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Chaos induced by coupling between Josephson junctions

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It is found that, in a stack of intrinsic Josephson junctions in layered high temperature superconductors under external electromagnetic radiation, the chaotic features are triggered by inter-junction coupling, i.e. the coupling between different junctions in the stack. While the radiation is well known to produce chaotic effects in the single junction, the effect of inter-junction coupling is fundamentally different and it can lead to the onset of chaos via a different route to that of the single junction. A precise numerical study of the phase dynamics of intrinsic Josephson junctions, as described by the CCJJ+DC model, is performed. We demonstrate the charging of superconducting layers, in a bias current interval corresponding to a Shapiro step subharmonic, due to the creation of a longitudinal plasma wave along the stack of junctions. With increase in radiation amplitude chaotic behavior sets in. The chaotic features of the coupled Josephson junctions are analyzed by calculations of the Lyapunov exponents. We compare results for a stack of junctions to the case of a single junction and prove that the observed chaos is induced by the coupling between the junctions. The use of Shapiro step subharmonics may allow longitudinal plasma waves to be excited at low radiation power.

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In high temperature superconductors such as $Bi_2Sr_2CaCu_2O_{8+\delta}$ (Bi-2212) the superconducting layers form intrinsic Josephson junctions (IJJs). A stack of coupled Josephson junctions shows more complex physics in comparison to the case of a single junction [1–3]. Chaos in such systems of coupled, identical oscillators has also attracted a great deal of recent theoretical and experimental interest (see, for example, Refs. [4, 5]). In such a state, different groups of oscillators can exhibit coexisting synchronous and incoherent behaviors (chimera states) despite homogeneous coupling. Probably, the IJJ which use for their description a model of coupled Josephson junctions may be an appropriate material for the experimental testing of this phenomenon. There is also a possibility to achieve so-called chaotic synchronization in these systems, i.e. when all the oscillations are synchronized and oscillating chaotically at the same time. The latter effect may have several applications of its own, including for secure communication, as it was recently discussed in Ref. [6].

The external radiation leads additionally to a series of novel effects related to the coupling between junctions, parametric instabilities and the excitation of a longitudinal plasma wave (LPW) propagating along the stack (c-axis) [7–9]. In general, the phase-locking of the Josephson oscillations with frequency ω_J to the frequency ω of external electromagnetic radiation leads to the appearance of Shapiro steps (SS) and their subharmonics (SSS) in the current voltage characteristics (IV-characteristics) [10]. Many devices in existence are based on traditional superconductors which exploit this effect [11]; notably voltage standards and terahertz radiation emitters/detectors [1]. Therefore, a detailed study of the SS and SSS in the intrinsic JJs under different resonance conditions presents important research questions with various potential applications. From the point of view of practical applications, a great number of junctions is often needed. For example, in a quantum voltage standard, thousands of junctions in series are required to obtain Shapiro steps at sufficiently high voltages up to 10 V [12]. The IJJs offer a new way to realize a large number of junctions in a very compact way [2, 13, 14]. As was mentioned in Refs. [15, 13], compared to the well-developed low-Tc dc voltage standards, a few advantages can be found using IJJs: (i) high operation temperatures, up to temperatures near 50 K; (ii) operation at higher frequencies up to some THz; and (iii) high density of junctions due to naturally compact atomic scale, which may simplify the instrumentation significantly.

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In this paper we present results on the effects of electromagnetic radiation on the phase dynamics of the intrinsic JJs and the temporal oscillations of the electric charge in the superconducting layers. For fixed parameters of JJs and parameters of simulations, we have studied the "charging" of the Shapiro step subharmonics in the $I-V$-characteristics, i.e. the charging of superconducting layers in the corresponding bias current intervals at fixed amplitude of external radiation. To escape the complexity related to the overlapping of the SS subharmonics, we consider here cases of small radiation amplitudes. With increase in the amplitude of the radiation a chaotic behavior is demonstrated. Comparison of the obtained results for stack of junctions with a case of single Josephson junction allows us to prove that the observed chaos is triggered by the coupling between junctions. An essential difference of the parametric instabilities in case of subharmonics and Shapiro step harmonics is a lower amplitude of radiation needed for the creation of the longitudinal plasma wave. This fact gives a unique possibility to create and control the longitudinal plasma wave in layered superconductors. It might be important for future applications.

In Ref. [9] it was shown that the variation of the amplitude and frequency of external electromagnetic radiation changes the wavelength of the longitudinal plasma in high temperature superconductors. This sensitivity to the external radiation might play an important role for the synchronization of Josephson oscillations in the stack [16] and paves the way for increasing the power of electromagnetic radiation in the terahertz region. However, there are still many open questions concerning the excitation of the longitudinal plasma wave. One such question, which requires further investigation, is related to the possibility of exciting the LPW at low power of radiation. Here we demonstrate that the use of SS subharmonics is one way to realize this possibility. For our calculations we used the well-known methods described in Ref. [17, 18].

To calculate the $I-V$-characteristics of the stack of the intrinsic JJ, we solve the system of nonlinear second-order differential equations (the one-dimensional CCJJ+DC model) for gauge-invariant phase differences $\varphi_l(t)$ between S-layers l and $l+1$ in the presence of electromagnetic irradiation:

$$\begin{cases} \dfrac{\partial \varphi_l}{\partial t} = V_l - \alpha(V_{l+1} + V_{l-1} - 2V_l), \\ \dfrac{\partial V_l}{\partial t} = I - \sin\varphi_l - \beta\dfrac{\partial \varphi_l}{\partial t} + A\sin\omega t + I_{\text{noise}}, \end{cases} \tag{1}$$

where t is the dimensionless time normalized to the inverse plasma frequency ω_p^{-1}, $\omega_p = \sqrt{2eI_c/\hbar C}$, C is the capacitance of the junctions, $\beta = 1/\sqrt{\beta_c}$, β_c is the McCumber parameter, α gives the coupling between junctions [7], and A is the amplitude of the radiation. To reflect the experimental conditions we add noise in the bias current. The noise is produced by random number generator (white noise) and its amplitude is normalized to the critical current value I_c [18].

In our simulations the fourth (also fifth) order Runge–Kutta method was used. We measure the voltage in units of $V_0 = \hbar\omega_p/(2e)$, the frequency in units of ω_p, the bias current I and the amplitude of radiation A in units of I_c. Numerical calculations have been done for a stack with the coupling parameter $\alpha = 0.05$, dissipation parameter $\beta = 0.2$ and periodic boundary conditions. We use the relatively small value of $\alpha = 0.05$ in our simulations to capture the principal features of the observed effect. Analysis of capacitive coupling parameter values in HTSC was presented in Ref. [19]. Estimations for BSCCO give alpha in the interval 0.05–1. Particularly, the value of $\alpha = 0.1$ was used in Ref. [20]. Coupling between junctions leads to the branch structure in $I-V$-characteristics of IJJs which is observed in the experiments [1, 2]. We note that the qualitative results are not very sensitive to these parameter values and boundary conditions. The details of the model and simulation procedure are presented in Refs. [17, 18].

The simulated $I-V$-characteristics of the stack with 10 JJs under radiation with frequency $\omega = 2$ for amplitude $A = 0.18$ (*1*) and 0.27 (*2*) are presented in Fig. 1a. The dissipation parameter $\beta = 0.2$ means that the junctions are underdamped, so the $I-V$-characteristics show large hysteresis. We note that the return current to the zeroth voltage state at $A = 0.27$ is larger than at $A = 0.18$ reflecting the decrease of hysteresis under radiation. Filled arrows indicate the end of the outermost branch of $I-V$-characteristics (all junctions are in rotating states [7]) and transition to the inner branches. We see the main SS on the outermost branch at $V = 20$ and few SS in inner branches.

There is also a manifestation, in the outermost branch of $I-V$-characteristics, of a subharmonic shown by a hollow arrow and indicated as SSS. To see it clearly, we first enlarge, in Fig. 1b, that part of $I-V$-characteristics marked by dashed rectangle in Fig. 1a, and then, in the inset of Fig. 1b, we enlarge that part surrounding the SS subharmonic. We use the separate axes for voltage in each $I-V$-characteristic to shift the curves and show clearly the features of SS subharmonic. Its voltage value $V \simeq 13.33$ indicates that it is a 2/3 SS subharmonic. We see that the subharmonics at $A = 0.27$ is fragmented possibly signaling the emergence of chaotic behavior in the stack. This frag-

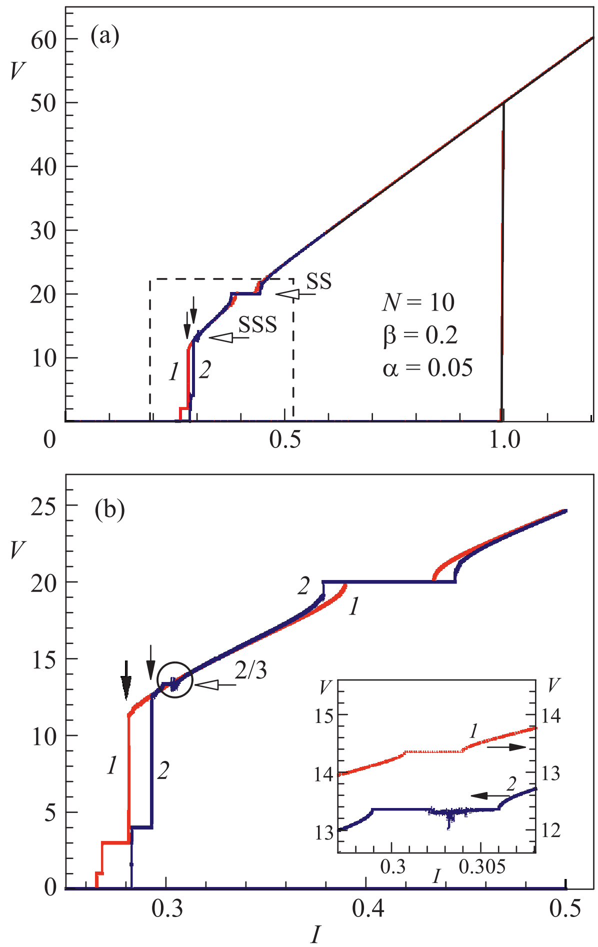

Fig. 1. (Color online) (a) – $I{-}V$-characteristics of the stack with 10 JJs at $\omega = 2$ and two different amplitude of radiation: $A = 0.18$ (*1*) and 0.27 (*2*). Filled arrows indicate the end of the outermost branch of $I{-}V$-characteristics, hollow ones indicate position of SS at $V = 20$ and SS subharmonic at $V \simeq 13.33$. (b) – Enlarged part of $I{-}V$-characteristics marked in a by dashed rectangular. Inset demonstrates enlarged part around of the 2/3 Shapiro step subharmonic. We use separate axes for voltage in each $I{-}V$-characteristic to shift the curves and show clearly the features of SS subharmonic at different amplitudes

mentation of the SS also occurs in the case of single JJ under high amplitude external radiation. To investigate the effect of coupling between junctions on this phenomenon, we have performed detailed simulations of the $I{-}V$-characteristics at different amplitude of radiation and analyzed the charging of superconducting layers in the current interval corresponding to the SS subharmonics.

The appearance of charge in the superconducting layers in the current interval corresponding to the SS harmonics was studied in Ref. [9]. However, the peculiarities of charging of the superconducting layers in the current interval corresponding to the SS subharmonics have not yet been studied.

The electric charge in the superconducting layer [18, 21] is determined by the difference between the voltages V_l and V_{l+1} in the neighbor insulating layers $Q_l = Q_0\alpha(V_{l+1} - V_l)$, where $Q_0 = \varepsilon_r\varepsilon_0 V_0/r_D^2$, ε_r is the relative permittivity, and ε_0 is the permittivity of free space. In the presented results we have normalized Q to Q_0. In Fig. 2 we demonstrate the appearance and growth of charge in the first superconducting layer of the stack, within a current interval corresponding to the subharmonic step 2/3, with an increase in the amplitude of radiation. In the other layers the appearance and growth of the charge is similar. We show charge-time dependence together with the $I{-}V$-characteristics at frequency $\omega = 2$ and four different values of amplitude Ae: 0.18 (a); 0.19 (b); 0.24 (c); 0.27 (d). At $A = 0.18$ (Fig. 2a) the charge is still at the noise level, but at $A = 0.19$ its value is much larger: $Q \simeq 0.1$. The character of charge oscillations is demonstrated in the inset. With increase in A the value of charge grows and it eventually occupies the total current interval corresponding to the SSS. We see this in Fig. 2d, which shows the result for $A = 0.27$. At this largest A we see fragmentation of the SSS, as mentioned above.

To confirm the chaotic behavior of the stack of Josephson junctions at $A = 0.27$, we calculate the Lyapunov exponent spectrum of the system, which consists of $2N$ coupled nonlinear equations. The Lyapunov exponent spectrum was calculated by using the algorithm that was originally proposed by Benettin et al. [22].

The simulated LE are presented in Fig. 3a which shows all non-trivial LEs together with $I{-}V$-characteristic as functions of the dc-bias current for the stack with 10 JJs at $\omega = 2$ and $A = 0.27$ around the subharmonic step 2/3. The LEs indicate the position of bifurcations (when they touch zero) and chaos (hyperchaos) when one (or more) becomes (become) positive. We see that one portion of the step 2/3 in $I{-}V$-characteristics is associated with hyperchaotic dynamics in the coupled junctions. Three LEs (blue, red, and green online) are positive within a subinterval of the current interval corresponded to this SS subharmonic.

The smooth curves seen in the non-chaotic regions of bias current correspond to the steps in the $I{-}V$-characteristic and the number of curves coincides with the denominator of the related term in the continued fraction sequence [23]. This feature reflects the origin of the 2/3 subharmonic, which appears when $3\omega_J = 2\omega$. Thus the step occurs when the Josephson frequency is such that 2 cycles of the phase correspond precisely to

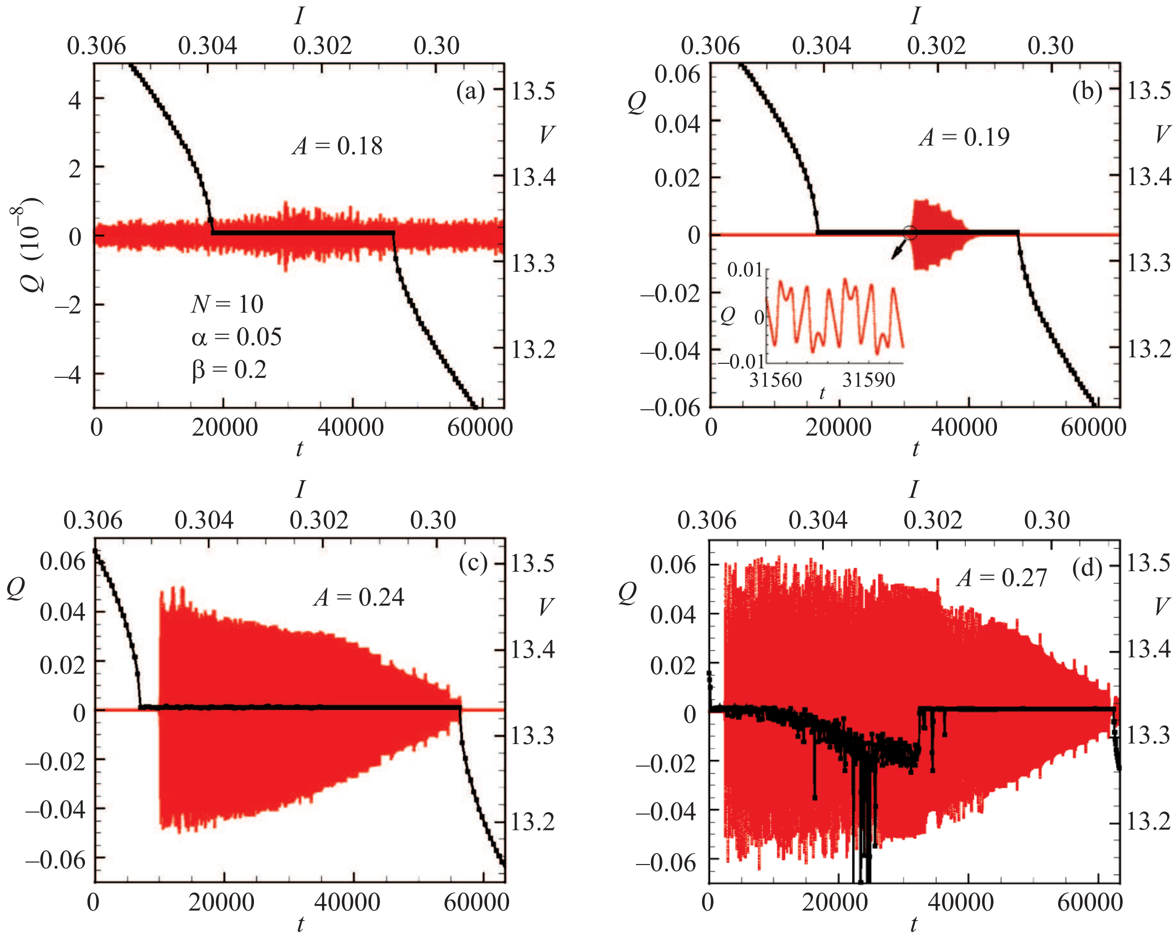

Fig. 2. (Color online) Charge-time dependencies (left and lower axes) together with $I-V$-characteristics (black curves, right and upper axes) demonstrating the appearance and growth of the charge Q in the first superconducting layer, within the current interval corresponding to the 2/3 step with increasing amplitude of radiation A: 0.18 (a); 0.19 (b); 0.24 (c); 0.27 (d). The inset to b demonstrates the character of the charge oscillation. Fig. d shows a manifestation of the chaotic features in the $I-V$-characteristic

3 cycles of the external radiation. The detailed investigations of the route to chaos and its features of coupled Josephson junctions in case of SS subharmonics will be done elsewhere.

To explain the obtained results, we have repeated the simulations for the case of a single JJ with the same parameters.

Fig. 4a shows the $I-V$-characteristic of the single JJ at $\omega = 2$ and $A = 0.3$. In difference with Fig. 1 it shows clearly the step at $V = 1$, which corresponds to the SS subharmonics 1/2 and was not fixed there because of branching of stack $I-V$-characteristic and decrease of hysteresis region. The circle marks the part of $I-V$-characteristic related to the step 2/3.

We have investigated the effect of the amplitude increase to see if we observe the same chaotic features in the JJ at $A = 0.27$ as was registered for the stack. In Fig. 4b we show the parts of $I-V$-characteristics of single JJ at five different amplitudes of radiation, up to $A = 0.5$. In all cases the chaos is absent in the current interval corresponded to the 2/3 SS subharmonic. But, as we have demonstrated above, in the case of stack of coupled JJs chaos manifested itself clearly at $A = 0.27$. So, we draw the conclusion that we have observed the effect of coupling, because the coupling between JJs is the main difference between stack and single JJ (or system of uncoupled JJs). This influence of coupling between JJs in the stack on chaos appearance is studied more detailed below.

From the results presented in Fig. 4 it follows that we can drive our system to chaos, or remove chaos, by increasing or decreasing the coupling parameter. I.e., the chaotic features of the stack of Josephson junctions might be triggered by coupling between junctions.

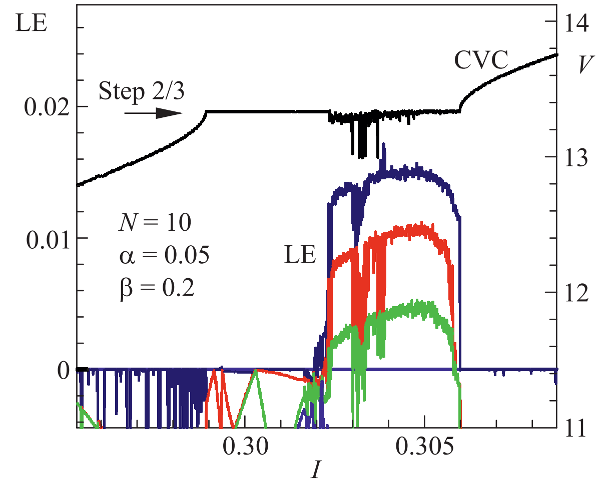

Fig. 3. (Color online) Lyapunov exponents (LE, left and lower axes) and $I{-}V$-characteristic (CVC, right and lower axes) of the stack with 10 JJs at $\omega = 2$ and $A = 0.27$ around of the Shapiro step subharmonic 2/3. The regions in bias current for which the LE become positive correspond to regions over which the phase-locked step in the underlying $I{-}V$-characteristic have been replaced by chaotic dynamics

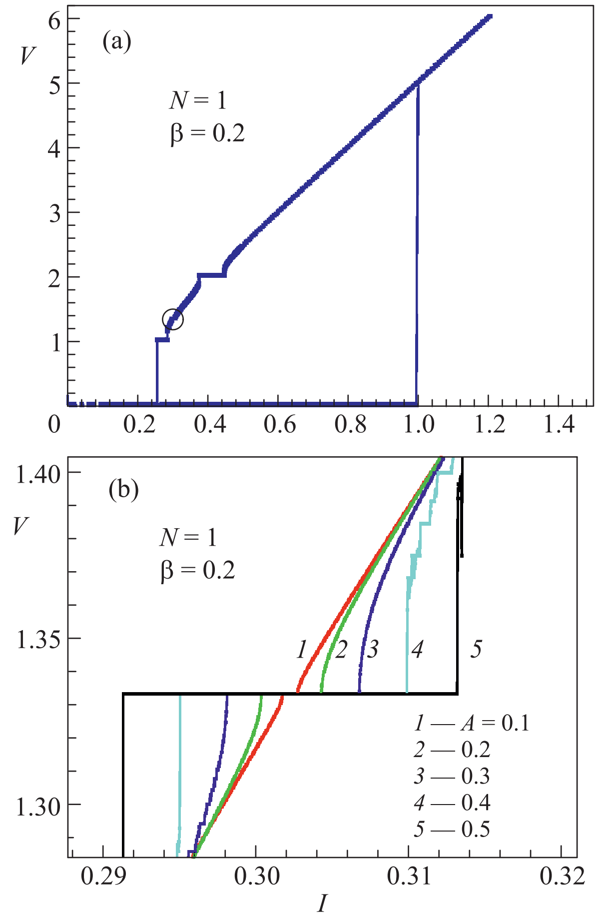

Fig. 4. (Color online) (a) – $I{-}V$-characteristic of the single JJ at $\omega = 2$ and $A = 0.3$. The circle marks the part of $I{-}V$-characteristic related to the step 2/3. (b) Demonstration of the chaos absence in the current interval corresponded to the 2/3 step with increase in radiation amplitude up to $A = 0.5$

To test it we demonstrate in Fig. 5 the effect of the coupling parameter α on the SS subharmonics 2/3 at amplitude $A = 0.27$. For clarity we show in Fig. 5a the $I{-}V$-characteristic of the stack at $A = 0.27$ and $\alpha = 0.05$ to see changes with decrease in α. We note that the part of the step which looks like it reflects regular behavior, actually does not. Under magnification the approximately straight line manifests some chaotic features. Fig. 5b enlarges that part of the $I{-}V$-characteristic marked by rectangular in Fig. 5a. We see in Figs. 5c and d that a decreasing in α leads to the disappearance of chaos in the stack of JJs. In the last case at $\alpha = 0.03$ the $I{-}V$-characteristic does not show any chaotic behavior at all. Based on these results we conclude that the observed chaos is indeed induced by coupling between JJs. We have tested this conclusion at $A = 0.25$ with an increase in α (not shown here) and obtained the same behavior.

Let us now briefly discuss the existing experimental results. An experimental survey of chaos related to the Josephson effect was presented by Noeldeke et al. [24], who analyzed a range of chaotic behavior that occurs in tunnel junctions and microbridges, subjected to dc and rf bias currents. Noeldeke et al. have furthermore shown, by their detailed analyses, that the experimental results agree with, firstly, simulations based on the resistively and capacitively shunted junction (RCSJ) model, and secondly, their theoretical predictions. Since the CCJJ+DC model which we employ is an improvement on the RCSJ model, the previous studies, such as those of Noeldeke et al. suggest that further numerical/experimental studies should be made to investigate, often subtle, the effects of the coupling on the observed chaotic behavior.

Experimental evidence of the strongly distorted steps and "wiggles", which are preceded by subharmonic voltage steps for lower rf powers in Pb/ox/PbIn tunnel junctions and in Pb microbridges irradiated by 70-GHz microwaves were reported in Ref. [25]. Fragmentation of the SS was also observed in our simulations [23], at amplitudes $A < 0.8$. The frequency range of chaotic behavior in tunnel junctions found in Refs. [26–28] are also in agreement with the simulations based on RCSJ model. Distorted $I{-}V$-characteristics and a strong noise rise in the distorted regions, were found in Ref. [29], where it was ascribed to chaotic noise. The authors of Ref. [30]

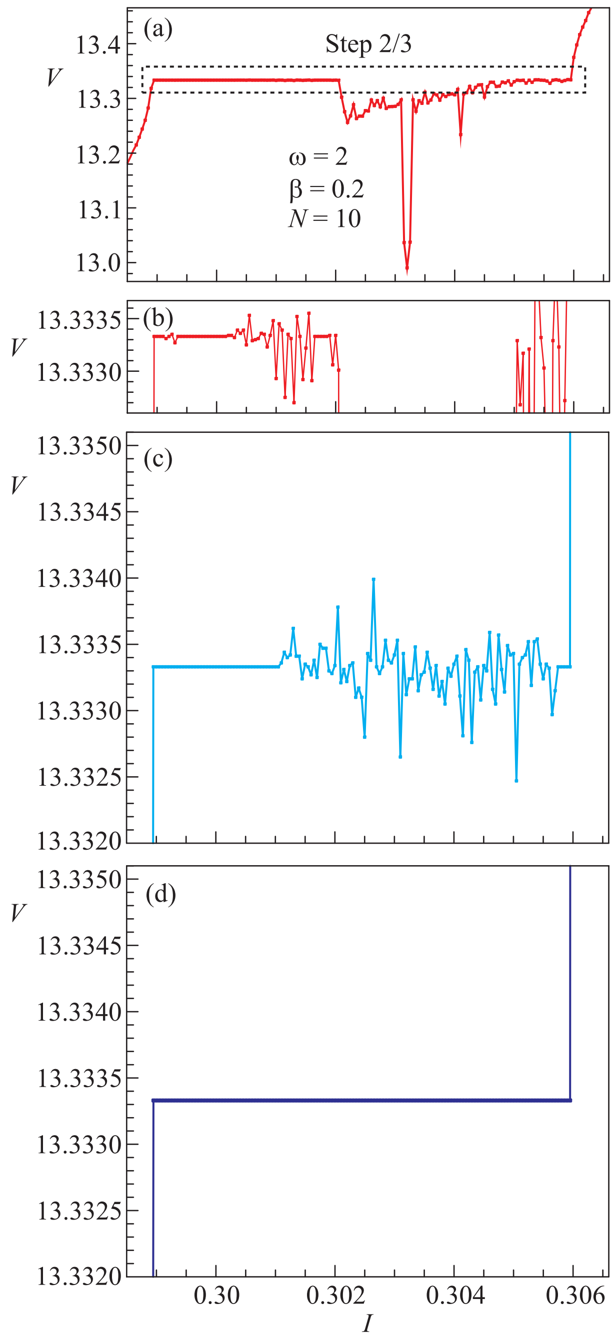

Fig. 5. (Color online) Effect of coupling between junctions on the Shapiro step subharmonics 2/3 at amplitude $A = 0.27$: $\alpha = 0.05$ (a); enlarged part around step 2/3 marked by rectangular in a (b); $\alpha = 0.04$ (c); $\alpha = 0.03$ (d)

observed tunnel-like behavior and oscillations in sync with the applied radiation, at integer and half-integer steps. We note that a detailed experimental investigation of the fragmented phases at different conditions and parameters of JJ is still lacking.

Lastly we mention that the importance of chaos in IJJs, and its effects on the CVCs in these systems, have been stressed in several more recent papers, such as [31, 32]; but, detailed experimental investigations of SS subharmonics in the $I{-}V$-characteristics of the stack of intrinsic Josephson junctions, under external electromagnetic radiation, are still absent. Since our results were obtained within the framework of the CCJJ+DC model, it would be very interesting to test, experimentally, the observed features which we have reported.

As a conclusion we note that have demonstrated new features of the phase dynamics related to the subharmonic Shapiro steps within stack of coupled JJs. Comparison of the obtained results for the stack of Josephson junctions to the case of a single junction has allowed us to prove that the observed chaos was induced by the coupling between junctions. We proposed that when the amplitude of the longitudinal plasma wave along the stack, i.e., the amplitude of charge oscillations in superconducting layers, becomes large enough, the charge oscillations destroy the phase locking between the Josephson oscillations and the external radiation.

An essential feature, which may be important for future applications, is the difference between the parametric instabilities in case of subharmonics in comparison to those of Shapiro step harmonics. In first case a much lower amplitude of radiation is needed for the creation of the longitudinal plasma wave along the stack of junctions. This fact gives a unique possibility to create and control the excitation of the longitudinal plasma wave in layered superconductors.

Another very interesting application of the physics of Shapiro step subharmonics might be found in topological superconductivity. Topological superconductors support Majorana fermions, which are expected to be used for the realization of quantum gates that are topologically protected from local sources of decoherence. Rokhinson et al. [33] report the observation of the fractional ac Josephson effect in a semiconductor–superconductor nanowire junction as a signature of Majorana quasiparticles. The use of subharmonics for the detection of Majorana fermions is a very important contemporary problem. Its solution may provide additional information on Majorana physics.

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