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The robust impact parameter profile of inelastic collisions

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It is shown that the impact parameter profile of inelastic hadron collisions is robust to admissible variations of the shape of the diffraction cone of elastic scattering. This conclusion is obtained using the unitarity condition and experimental data only with no phenomenological model inputs.

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The impact parameter profile of inelastic high energy hadron collisions is determined as the probability of such reactions to take place at definite impact parameters at a given energy (see, e.g., [1]). It can be derived from the unitarity condition if the properties of the elastic scattering amplitude are known. We show that its general features are robust to variations of the shape of the differential cross section of elastic scattering with the transferred momentum and total energy measured experimentally.

The impact parameter profiles of elastic and inelastic hadron collisions are not directly measurable but they help us visualize the geometrical picture of partonic interactions indicating their space extension and the intensity. Our intuitive guesses about the space-time development of these processes can be corrected in this way. The inelastic profile $G(s,b)$ is a function of the energy $s=4E^2$, where E is the total energy of colliding particles in the center of mass system, and of the impact parameter b, which represents the transverse distance between their centers. It is determined from the unitarity condition in a following way

$$G(s,b) = 2\mathrm{Re}\,\Gamma(s,b) - |\Gamma(s,b)|^2, \tag{1}$$

where

$$i\Gamma(s,b) = \frac{1}{2\sqrt{\pi}}\int_0^\infty d|t| f(s,t) J_0(b\sqrt{|t|}) \tag{2}$$

is the elastic profile defined by the Fourier–Bessel transform of the elastic scattering amplitude $f(s,t)$ which depends on energy and the transferred momentum squared

$$-t = 2p^2(1-\cos\theta) \tag{3}$$

with θ denoting the scattering angle in the center of mass system and p the momentum.

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The left-hand side of (1) called the overlap function describes the impact parameter profile of inelastic collisions of protons. Its widths shows the spatial extension of the region of inelastic interactions. It satisfies the inequalities $0 \le G(s,b) \le 1$ and determines how absoptive is the interaction region depending on the impact parameter (with $G=1$ for full absorption). If integrated over the impact parameters, (1) leads to the general statement that the inelastic cross section equals to the difference of the total and elastic cross sections.

The differential cross section of elastic scattering $d\sigma/dt$ measured in experiments is related to the scattering amplitude $f(s,t)$ in a following way

$$\frac{d\sigma}{dt} = |f(s,t)|^2. \tag{4}$$

The shape of $d\sigma/dt$ varies with energy. However, there are some common features typical at high energies. Particles are elastically scattered mostly at small transferred momenta within the so-called diffraction peak. It is roughly approximated by the exponential shape

$$\frac{d\sigma}{dt} \propto e^{-B|t|} \tag{5}$$

with the slope B depending on energy s and slightly varying with the transferred momentum t.

Moreover, the real part of the amplitude is small compared to the imaginary part within the diffraction cone at high energies. At the LHC, their ratio in forward direction ρ_0 is equal to 0.1 [2]. It decreases within the cone and crosses the abscissa axis according to all phenomenological models and general statements of Ref. [3]. That is why it is possible to neglect this ratio in Eq. (2) where it enters weighted by the suppressing exponential factor. The corresponding corrections to $G(s,b)$ are quadratic in ρ. Surely, they are smaller than one percent and will not be considered in what follows.

If one neglects for some time by the dependence B on t, the inelastic profile looks as

$$G(s,b) = \frac{2}{Z} \exp\left(-\frac{b^2}{2B}\right) - \frac{1}{Z^2} \exp\left(-\frac{b^2}{B}\right), \qquad (6)$$

where $Z = 4\pi B/\sigma_t$.

It is important that at any high energy from ISR to LHC the differential cross section becomes 4 or 5 orders of magnitude smaller before the exponential regime (5) is replaced by another slower decreasing behavior at larger transferred momenta (the Orear region). Therefore, the role of this tail is negligible for the profile $G(s,b)$ since its contribution to the integral in $\Gamma(s,b)$ (2) is extremely small.

The variations of the slope within the diffraction cone can be only important. As was observed in experiments, they are twofold. The slope itself can change its value with the transferred momentum or/and there appear some oscillations imposed over its smooth shape. At ISR energies, it was shown [4–7] that the slope becomes smaller at $|t| > (0.12-0.15)\,\mathrm{GeV}^2$ and the exponent in (5) can be approximated more accurately by $Bt + Ct^2$ with positive C or by the sum of two exponential terms with exponents differing by about $1.5\,\mathrm{GeV}^{-2}$. The accuracy of the data is not enough to distinguish between these fits. At LHC energies, the slope becomes larger at $|t| > 0.36\,\mathrm{GeV}^2$ [8] so that $C < 0$ or, in the case of two exponential terms, the exponents differ approximately by the same amount but with the opposite sign. Anyway, the impact of these variations on the inelastic profile at the LHC is very small as shown in Fig. 1a of Ref. [9] where its shapes are calculated either directly from experimental data or from their simple approximation by (5). They are almost indistinguishable.

Another interesting feature of the slope behavior was studied at the energy $\sqrt{s} \approx 11\,\mathrm{GeV}$ in Refs. [10, 11] reviewed in Ref. [12]. Slight oscillations with t in the behavior of B at the level of 5–10 % were noticed. Some decline from the simple exponential form can be also seen at ISR energies if carefully studied. It is intended to be studied with more precision again at Protvino energies about 11 GeV. This effect should be looked for at the LHC energies as well.

The corrections ΔG to the profile $G(s,b)$ are connected with the corrections $\Delta\Gamma$ to Γ in a following way

$$\Delta G(s,b) = 2\Delta\Gamma(1-\Gamma) = 2\Delta\Gamma \left[1 - \frac{1}{Z}\exp(-b^2/2B)\right]. \qquad (7)$$

At the LHC, where $Z = 1$, no corrections appear at the center $b = 0$ but all of them are shifted to the tail of the impact parameter distribution. That shows their peripheral origin.

The impact of oscillations on the behavior of $G(s,b)$ can be estimated if we approximate this decline by the simplest oscillating function inserted in (2), (1) so that $G(s,b)$ is changed by the amount

$$\Delta G(s,b) = a\left[1 - \frac{1}{Z}\exp(-b^2/2B)\right] \times$$
$$\times \int_0^\infty d|t| \exp(Bt/2) J_0(b\sqrt{|t|}) \cos[\kappa(|t| - |t_0|)] =$$
$$= a\left[1 - \frac{1}{Z}\exp(-b^2/2B)\right] \frac{2B}{B^2 + 4\kappa^2} \times$$
$$\times \exp\left[\frac{-b^2 B}{2(b^2 + 4\kappa^2)}\right] \left(\cos u - \frac{\kappa}{2B}\sin u\right), \qquad (8)$$

where $u = \kappa(|t_0| + \frac{b^2}{B^2 + 4\kappa^2})$. The amplitude a, positions of zeros and period of oscillations are estimated from approximations of Figures shown in Refs. [10–12]. They are $a \approx 0.1$; $x_0^2 \approx 0.07\,\mathrm{GeV}^2$; $\kappa \approx 5\pi\,\mathrm{GeV}^{-2}$. These corrections to the inelastic profile at the LHC with $B \approx 20\,\mathrm{GeV}^{-2}$ are of the order of one percent or even less. They can reveal themselves at very high impact parameters where the profile values are small. The oscillations were ascribed in Ref. [13] to the inelastic diffraction processes possessing the peripheral origin.

Thus, the main structure of the inelastic profile in proton collisions remains quite intact. Its general feature at LHC energies is the widely spread black region at $b \le 0.5\,\mathrm{Fm}$ which reveals itself in properties of jets produced in very high multiplicity events [9]. Even though the corrections are small, the fine structure of the profile should be further studied. It can open ways to identification of various classes of inelastic processes with different regions of impact parameters. More precise data about the substructure of the diffraction peak in t-variable are necessary to relate them with inelastic processes of different kinds.

Let us stress once more that the unitarity condition in combination with experimental data about elastic scattering within the diffraction cone was only used without any reference to QCD ideas or phenomenological models.

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