
$$d\alpha = \left[-i\omega\alpha - \frac{\kappa}{2}\alpha - \frac{ig_0^2}{\Delta}\psi_i\left(\alpha[U^2]_{ik} + \eta U_{ik}\right)\varphi_k\right]dt +$$
$$+ \frac{1}{2}\sqrt{\frac{g_0^2}{\Delta}}(dW_1 - idW_2),$$
$$d\beta = \left[i\omega\beta - \frac{\kappa}{2}\beta + \frac{ig_0^2}{\Delta}\varphi_i\left(\beta[U^2]_{ik} + \eta^* U_{ik}\right)\psi_k\right]dt +$$
$$+ \frac{1}{2}\sqrt{\frac{g_0^2}{\Delta}}(dW_3 + idW_4),$$
$$\Pi_{ik} = \frac{\hbar g_0^2}{\Delta}\left(\eta^*\alpha + \eta\beta\right)U_{ik} + \frac{\hbar g_0^2}{\Delta}\left(1 + \Gamma\right)\alpha\beta[U^2]_{ik}. \quad (6)$$

Here φ_i, ψ_i are the pair of phase-space variables corresponding to ith mode of the atomic field, while α and β are the phase-space variables of the cavity field. All these quantities are complex numbers. The various real Wiener increments dW_i are assumed to be independent, satisfying $dW_i dW_j = dt\delta_{i,j}$. The matrix elements of the kinetic energy are given by

$$K_{ik} = \int dx w_i(x)\left[-\frac{\hbar^2}{2m}\partial_x^2\right]w_k(x). \quad (7)$$

Analogously one defines the matrix elements representing the cavity mode function U_{ik} and the square of the mode function $[U^2]_{ik}$, which determines the coordinate dependence of the atomic potential. We assume that the mode functions $w_i(x)$ are real and, as a consequence, the matrices are symmetric.

As is seen from Eq. (6) the noise does not depend on the feedback gain Γ so the feedback does not introduce excess noise into the system. This is though expected but non-trivial result of the paper. The collective nature of the atom-field interaction is reflected by the fact that the same noise drives all the atomic modes and the cavity field.

Now we neglect the noise in Eq. (6) and study semiclassical dynamics. In the absence of the noise that can disrupt the complex conjugate of φ_i and ψ_i as well as α and β we can consider only the equations for φ_i and α, since $\psi_i = \varphi_i^*$ and $\beta = \alpha^*$.

Since the aim of the paper is to demonstrate the principal applicability of the feedback enhancement, we consider below rather rough but simplest approximation. As the first step we specify the mode functions $w_m(x)$ as step functions representing the atoms localized in a spacial domain of a certain width Δx

$$w_i(x) = \left[\Theta(x - x_i) - \Theta(x - x_i - \Delta x)\right]/\sqrt{\Delta x}, \quad (8)$$

where $\Theta(x)$ is the Heaviside step function. The matrix elements of the potential corresponding to the maximum/minimum and $\Delta x = \lambda/4$ will be approximated as $U_{\max}^2 = 1$ and $U_{\min}^2 = 0$.

Furthermore, we restrict the number of the atomic degrees of freedom to only three modes. In other words, we consider three spacial domains: one corresponding to all the odd sites of the cavity potential, another to all the even sites, and the last to the space between the cavity potential minima. The graphical representation of this approximation is shown in Fig. 2, where the

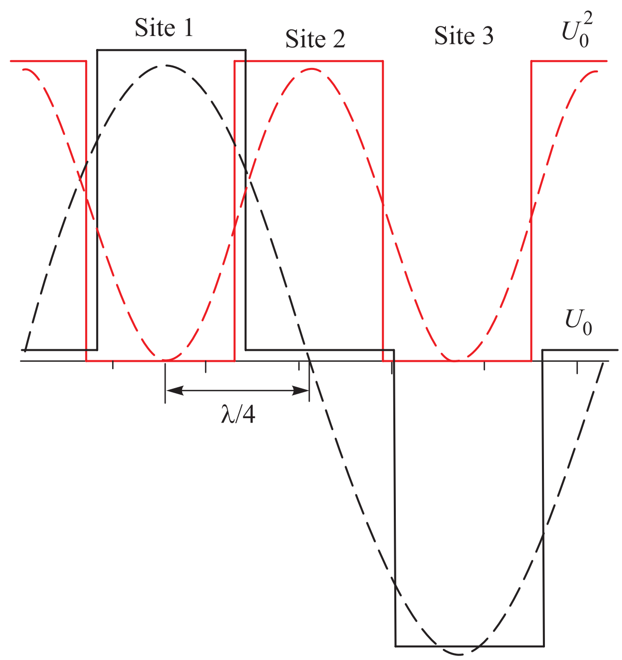

Fig. 2. Three-site approximation for the distribution of the atoms. The approximate stepwise cavity mode function U_0 and the cavity potential U_0^2 (solid curves) are shown and compared with original ones (dashed curves)

stepwise cavity mode function U_0 and the cavity potential U_0^2 corresponding to the made three-site approximation are shown. This approximation greatly simplifies SDE (6), but ignores long-wavelength excitations in BEC. It, however, captures the spatial periodicity of the forces acting on the atoms, thus it should correctly describe the essential features of the atomic motion. We scale the time with the quantity τ that is the time interval required to travel a wavelength with the recoil velocity: $\tau = \lambda^2 m/2\pi\hbar$. Then the combination $\varepsilon \equiv g_0^2\tau/\Delta$ represents the strength of the atom-field coupling with respect to the atomic kinetic energy. The cavity decay rate κ is also scaled as $\kappa \to \kappa/\tau$. To ensure the uniform distribution to be a valid solution we require zero-flux boundary conditions on sites 1 and 3.

The set of evolution equations for the three-mode approximation then reads

$$\begin{aligned} \dot{\varphi}_1 &= i(\varphi_2 - \varphi_1) - i\varepsilon\eta(\alpha + \alpha^*)\varphi_1, \\ \dot{\varphi}_2 &= i(\varphi_1 + \varphi_3 - 2\varphi_2) - i\varepsilon(1+\Gamma)|\alpha|^2\varphi_2, \\ \dot{\varphi}_3 &= i(\varphi_2 - \varphi_3) + i\varepsilon\eta(\alpha + \alpha^*)\varphi_3, \\ \dot{\alpha} &= -\tfrac{\kappa}{2}\alpha - i\varepsilon|\varphi_2|^2\alpha - i\varepsilon\eta(|\varphi_1|^2 - |\varphi_3|^2). \end{aligned} \tag{9}$$

Let us look at the limit of small coupling $\varepsilon \to 0$, as the feedback is assumed to be useful in this regime. It is seen from Eq. (9) that to have non-trivial dynamics in this case one should compensate for small coupling by increasing the feedback gain Γ so that the product $\varepsilon\sqrt{\Gamma+1}$ is kept constant. However, even then the last terms in the first and the third equations of the system (9) vanish. These terms represent the collective recoil of the atoms during the scattering of pump photons into the cavity mode.

As the scattering depends on the position of the atoms the collective recoil [27, 28] may be very important to initiate the self-organization. Another constituent of the atomic dynamics is the motion in the feedback-controlled potential, which is represented by the term proportional to Γ. Below we analyze the role of these effects in the formation of the self-organized state.

Having the three-site model with only four bosonic modes we can perform straightforward numerical analysis of the nonlinear problem. Since the number of degrees of freedom is small the simulations can be done on a conventional workstation computer. In particular, we numerically solve the system Eq. (9) with the help of XMDS2 [29].

We ignore the fact that the intensity of the feedback laser is technically limited and assume that the feedback laser has enough power to provide any required potential amplitude.

We perform simulations for several values of the transverse pump η. For each value of η a series of simulations with different gain Γ is made to approximately determine the feedback gain that corresponds to the self-organization threshold. The numerical results are obtained for $\varepsilon = 1.3\cdot 10^{-3}$, $\kappa = 5\cdot 10^3$, and $N = 10^4$, which should be easily realized in an experiment as they correspond to $\kappa = 38$ MHz, $g_0 = 32$ kHz, and $\Delta = 100$ MHz (compare with $g_0 \approx 68$ MHz in [30]). The example of the simulation results is shown in Fig. 3. The transverse

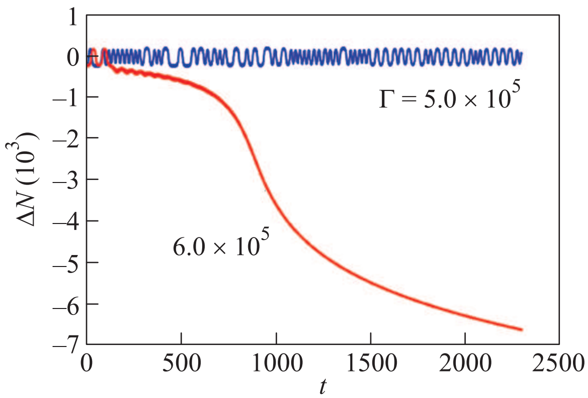

Fig. 3. The atom number difference ΔN between the odd and even sites of the potential for $\Gamma = 5\cdot 10^5$ and $6\cdot 10^5$

pump is taken to be $\eta = 1000$ as it is somewhat below the self-organization threshold without feedback. The figure shows the time dependence of the atom number difference between the odd and even sites ΔN. The upper curve corresponds to the gain $\Gamma = 5\cdot 10^5$ and shows completely oscillatory behavior with uniform (on average) distribution. The lower plot corresponds to $\Gamma = 6\cdot 10^5$ and demonstrates the presence of the organized phase since the field and the number difference approach the steady-state values far from that in the uniform distribution. The atoms start to collect themselves in odd or even potential minima forming a sort of Bragg grating that enhances further light scattering. This is the self-organization.

Similar numerical calculations can be done for smaller values of the transverse pump η. The range of gain values containing the self-organization threshold can be determined in a series of numerical tests. The results of these tests are summarized in Fig. 4. The error bars on the plot represent the range of Γ values containing the threshold value. The important consequence of these results is the possibility to observe the self-organization even for very small pump η by appropriate choice of the gain Γ. Physically, this means that the collective recoil effect, that gets small in this limit, is not necessary for the self-organization to occur. The feedback control of the potential seem to be able to organize the atoms into the Bragg-grating even without the help of collective recoil.

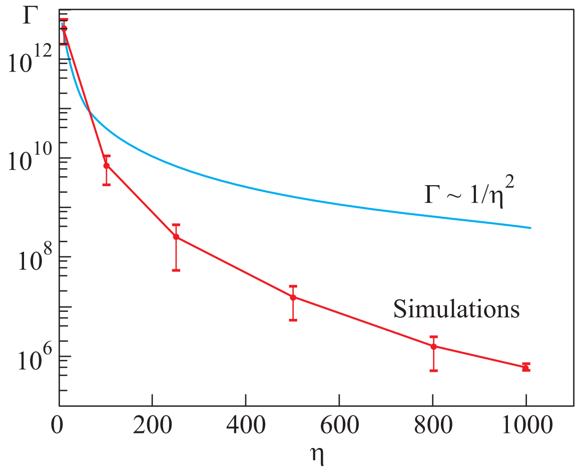

Fig. 4. The lower curve represents the estimation of the threshold gain Γ as a function of pump η based on the simulations results. The upper curve is $\Gamma \sim 1/\eta^2$, that corresponds to the dependence of the threshold gain value on the pump without collective recoil

On the other hand, the recoil effect considerably stimulates the transition to the self-organized phase. Artificially omitting the collective recoil effect by removing the corresponding terms from Eq. (9) one can see that the single parameter that describes the system can be introduced. This parameter is the combination $\varepsilon\eta\sqrt{\Gamma}$. Thus the threshold in the absence of collective recoil should depend on the whole combination. Then the dependence of the threshold value of the gain Γ on η should be of the form $\Gamma \sim 1/\eta^2$. This dependence is shown in Fig. 4 with the dashed curve. The simulation results that take the collective recoil into account, solid line with error bars in Fig. 4, considerably deviate from the dashed curve for large transverse pump η. One sees that smaller gain Γ is required if the collective recoil effect is strong. For very small η the collective recoil is negligible and the simulated dependence $\Gamma(\eta)$ approaches the simple limit $1/\eta^2$.

In conclusion, we propose to use feedback to enhance the self-organization of the atoms in a cavity-induced dipole potential. The feedback is based on the measurement of the field leaking the cavity and the appropriate change of the cavity-induced potential. Starting with the fully quantum theory we identify quantum noise sources that appear to be independent on the feedback gain. Then we performed numerical simulations in the classical approximation with only three atomic modes, corresponding to odd and even potential wells and the intermediate space.

Numerical simulations showed that the feedback allows to reach atomic self-organization in the cases where it is otherwise impossible. Furthermore, numerical tests made for different transverse pump amplitude η showed that providing necessary feedback gain the self-organization can be observed even for rather small transverse pump. This indicates that the effect of the atomic collective recoil is not crucial for the self-organization.

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