Radiative parton energy loss in expanding quark-gluon plasma with magnetic monopoles

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We study radiative parton energy loss in an expanding quark-gluon plasma with magnetic monopoles. We find that for realistic number density of thermal monopoles obtained in lattice simulations parton rescatterings on monopoles can considerably enhance energy loss for plasma produced in AA collisions at RHIC and LHC energies. However, contrary to previous expectations, monopoles do not lead to the surface dominance of energy loss.

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I. Introduction. It is widely accepted that the jet quenching phenomenon in AA collisions observed at RHIC and LHC is a manifestation of parton energy loss in the hot quark-gluon plasma (QGP) produced in the initial stage of AA collisions. The dominating contribution to parton energy loss comes from induced gluon radiation due to parton multiple scattering in the QGP [1–7]. The effect of collisional energy loss is relatively small [8]. The RHIC and LHC data on suppression of the high-p_T hadrons in AA collisions can be reasonably well described within the light-cone path integral (LCPI) approach to induced gluon emission [3] in a scenario of purely perturbative QGP (pQGP) [9–12] with the quasiparticle parton masses borrowed from the quasiparticle fit [13] to lattice results (which are close to that predicted by the HTL scheme [14]). Although, in the relevant range of the plasma temperatures $T \lesssim T_c(2{-}3)$, the non-perturbative effects may be important, one could hope that the pQGP model is reasonable since radiative energy loss is mostly sensitive to the number density of the color constituents of the QGP. And the internal dynamics of the matter is practically unimportant from the standpoint of the energy loss calculations. This assumption may be wrong, however, if the non-perturbative effects lead to formation of new effective scattering objects that are absent in the pQCD picture. Evidently, thermal magnetic monopoles in the so called "magnetic scenario" of the QGP [15–17] are such objects that can be potentially very important for parton energy loss.

The thermal magnetic monopoles are now under active investigation [18–21] (and references therein). Lattice calculations show that monopoles in the QGP are compact and heavy objects [19]. For this reason from the point of view of parton rescatterings they can act as practically point-like static scattering centers. Similarly to QED (for a review on monopoles in QED see, for instance, [22]) the differential cross section for parton scattering off thermal monopoles has the Rutherford form. It is important that, contrary to the ordinary pQCD parton cross sections, for monopoles, due to the Dirac charge quantization condition constraint, there are no the running and thermal effects. Lattice results show that the monopole number density, n_m, may be quite large $n_m/T^3 \sim 0.4 - 0.9$ at $T \sim T_c(1{-}3)$ [20, 19]. Although, it is smaller by a factor of $\sim 5{-}10$ than the number density of ordinary thermal partons in pQGP, the scattering cross section for monopoles is considerably higher than that for thermal quarks and gluons. As a result, the monopoles can give a considerable contribution to induced gluon emission (and to photon emission from quarks). In [23] within the classical non-relativistic approach it was shown that interaction of quarks with monopoles may be important for photon emission from the QGP. The effect of monopoles on jet quenching in AA collisions has been addressed in recent analysis [24] within the GLV approach [6] in the approximation of $N = 1$ rescattering.

In the present paper we address within the LCPI scheme [3] the question to which extent monopoles can be important for parton energy loss in the expanding QGP for RHIC and LHC conditions. The advantage of the LCPI formalism is that it includes any number of parton rescatterings (that is very important for the QGP with monopoles (below we denote it as mQGP) due to large cross section of parton interaction with monopoles). The LCPI approach treats accurately the

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mass and finite-size effects, and is valid beyond the soft gluon approximation.

II. Theoretical framework. In the LCPI approach [3] the induced gluon x-spectrum for a fast quark (or gluon) may be written through the in-medium light-cone wave function of the $gq\bar{q}$ (or ggg) system in the coordinate ρ-representation. The z-dependence of this light-cone wave function is governed by a two-dimensional Schrödinger equation in which the longitudinal coordinate z (z-axis is chosen along the fast parton momentum) plays the role of time. We use the representation for the gluon spectrum obtained in [25] which is convenient for numerical calculations. For a fast quark (produced at $z=0$) the gluon spectrum reads

$$\frac{dP}{dx}=\int\limits_0^L dz\, n(z)\frac{d\sigma_{\text{eff}}^{\text{BH}}(x,z)}{dx}, \quad (1)$$

where $n(z)$ is the medium number density, $d\sigma_{\text{eff}}^{\text{BH}}/dx$ is an effective Bethe–Heitler cross section accounting for both the LPM and finite-size effects. The $d\sigma_{\text{eff}}^{\text{BH}}/dx$ reads

$$\frac{d\sigma_{\text{eff}}^{\text{BH}}(x,z)}{dx}=-\frac{P_q^g(x)}{\pi M}\times$$
$$\times\,\text{Im}\int\limits_0^z d\xi\alpha_s[Q^2(\xi)]\,\frac{\partial}{\partial\rho}\left[\frac{\Psi(\xi,\rho)}{\sqrt{\rho}}\right]\Bigg|_{\rho=0}. \quad (2)$$

Here $P_q^g(x)=C_F[1+(1-x)^2]/x$ is the usual splitting function for $q\to gq$ process, $M=Ex(1-x)$ is the reduced "Schrödinger mass", E is the initial parton energy, $Q^2(\xi)=aM/\xi$ with $a\approx1.85$ [8], Ψ is the solution to the radial Schrödinger equation for the azimuthal quantum number $m=1$

$$i\frac{\partial\Psi(\xi,\rho)}{\partial\xi}=\left[-\frac{1}{2M}\left(\frac{\partial}{\partial\rho}\right)^2+\right.$$
$$\left.+\,v(\rho,x,z-\xi)+\frac{4m^2-1}{8M\rho^2}+\frac{1}{L_f}\right]\Psi(\xi,\rho) \quad (3)$$

with the boundary condition $\Psi(\xi=0,\rho)=\sqrt{\rho}\sigma_{q\bar{q}g}(\rho,x,z)\epsilon K_1(\epsilon\rho)$ (K_1 is the Bessel function), $L_f=2M/\epsilon^2$ with $\epsilon^2=m_q^2x^2+m_g^2(1-x)^2$, $\sigma_{q\bar{q}g}(\rho,x,z)$ is the cross section of interaction of the $q\bar{q}g$ system (in the ρ-plane $\bar{q}$ is located at the center of mass of qg) with a medium constituent located at z. The potential v in (3) reads

$$v(\rho,x,z)=-i\frac{n(z)\sigma_{q\bar{q}g}(\rho,x,z)}{2} \quad (4)$$

(summing over the species of the medium constituents is implicit here). The $\sigma_{q\bar{q}g}$ may be written through the dipole cross section $\sigma_{q\bar{q}}$ [26]

$$\sigma_{q\bar{q}g}(\rho,x,z)=\frac{9}{8}\{\sigma_{q\bar{q}}(\rho,z)+\sigma_{q\bar{q}}[(1-x)\rho,z]\}-\frac{1}{8}\sigma_{q\bar{q}}(x\rho,z). \quad (5)$$

The dipole cross section for scattering of the $q\bar{q}$ pair on a medium constituent c may be written as

$$\sigma_{q\bar{q}}(\rho,z)=\frac{2}{\pi}\int d\mathbf{q}[1-\exp(i\mathbf{q}\boldsymbol{\rho})]\frac{d\sigma_{qc}}{dq^2}, \quad (6)$$

where $d\sigma_{qc}/dq^2$ is the $qc\to qc$ differential cross section. For scattering on thermal quarks and gluons the differential cross section (in the approximation of static Debye screened color centers [1]) reads

$$\frac{d\sigma_{qc}}{dq^2}=\frac{C_TC_F}{2}\frac{\pi\alpha_s^2(q^2)}{[q^2+m_{\text{D}}^2]^2}, \quad (7)$$

where $C_{F,T}$ are the color Casimir for the quark and thermal parton (quark or gluon), m_{D} is the local Debye mass. For the QGP with monopoles we should account for in the potential (4) the contribution from rescatterings on monopoles. The formula (6) is valid for monopoles as well. In QCD there are two different species of monopoles related to the Cartan generators T_3 and T_8 of the $SU(3)$ group. Lattice calculations [20] show that both the species of monopoles have the same number density. For fast quarks thermal monopoles $M_{3,8}$ act as Abelian scattering centers. For gluons in the color basis of definite color isospin and hypercharge (see below) it is true as well. In vacuum the differential cross section for scattering of a charged particle with electric charge q_e off a monopole with magnetic charge q_m has the Rutherford form [27]

$$\frac{d\sigma}{dq^2}=\frac{4\pi D^2}{q^4}, \quad (8)$$

where $D=q_eq_m/4\pi$. The Dirac charge quantization condition says that $|D|=n/2$ where n is an arbitrary integer. We will assume that in the QGP for both the color species of monopoles $|D|=1/2$ (here we mean the minimal value of $|D|$ for parton-monopole interactions, for some parton color states it can be bigger). This value is supported by extraction of the magnetic coupling $\alpha_m=q_m^2/4\pi$ from the monopole-(anti)monopole correlations in lattice simulations [19, 20] which give $\alpha_m\sim2{-}4$ at $T/T_c\sim1-2$. Making use this α_m by inspecting the qM-scattering one can easily obtain that the condition $|D|=1/2$ gives $\alpha_s=1/\alpha_m\sim0.25{-}0.5$ which is quite reasonable for α_s in the QGP at $T/T_c\sim1{-}2$. The value $|D|=1$ leads to a four times bigger α_s which seems to be unrealistic. For the $qM_{3,8}$ scattering the coupling to the vector potential of the monopole color field is given by $g\hat{\lambda}_{3,8}/2$, and the possible values of $|D|$ are 1/2 and 1. We write the $qM(\bar{M})$ differential cross section in the form

$$\frac{d\sigma_{qM}}{dq^2} = \frac{C_F \pi F^2(q^2)}{(q^2 + m_{\rm D}'^2)^2}. \tag{9}$$

Here we introduced a phenomenological form-factor F accounting for the finite size of the monopole, $m_{\rm D}'$ is the magnetic Debye mass, C_F is the color factor arising from averaging over the quark and monopole color states which for the $SU(3)$ group gives $\{[1^2+(-1)^2]/3+[1^2+1^2+(-2)^2]/3\}/2 = 4/3$. From (7) and (9) one can see that in comparison to qq scattering the $qM(\bar{M})$ cross section is enhanced by a factor of $3/2\alpha_s^2$ (if we ignore F and possible difference in the electric and magnetic Debye screening masses).

For energetic partons energy loss is dominated by the small x-region. In the limit $x \to 0$ the three-body cross section (5) reduces to the cross section for the gg-color dipole. In pQCD for scattering on thermal partons $\sigma_{gg} = \frac{C_A}{C_F}\sigma_{q\bar{q}}$. One can show that this relation is valid for scattering off monopoles as well. Indeed, similarly to our analysis of the synchrotron-like gluon emission [28], the scattering amplitude for interaction of gluons with monopoles may be diagonalized by introducing the gluon fields having definite color isospin, Q_A, and color hypercharge, Q_B. In terms of the usual gluon vector potential, G, the diagonal color gluon states read (we denote $Q = (Q_A, Q_B)$) $X = (G_1 + iG_2)/\sqrt{2}$ $(Q = (-1, 0))$, $Y = (G_4 + iG_5)/\sqrt{2}$ $(Q = (-1/2, -\sqrt{3}/2))$, $Z = (G_6 + iG_7)/\sqrt{2}$ $(Q = (1/2, -\sqrt{3}/2))$. The neutral gluons $A = G_3$ and $B = G_3$ with $Q = (0, 0)$, do not interact with monopoles at all. Then using this basis one can easily show that the averaged over the color states of the gluon and of the monopole differential cross section for gM scattering reads

$$\frac{d\sigma_{gM}}{dq^2} = \frac{C_A \pi F^2(q^2)}{(q^2 + m_{\rm D}'^2)^2}. \tag{10}$$

One can see that similarly to scattering on thermal partons for monopoles the ratio of the cross sections for gluons and quarks equals C_A/C_F. At $x \to 1$ the three-body cross section (5) reduces to the dipole cross section $\sigma_{q\bar{q}}$. From the above consideration of the cross section for qM and gM scatterings we can conclude that our formula for the three-body cross section (which has been derived for the double gluon exchanges [26]) is valid for scattering on monopoles in the limits $x \to 0, 1$. In principle for monopoles at moderate values of x it may be invalid. However, due to the fact that its variation between $x \sim 0$ and $x \sim 1$ is not very strong and the dominating region is $x \sim 0$ the errors due to use of (5) for monopoles cannot be significant.

For $g \to gg$ one should just replace the splitting function and m_q by m_g in ϵ^2. The three-body cross section $\sigma_{q\bar{q}g}$ for $g \to gg$ is replaced by the cross section for the color singlet ggg state, that can be written in terms of the dipole cross section $\sigma_{q\bar{q}}$ as

$$\sigma_{ggg}(\rho, x, z) = \frac{9}{8}\{\sigma_{q\bar{q}}(\rho, z) + \sigma_{q\bar{q}}[(1-x)\rho, z] + \sigma_{q\bar{q}}(x\rho, z)\}. \tag{11}$$

We use the electric Debye mass obtained in the lattice analysis [29] giving $m_{\rm D}/T$ slowly decreasing with T ($m_{\rm D}/T \approx 3.2$ at $T \sim T_c$, $\mu_{\rm D}/T \approx 2.4$ at $T \sim 4T_c$). For the magnetic Debye mass we use predictions of the lattice simulations of [30] that also gives decreasing with T ratio $m_{\rm D}'/T$ ($m_{\rm D}'/T \approx 2.8$ at $T \sim T_c$, and $m_{\rm D}'/T \approx 1.2$ at $T \sim 4T_c$). However, the energy loss is not very sensitive to the Debye masses [8]. For the quasiparticle masses of light quarks and gluon we take $m_q = 300$ and $m_g = 400$ MeV supported by the analysis of the lattice data [13]. Our results are not very sensitive to m_g, and are practically insensitive to the value of m_q.

We used running α_s frozen at some value α_s^{fr} at low momenta for gluon emission in vacuum a reasonable choice is $\alpha_s^{fr} \approx 0.7$ [31, 32]. However, in the QGP the thermal effects can suppress the α_s^{fr}, and we regard it as a free parameter. The data on the nuclear modification factor R_{AA} support $\alpha_s^{fr} \sim 0.5-0.6$ for RHIC energies and $\alpha_s^{fr} \sim 0.4-0.5$ for LHC energies [10–12].

For the monopole density n_m we use predictions of the recent lattice simulations [20] of the $SU(3)$ gluondynamics. In [20] it was obtained that the ratio $\bar{n}_m = n_m/T^3$ is a decreasing function of T, $\bar{n}_m \sim 0.9$ at T/T_c slightly above 1 and $\bar{n}_m \sim 0.45$ at $T/T_c \approx 2$. At $T/T_c \gtrsim 2$ the authors have found that their results may be approximated as $\bar{n}_m \approx 2A/[\ln(T/\Lambda)]^3$ with $A = 3.66$ and $\Lambda/T_c = 0.163$. The difference in the monopole density for the $SU(3)$ gluondynamics studied in [20] and for full QCD should not be large since the lattice simulations performed in [21] show that at $T \sim T_c$ the monopole density for the gluondynamics and for full QCD are close to each other. We perform the computations for point-like monopoles, i.e., for $F(q^2) = 1$, and for monopoles with a Gaussian form-factor $F(q^2) = \exp(-q^2 R_m^2/6)$, where R_m may be viewed as a mean square monopole radius. Lattice results on the r-dependence of the monopole-(anti)monopole correlation functions show [19] that $R_m \sim 0.05-0.1$ fm. We are fully aware that the Gaussian form has not any theoretical justification (especially for $F \ll 1$). But it allows one to understand how the monopole internal structure can suppress energy loss. We perform calculations for $R_m = 0.1$ and 0.15 fm. The first value is consistent with the $SU(3)$ lattice simulations of [20]. The second one also may be reasonable since the estimation of the monopole radius via the r-dependence of the monopole-(anti)monopole correlation functions has qualitative character rather than quantitative.

One remark is in order here on the $q(g)M$ differential cross sections. The formulas (9), (10) do not account for the possible electric charge of the monopole. The lattice calculations for the $SU(2)$ gluondynamics performed in [18] show that the thermal monopoles are dyons, i.e., besides magnetic color charge they have electric color charge as well. It is known [27] that for scattering of a charged particle (with electric charge q_{1e}) on a dyon with electric and magnetic charges q_{2e} and q_{2m} one should use in the Rutherford formula the sum $(q_{1e}q_{2m})^2 + (q_{1e}q_{2e})^2$. Here only the first (electric-magnetic) term obeys the Dirac charge quantization conditions. The second (electric-electric) term is similar to that for ordinary pQCD parton-parton interactions. One can neglect it since in the QGP the number density of monopoles (dyons) is much smaller that of thermal quarks and gluons.

We have performed calculations for Bjorken's 1+1D expansion of the QGP [33], which gives $T_0^3\tau_0 = T^3\tau$. As in our previous analyses of jet quenching [10–12] we take $\tau_0 = 0.5$ fm. For $\tau < \tau_0$ we take the number density $\propto \tau$. We consider the situation for production of a fast parton in the central rapidity region. We assume that the fast parton produced in a hard process at $z = \tau = 0$, passes through a length L of an expanding QGP (our z-axis lies in the impact parameter plane for AA collision, and L corresponds to the transverse parton path length in the QGP that equals the proper time τ). We define the energy loss as

$$\Delta E = E \int\limits_{x_{\min}}^{x_{\max}} dx x \frac{dP}{dx}, \qquad (12)$$

where E is the initial parton energy. Since for hard gluons with $x \gtrsim 0.5$ the jet really does not disappear, from the point of view of the jet quenching, a reasonable choice for the upper limit of x-integration is $x_{\max} = 0.5$. For $x_{\min}$ we use the value m_g/E.

III. Numerical results. We performed computations for $\alpha_s^{fr} = 0.5$. We present results for $T_0 = 320$ MeV corresponding to central Au + Au collisions at $\sqrt{s} = 0.2$ TeV, and for $T_0 = 420$ MeV corresponding to central Pb + Pb collisions at $\sqrt{s} = 2.76$ TeV (the procedure that leads to these values of T_0 is described in [12]). Fig. 1 shows the radiative energy loss ΔE for gluons and quarks vs the initial parton energy E for

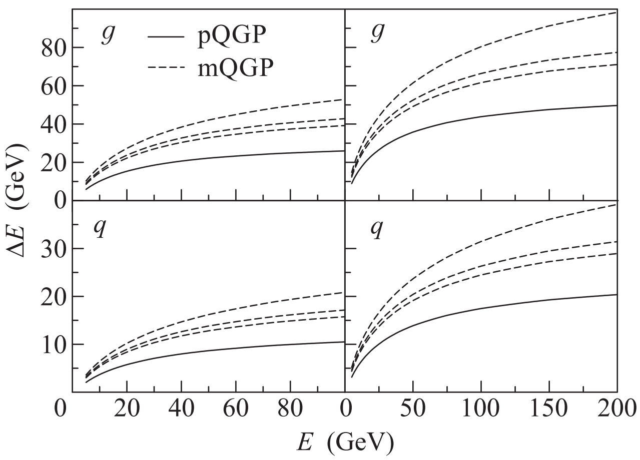

Fig. 1. Energy dependence of the energy loss of gluons (upper panels) and light quarks (lower panels) for the expanding QGP with $T_0 = 320$ MeV (left) and 420 MeV (right) for $L = 5$ fm. Solid line – radiative energy loss for pQGP without monopoles; dashed line – radiative energy loss for mQGP obtained (top to bottom) for the monopole radii $R_m = 0$, 0.1, and 0.15 fm

the plasma thickness $L = 5$ fm (which approximately corresponds to the dominating parton path length for central Au + Au and Pb + Pb collisions). We show the results for the ordinary QGP (pQGP) and QGP with monopoles (mQGP). For monopoles we present the results obtained for the monopole radii $R_m = 0$, 0.1, and 0.15 fm. From Fig. 1 one sees that monopoles may increase considerably the energy loss. Fig. 1 shows that the enhancement factor $\Delta E(\text{mQGP})/\Delta E(\text{pQGP})$ due to monopoles at $E \lesssim 100$ GeV is $\sim 1.6{-}2$ for $R_m = 0$ (point-like monopoles). From the curves for $R_m = 0.1$ and 0.15 fm one can see that the finite-size effects can reduce the enhancement to $\sim 1.4{-}1.6$. As one could expect the effect of monopoles is somewhat stronger for lower T_0 since the monopole density decreases with temperature. However, the difference between the ratio $\Delta E(\text{mQGP})/\Delta E(\text{pQGP})$ for $T_0 = 320$ and 420 MeV is not big.

To illustrate the L-dependence of parton energy loss in Fig. 2 we show the results for radiative quark energy loss vs the path length L for $E = 20$ and 50 GeV for $T_0 = 320$ and 420 MeV. One can see that monopoles change the normalization of the curves, but they practically do not change the form of the L-dependence of ΔE (which at $L \gtrsim \tau_0$ is approximately linear). At $L < \tau_0$ $\Delta E \propto L^3$ (since the leading $N = 1$ rescattering contribution to the effective Bethe–Heitler cross section is $\propto L$ [34, 35] and integration over the longitudinal coordinate of the scattering center gives additional two powers of L). For the curves in Fig. 2 at $L \sim 4{-}5$ tem-

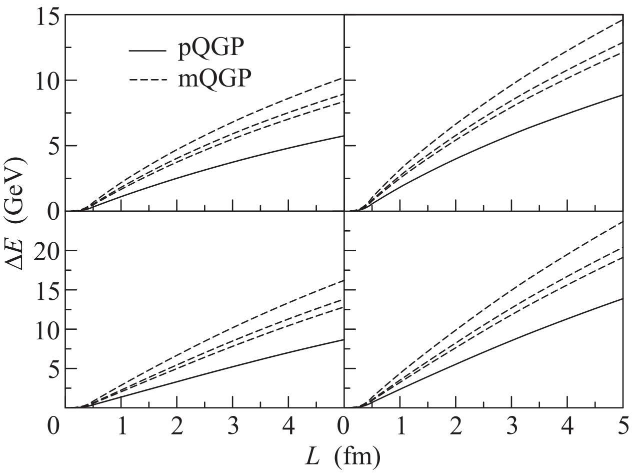

Fig. 2. Radiative energy loss for quarks with $E = 20$ (upper part) and 50 (lower part) GeV vs parton path length L for the initial QGP temperature $T_0 = 320\,\mathrm{MeV}$ (left) and 420 MeV (right). Solid line – radiative energy loss for pQGP without monopoles; dashed line – radiative energy loss for mQGP obtained (top to bottom) for the monopole radii $R_m = 0$, 0.1, and 0.15 fm

perature becomes close to T_c (we take $T_c = 160\,\mathrm{MeV}$) i.e., this region corresponds to the QCD phase transition. In [36], within a purely phenomenological model of the jet interaction with the QGP, it has been suggested that in the magnetic scenario of the QGP the parton energy loss should be strongly enhanced in the near-T_c region, and this should result in the surface dominance of jet energy loss. In terms of our curves for $\Delta E(L)$ in Fig. 2 it would mean that $d\Delta E/dL$ must rise steeply at $L \gtrsim 3\,\mathrm{fm}$. However, our calculations performed with accurate treatment of induced gluon emission for realistic lattice monopole density [20], show that $\Delta E(L)$ does not exhibit anything special in the near-T_c region for the scenario with monopoles. The curves for mQGP may be well reproduced in the pQGP scenario by taking somewhat bigger α_s^{fr} (for predictions with the form-factor it is $\alpha_s^{fr} \sim 0.8$).

IV. Conclusion. We have performed the comparison of radiative energy loss of energetic quarks and gluons for the expanding QGP for a purely perturbative scenario of plasma consisting of quarks and gluons and for a scenario of plasma with magnetic monopoles. Our results show that for RHIC and LHC conditions, for the monopole number densities in the QGP predicted by lattice calculations, monopoles can enhance considerably radiative energy loss. For point-like monopoles the enhancement factor is $\sim 1.6-2$. Our qualitative analysis with a Gaussian monopole form-factor shows that for the monopole radius $\sim 0.1\,\mathrm{fm}$, supported by lattice analyses, the finite-size effects can reduce the enhancement factor to $\sim 1.4-1.6$.

Our calculations show that for RHIC and LHC conditions the L-dependence of energy loss is similar to that for ordinary plasma (except for its magnitude). It means that monopoles do not lead to any strong surface dominance of energy loss, as was suggested in [36]. For this reason it is difficult to discriminate the mQGP scenario from the pQGP one using data on the azimuthal anisotropy of high-p_T hadrons in non-central AA collisions. Nevertheless, the effect of monopoles on the induced gluon emission and jet quenching may be quite big and the magnetic scenario deserves further investigation. In particular, it would be interesting to perform a quantum analysis of the photon emission from the mQGP, addressed in [23] within the classical nonrelativistic formalism. This process also can be studied consistently within the LCPI approach (in the form given in [37]). Also, the effect of monopoles may be important for jet quenching in pp and pA collisions (where the plasma temperature is smaller than in AA collisions, and monopoles may be more important), discussed recently in the pQGP scenario in [38–40]. We leave it for future studies.

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