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# Magnetic susceptibility of the quark matter in QCD

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Magnetic susceptibility in the deconfined phase of QCD is calculated in a closed form using a recent general expression for the quark gas pressure in magnetic field. Quark selfenergies are entering the result via Polyakov line factors and ensure the total paramagnetic effect, increasing with temperature. A generalized form of magnetic susceptibility in nonzero magnetic field suitable for experimental and lattice measurements is derived, showing a good agreement with available lattice data.

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**I. Introduction.** A possibility of strong magnetic fields (MF) in astrophysics [1, 2] as well as in heavy ion collisions [3, 4], see [5] for a review, poses an important question: how the QCD matter react to MF and, in particular, whether it is paramagnetic or diamagnetic.

This topic has caused a vivid interest in the physical community recently [6–9] and the first numerical results have been obtained for the magnetic susceptibility at zero and finite temperature in [6], magnetization in [7], magnetic susceptibility as a function of temperature in [8], and pressure in MF at finite temperature [9].

In the analytical approach one should derive these results from the quark pressure $\bar{P}_q(B,T)$ in MF $B$ and temperature $T$ in the deconfined phase of QCD for $T > T_c$ and from the corresponding hadron pressure in the confining region.

The magnetic contribution to the quark pressure was considered mostly in the framework of effective field theories [10]. We shall follow the standard approximation [11], generalized with inclusion of the vacuum QCD effects.

Recently the quark pressure in MF $\bar{P}_q(B,T)$ was calculated in a simple closed form, in [12], where the sum over all Landau levels was expressed in terms of modified Bessel functions with correct limits for large and small MF. It is important, that in our approach the effect of the QCD vacuum enters in the form of Polyakov lines, which correct the free quark contribution. Moreover in [13] a further analysis of quark mass dependence of the transition temperature $T_c(B)$ was done, explaining the observed [14] decreasing behavior of $T_c(B)$ for small masses $m_q$.

It is a purpose of the present paper to study magnetic susceptibility (MS) of the quark matter as a function of temperature using the approach of [12, 13], and to compare the resulting curves with the numerical data of [7–9].

The paper is organized as follows. In the next section we define the basic quantities and discuss their dependence on $B, T$, and $m_q$ in the case of zero chemical potential $\mu$. In section III a detailed comparison with lattice data is done and in the final section a summary and prospectives are given.

**II. General formalism.** We consider the quark gas in MF, where each quark undergoes the influence of background color fields, which can be expressed in terms of field correlators (FC). The full thermal theory based on FC was suggested in [15] and finally formulated in [16, 17].

In the deconfined phase the quark pressure in MF is [12]

$$\bar{P}_q(B,T) = \sum_q P_q(B,T), \quad e_q \equiv |e_q|,$$

$$P_q(B,T) = \frac{N_c e_q BT}{\pi^2} \sum_{n=1}^{\infty} \frac{(-)^{n+1}}{n} L^n \sum_{n_\perp,\sigma} \varepsilon^\sigma_{n_\perp} K_1\left(\frac{n\varepsilon^\sigma_{n_\perp}}{T}\right), \tag{1}$$

Where

$$\varepsilon^\sigma_{n_\perp} = \sqrt{m_q^2 + e_q B(2n_\perp + 1 - \sigma)}. \tag{2}$$

It is expressed in terms of Polyakov loops $L(T)$, which contain the FC contribution [16, 17], namely, when one neglects the bound $q\bar{q}$ pairs, appearing close to $T_c$, then one can take into account only the large distance term $V_1(\infty, T)$, calculated via FC in [17, 18], and the fundamental Polyakov loop in this approximation (called in [16] the Single Line Approximation) is

$$L(T) \equiv L^{(V)}(T) = \exp\left[-\frac{V_1(\infty,T)}{2T}\right], \tag{3}$$

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where $V_1(\infty,T)$ was found in [17], [18] from the field correlators. Note, that $V_1(\infty,T)$ in $L^{(V)}(T)$ entering (1), is actually the static $q\bar{q}$ interaction at large distances, which was measured recently on the lattice [19] to be approximately 0.5 GeV for $T > T_c$. We shall use the form (3) with this value $V_1(\infty,T) = 0.5$ GeV in what follows, as well as direct lattice calculations from [20] for the Polyakov loop

$$L^{(F)}(T) = \exp\left[-\frac{F_1(\infty,T)}{2T}\right], \qquad (4)$$

where $F_1(\infty,T)$ is the free energy, containing all excitations. It was shown in [16], that $V_1(\infty,T) > F_1(\infty,T)$ and hence $L^{(F)}(T) > L^{(V)}(T)$.

As it is shown in the appendix of [12], the sum over $n_\perp,\sigma$ can be done explicitly in (1) with the result

$$P_q(B) = \frac{N_c e_q BT}{\pi^2}\sum_{n=1}^{\infty}\frac{(-)^{n+1}}{n}L^n\left[m_q K_1\left(\frac{nm_q}{T}\right) + \right.$$
$$+\frac{2T}{n}\frac{e_qB+m_q^2}{e_qB}K_2\left(\frac{n}{T}\sqrt{e_qB+m_q^2}\right) -$$
$$\left. -\frac{ne_qB}{12T}K_0\left(\frac{n}{T}\sqrt{m_q^2+e_qB}\right)\right\}. \qquad (5)$$

Eq. (5) gives correct limits of $P_q(B)$ for small and large $B$. The quark pressure (5) depends on $B, T$, and $m_q$. We shall be first of all interested in the region of parameters, when $eB \ll T$ and $m_q \ll T$, corresponding to the area studied on the lattice. In this case one can define the magnetic susceptibility $\hat{\chi}_q(B,T)$

$$P_q(B,T) - P_q(0,T) = \frac{\hat{\chi}_q}{2}(e_qB)^2 + O[(e_qB)^4]. \qquad (6)$$

To proceed one can expand the r.h.s. of (5) in the Taylor series in powers of $(e_qB)$. To this end one can exploit the relation

$$K_\nu(z) = \frac{1}{2}\left(\frac{z}{2}\right)^{-\nu}\int_0^\infty dt e^{-t-z^2/4t}t^{\nu-1} \qquad (7)$$

and obtains

$$P_q(B,T) - P_q(0,T) = \frac{N_c(e_qB)^2}{2\pi^2}\sum_{n=1}^{\infty}(-)^{n+1}L^n \times$$
$$\times\sum_{k=0}\left(\frac{e_qBn}{2Tm_q}\right)^k\frac{(-)^k}{k!}K_k\left(\frac{nm_q}{T}\right)\times$$
$$\times\left[\frac{1}{(k+1)(k+2)} - \frac{1}{6}\right]. \qquad (8)$$

Note, that the first two terms $O[(e_qB)^0]$ and $O[(e_qB)^1]$ in (8) identically vanish as well as the cubic terms, while the quadratic terms can be written as

$$\frac{\hat{\chi}_q}{2} = \frac{N_c}{6\pi^2}\sum_{n=1,2,\ldots}(-)^{n+1}L^nK_0\left(\frac{nm_q}{T}\right). \qquad (9)$$

As one can see in (9) the quark system retains its paramagnetic nature for any $m_q$, $T$ provided the Matsubara series over $n$ is convergent. We shall see however that in (9) a strong compensation of different terms in the series occurs.

Indeed, if $L \sim O(1)$ and $m_q/T < 1$, one should keep in (9) the sum over Matsubara frequencies, which yields for the total MS

$$\hat{\chi}(T) = \sum_q\left(\frac{e_q}{e}\right)^2\hat{\chi}_q(T) = \frac{N_c}{3\pi^2}\sum_q\left(\frac{e_q}{e}\right)^2 J_q,$$
$$J_q \equiv \sum_{n=1,2,\ldots}(-)^{n+1}L^nK_0\left(\frac{nm_q}{T}\right). \qquad (10)$$

To find $J_q$ we use the integral representation for $K_0$

$$K_0\left(\frac{m_qn}{T}\right) = \frac{1}{2}\int_0^\infty\frac{d\omega}{\omega}e^{-n(m_q^2/2T^2\omega+\omega/2)}, \qquad (11)$$

and summing over $n$ in (10) one obtains the following form for $J_q$,

$$J_q = \frac{1}{2}\int_0^\infty\frac{dx}{x}\frac{y(x)}{1+y(x)},\ \ y(x) = L\exp\left(-\frac{1}{x} - \frac{m_q^2x}{4T^2}\right). \qquad (12)$$

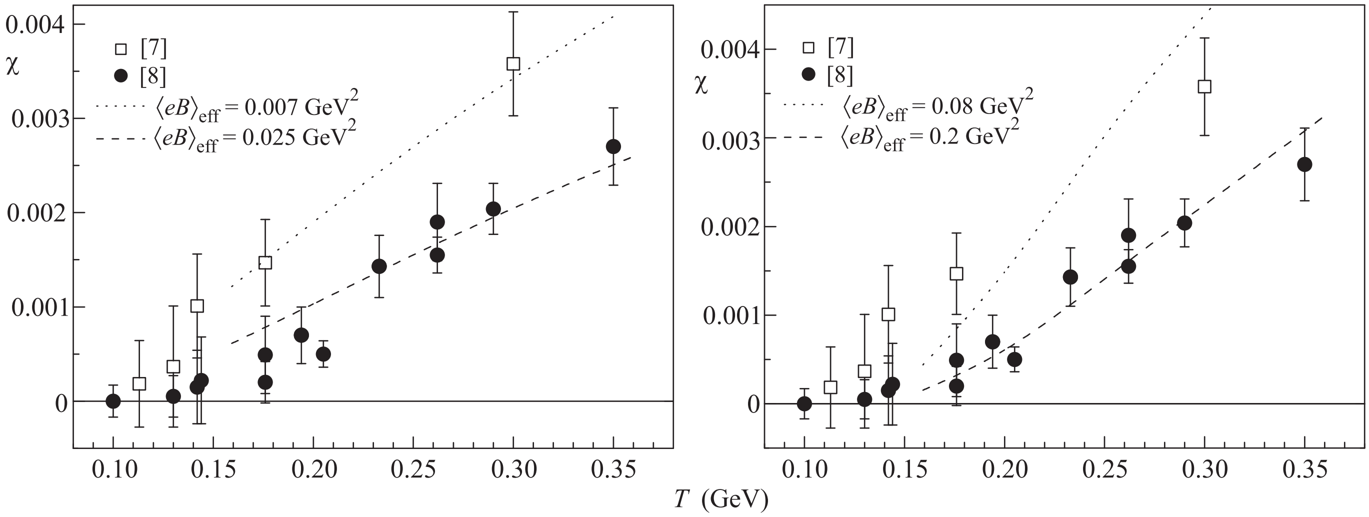
**III. Results and discussion.** Our resulting formula for $\hat{\chi}(T)$ is given in (10), where the integral $J_q$ is defined in (12). One can see in (12) that the temperature dependence of $\hat{\chi}(T)$ is defined mostly by the Polyakov line factor $L(T)$, which grows strongly in the considered region (see e.g. Fig. 3 of the first ref. in [20]). In addition there is a weak logarithmic dependence from the upper limit of integration $\sim \ln(T/m_q)$. One can see qualitatively the same type of behavior of $\hat{\chi}(T)$ in the lattice data of [7–9]. However, to compare (10) with lattice results one must take into account, that MF on the lattice is quantized and has a minimal value, dependent on the lattice size, so that one actually refers to the generalized MS $\hat{\chi}(B,T)$,

$$\frac{1}{2}\hat{\chi}_q(B,T) = \frac{P_q(B,T) - P_q(0,T)}{(e_qB)^2} \equiv f\left(\frac{\sqrt{e_qB+m_q^2}}{T}\right), \qquad (13)$$

where $m_q^2$ enters always as $m_q^2 + e_qB$, and therefore one can introduce in (12) the effective quark mass

$$m_{q_{\rm eff}}^2 = m_q^2 + \langle e_qB\rangle_{\rm eff}, \qquad (14)$$

where $\langle e_qB\rangle_{\rm eff}$ depends in general on the experimental setup or lattice configuration. One can estimate the minimal value of $\langle eB\rangle_{\rm eff}$, on the lattice, $\langle eB\rangle_{\min} \approx 6\pi/(L_sa)^2$, which gives for the measurements in [8],




Magnetic susceptibility in SI units ($\chi = \frac{4\pi}{137}\hat{\chi}$) as a function of temperature for different $L(T)$, as obtained from (3) with $V_1(\infty, T) = 0.5\,\mathrm{GeV}$ (left plot) and $L^{(F)}(T)$ from lattice data [20] (right plot), for different values of $\langle eB\rangle_{\mathrm{eff}}$ in comparison with lattice data [7] and [8]

$\langle eB\rangle_{\min} \approx 0.023\,\mathrm{GeV}^2$, and for those in [7] $\langle eB\rangle_{\min} \approx \approx 0.005\,\mathrm{GeV}^2$. Therefore we keep in (12) $m_q \to m_{q_{\mathrm{eff}}}$ as in (14) with $\langle eB\rangle_{\mathrm{eff}}$ as a fitting parameter in the interval $0.005{-}0.04\,\mathrm{GeV}^2$. As a result we obtain two sets of curves in Figure for $\chi = \frac{4\pi}{137}\hat{\chi}$, which follow closely the data points; one set, corresponding to $V_1(\infty, T) = = 0.5\,\mathrm{GeV}$ gives the best fit for $\langle eB\rangle_{\mathrm{eff}} \approx 0.025\,\mathrm{GeV}^2$ for the data of [8] and $\langle eB\rangle_{\mathrm{eff}} \approx 0.007\,\mathrm{GeV}^2$ for the data of [7]. Another set of curves, corresponding to $L^{(F)}(T)$, taken from lattice data [20], gives larger values of effective field $\langle eB\rangle_{\mathrm{eff}}$, 0.08 and $0.2\,\mathrm{GeV}^2$ for the data of [8] and [7] correspondingly. One can see a good agreement of our theoretical predictions and lattice results, note also a close correspondence of the $\langle eB\rangle_{\min}$ with the fitted values of $\langle eB\rangle_{\mathrm{eff}}$ for the curves on the left graph. This fact shows a usefulness of our definition of the $m_{q_{\mathrm{eff}}}$ and of our approach in general, where the main features of the QCD quark matter are incorporated in Eq. (1), derived from the quark path integrals in the QCD vacuum and containing quark selfenergies in the form of the Polyakov lines.

**IV. Conclusions.** We have succeeded in obtaining simple formulas for the MF dependence of the pressure and the MS of the quark matter. Our resulting Eq. (10) contains a simple tabulated integral $J_q$. One can see, that $\hat{\chi}(T)$ strongly depends on $T$ due mostly to the Polyakov loop and is almost insensitive to quark masses when $T \gg m_q$. Both features are supported by the data of [8]. The resulting magnitude of $\hat{\chi}(T)$ is strongly reduced by the oscillating Matsubara series as compared to the leading $n = 1$ term, and is in a good quantitative agreement with lattice computations in [7], [8], and [9], where the result of [7] is somewhat higher, due to smaller effective mass values $m_{q_{\mathrm{eff}}}$.

A good agreement of our predictions for MS with available data is in line with similar agreement of the transition temperature in MF in [12, 13], obtained in the framework of our theoretical approach [15–18], which can be a good starting point for detailed analysis of the quark-hadron transition.

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