Electromagnetic production of electron and positron in collisions of heavy nuclei

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We consider the cross-section of electromagnetic production of e^+e^- pair in adiabatic scattering of heavy nuclei.

It is well-known that for a point-like nucleus of charge Z the ground state energy [1–3]

$$\varepsilon_0 = m\sqrt{1 - Z^2\alpha^2} \tag{1}$$

of a single-electron ion turns to zero for $Z = Z_c = 137$; here and below m is the electron mass, $\alpha = 1/137$. The account for the final size of nucleus shifts the critical value to $Z_c \simeq 170$ [4, 5]. At higher Z, such superheavy nucleus gets unstable with respect to the decay

$$Z \to Z + e^- + e^+, \tag{2}$$

as well as to the decay into single-electron ion (Ze^-) with subcritical charge $Z - 1$ and positron [6]:

$$Z \to (Ze^-) + e^+. \tag{3}$$

Of course, the creation of such superheavy nuclei by itself does not look realistic. Still, the phenomenon could be observed under the adiabatic rapprochement of two sufficiently heavy common nuclei [7, 8]. Calculations performed in Ref. [8] result in the estimate

$$\sigma_P \sim 10^{-25}\ \text{cm}^2 \tag{4}$$

for the cross-section of positron production in collisions of uranium nuclei. As to the electron produced simultaneously, it is captured here by one of the nuclei. This effect is essentially nonperturbative.

In fact, the rapprochement of two sufficiently heavy nuclei is accompanied also by the production of e^+e^- pairs due to rather common QED effects. We analyze these effects, and demonstrate that they are quite comparable with (4).

So, let us address the diagram presented in Figure. Double lines therein describe the propagation of heavy

nuclei, single and wavy lines refer to $e^\pm$ and photons, respectively. The essential property of this diagram is that it does not vanish in the limit of heavy mass of the nuclei, i.e. for $M \to \infty$.

The corresponding cross-section looks as follows:

$$d\sigma = 32\,\frac{Z^4\alpha^4}{\pi^4}\,v^3\,\frac{d^3p_-}{2p_{-0}}\,\frac{d^3p_+}{2p_{+0}}\,\delta(p_{-0} + p_{+0} - \varepsilon) \times$$

$$\times \int \frac{d^3q_1}{\bar{q}_1^{\,4}}\,\frac{d^3q_2}{\bar{q}_2^{\,4}}\,\delta(\bar{q}_1 + \bar{q}_2 + \bar{p}_- + \bar{p}_+) \times$$

$$\times \frac{\mathrm{Sp}\,\gamma_3(\hat{p}_- + m)\gamma_3(\hat{p}_- - \hat{q}_1 + m)\gamma_3(\hat{p}_+ - m)\gamma_3(\hat{p}_+ + \hat{q}_2 + m)}{(\bar{q}_1^2 - 2\bar{q}_1\bar{p}_-)(\bar{q}_2^2 + 2\bar{q}_2\bar{p}_+)}. \tag{5}$$

Here $\bar{p}_-$, $\bar{p}_+$ are the three-dimensional components of the momenta of produced electron and positron; p_{-0}, p_{+0} are the time components of these momenta; ε is the total energy extracted by the created electron–positron pair from the colliding heavy nuclei. The virtual quanta $\bar{q}_{1,2}$, emitted by the nuclei, produce then the e^+e^- pair. In particular, these vector fields enter as $1/\bar{q}_{1,2}^2$ the amplitude of the discussed process, and as $1/\bar{q}_{1,2}^4$ its cross-section. As to the velocity of colliding nuclei, we assume $v \simeq 0.1$; we assume also $Z = 92$.

To our approximation, the heavy nuclei can be treated classically. We assume that their velocities v at the moment of the maximum rapprochement are comparable to those at infinity. The photons (wavy lines) are produced here by electromagnetic currents of heavy nuclei, i.e., by Zev. One more power of e is due to the charges of the created electron and positron. This flux of the colliding nuclei is proportional to v, and enters the result in the denominator. This is the origin of factor $Z^4\alpha^4v^3$ in formula (5).

As to rather tedious spinor structure

$$\gamma_3(\hat{p}_- + m)\gamma_3(\hat{p}_- - \hat{q}_1 + m)\gamma_3(\hat{p}_+ - m)\gamma_3(\hat{p}_+ + \hat{q}_2 + m) \tag{6}$$

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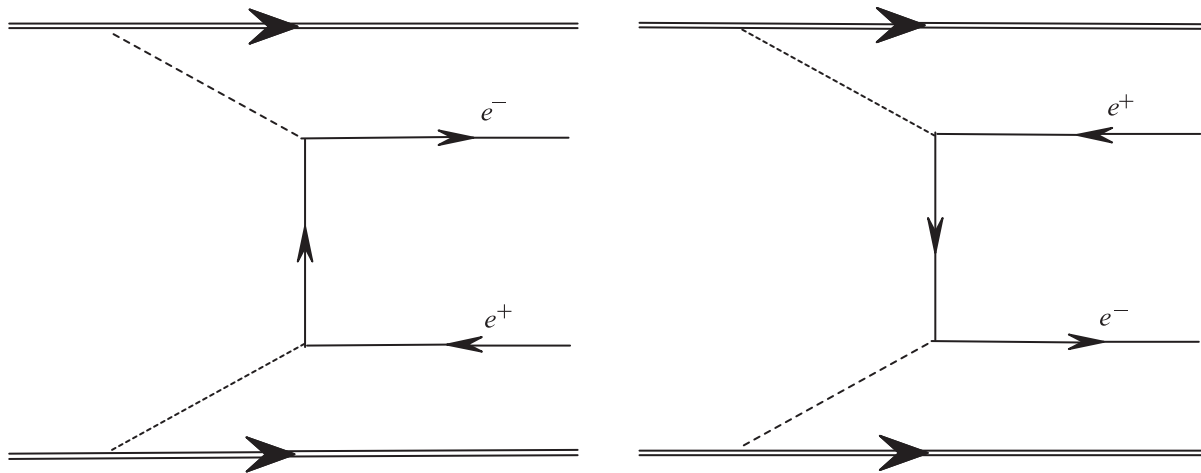
in formula (5), with eight γ-matrices and common momentum operators $\hat{p}_{\pm} = \gamma_0 p_{\pm 0} - \gamma_1 p_{\pm 1} - \gamma_2 p_{\pm 2} - \gamma_3 p_{\pm 3}$, it corresponds to the situation when the velocities of nuclei are aligned along the 3 axis. This structure can be rewritten as

$$(\tilde{p}_- + m)(\hat{p}_- - \hat{q}_1 + m)(\tilde{p}_+ - m)(\hat{p}_+ + \hat{q}_2 + m), \quad (7)$$

with four γ-matrices and

$$\tilde{p}_{\pm} = \gamma_0\, p_{\pm 0} - \gamma_1\, p_{\pm 1} - \gamma_2\, p_{\pm 2} + \gamma_3\, p_{\pm 3}. \quad (8)$$

Let us consider now the integration range for the momenta $\hat{q}_{1,2}$. The energy carried by each of these momenta is obviously

$$\bar{p}^2/2M - (\bar{p} - \bar{q}_{1,2})^2/2M \simeq (\bar{p}\bar{q}_{1,2})/M = \bar{v}\bar{q}_{1,2}. \quad (9)$$

Then, the total energy squared of the two quanta equals $[\bar{v}(\bar{q}_1 + \bar{q}_2)]^2$, and their total momentum squared is $(\bar{q}_1 - \bar{q}_2)^2$. The invariant mass of the two virtual photons, and therefore of the produced pair e^+e^-, is

$$\xi = [\bar{v}(\bar{q}_1 + \bar{q}_2)]^2 - (\bar{q}_1 - \bar{q}_2)^2 =$$
$$= [\bar{v}(\bar{p}_- + \bar{p}_+)]^2 - (\bar{q}_1 - \bar{q}_2)^2 \geq 4m^2, \quad (10)$$

i.e., for the real creation of the pair e^+e^-, this invariant should exceed $4m^2$. To simplify the further analysis, we average Eq. (10) over the orientation of the total momentum $\bar{p}_- + \bar{p}_+$ of the pair with respect to the velocity $\bar{v}$ of the nuclei. It results in

$$\xi = \frac{1}{3}\, v^2(\bar{p}_- + \bar{p}_+)^2 - (\bar{q}_1 - \bar{q}_2)^2 \geq 4m^2. \quad (11)$$

Thus, the total momentum squared of the e^+e^- pair is bounded from below as follows:

$$(\bar{p}_- + \bar{p}_+)^2 > (3/v^2)\, 4m^2 = 12\, m^2/v^2. \quad (12)$$

In other words, the typical momenta $(3-4)\, m/v$ of thus produced electrons and positrons are about 15 MeV. With such high momenta, we can neglect in our estimates not only the Coulomb final state interaction between the electron and positron, but also the Coulomb final state interaction of e^+e^- with the nuclei.

Let us estimate now the total cross section of e^+e^- production in the collision of heavy ions. Just for dimensional reasons, this cross section should be proportional to the ratio $v^2/12m^2$. Then, the total cross section should obviously contain the factor $32(Z\alpha)^4v^3/\pi^4$ and, as it follows from formula (5), factors $(4\pi/2)^2 = 4\pi^2$ from $(d^3p_-/2p_{-0})(d^3p_+/2p_{+0})$, and 4π from d^3q_1 (one power of 4π disappears here, together with $\delta(\bar{q}_1 + \bar{q}_2 + \bar{p}_- + \bar{p}_+)$). In this way we arrive at the following estimate for cross-section (5):

$$\sigma = 32(Z\alpha)^4(v^3/\pi^4)\,(4\pi/2)^2\,(4\pi)\,(v^2/12m^2) =$$
$$= \frac{128}{3\pi}(Z\alpha)^4v^5/m^2 \sim 0.4 \cdot 10^{-25}\ \text{cm}^2. \quad (13)$$

This result for the e^+e^- production is on the same order of magnitude as the estimate $\sigma_P \sim 10^{-25}$ cm^2 made in [8] for the positron production cross-section, with the electron captured by one of the nuclei.

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