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One-dimensional strongly interacting electrons with single impurity: conductance reemergence

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We show that conductance of 1D channel with one point-like impurity critically depends on asymptotic behavior of $e-e$ interaction at small momenta k (about inverse length of a channel). Conductance reemerges (contrary to the case of point-like repulsive potential) if potential $V\ (k\ =\ 0)\ =\ 0$. For example, this happens if the bare $e-e$ interaction is screened by the charges in the bulk. The relation of this phenomena to the long-range order present in the Luttinger model is discussed. We consider spinless electrons but generalization is straightforward.

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Introduction. Theory of one-dimensional interacting electrons is under investigation for a long time [1–3]. Its relativistic analog, two-dimensional QED, also attracted a lot of attention [4] in the past, since it is a simplest field theory with confinement. During the time it was understood that one-dimensional pure electronic systems (in particular, the Luttinger model [2]) are exactly solvable. To date the properties of the clean systems are very well understood. Situation is different for the 1D channels with some impurities that are understood as short-range barriers with transition coefficient K and reflection coefficient R. The simplest system of this kind (with only one impurity) was considered for the first time in Ref. [5]. It turned out that properties of such system depend critically on the sign of the electron-electron ($e-e$) interaction. Conductance for attractive potentials is equal to the ballistic one and it is not affected by $e-e$ interaction (only the Fermi speed should be renormalized). Conductance for repulsive potentials vanishes. These results were obtained in [5] by the bosonization method.

Another approach with similar results was developed in [6]. The authors returned to the fermion language. Assuming that the interaction is short-range, $V = V_0\delta(x)$ and small $V_0 \ll 1$, they summed up the leading infrared logarithms of frequency ω by the renormalization group method. Next-to-leading corrections to the conductivity were also found in [7] by the methods of current algebra.

The approaches of [5] and [6] have different (but overlapping) regions of applicability. The first approach employs perturbation theory in reflection (transition) coefficient for an arbitrary attractive (repulsive) potential, while the second one employs perturbation theory in potential for an arbitrary reflection or transition coefficients. A point-like $e-e$ interaction is assumed in both approaches.

We suggested in [8] an alternative approach to the problem based on the path integral formalism. Using the well-known trick [9] we see that Luttinger model can be interpreted as the system of non-interacting electrons in a random external field. Green functions of one-dimensional electrons in any external field can be found exactly. We used this fact to construct perturbatively a Green function of the system with impurity. At the end we integrate out fermions and arrive at a 0+1-dimensional field theory. This theory describes the evolution with time of the electron phase at the point where the impurity is located. It is completely equivalent to the original Luttinger model with one impurity. For the sake of simplicity we will consider here only electrons without spin.

Using this theory we were able to prove two theorems. First, conductance of the system is zero (for repulsion) or maximal (for attraction) for a wide class of potentials. The arguments in favor of this statement for a point-like potential were given earlier in both approaches of [5] and [6]. We will see below that a necessary condition for such behavior of conductance is that the Fourier transform of the potential $V(k)$ has a non-vanishing limit at $k \to 0$. The second theorem is a general exact property of the theory which one can call *duality*. It states that the effective reflection coefficient

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$|\mathcal{R}_\omega|^2$ in a theory with an attractive potential is equivalent to the effective transition coefficient $|\mathcal{K}_\omega|^2$ in a theory with repulsion if one exchanges $K \leftrightarrow R$ (for a precise formulation of duality transformation of potential, see below). The traces of this property were seen in the perturbation theory in [5] where duality transformation reduces to $v_c \to v_c^{-1}$ (v_c is the renormalized Fermi speed). However, this statement is far more general. It means that it is enough to consider, say, only repulsive potentials.

For a repulsive potential conductance restores if potential vanishes at $k \to 0$. Such a situation takes place in the systems with a small density of carriers when the screening radius is large. In this case $e{-}e$ interaction is not point-like from one-dimensional point of view, and it is screened by the image charges on 3-dimensional gates, edges of the channel, etc. Renormalization of the ballistic conductance in such system is finite and will be calculated below. Pay attention that the form and value of conductance is determined not by small k but by the whole region $V(k)$ where the potential is not small. So one needs an approach which is valid for an arbitrary $e{-}e$ potential, not only for a point-like one as in [5, 6].

The physical reason for critical phenomena of the conductance in the Luttinger model with an impurity is a long-range order which is present in a system of one-dimensional electrons. It is well-known that its analogue — the Schwinger model — exhibits the anomalous breakdown of chiral symmetry. The strength of the interaction in the repulsive Luttinger model is smaller: the system is in the Berezinskii–Kosterlitz–Thouless (BKT) phase [10]. Chiral condensate (consisting of pairs of R electron and L hole with finite density) arises only in the limit of an infinitely large interaction. In the case of an attractive potential there is a charged condensate of Cooper pairs with vanishing density (for the channel with infinite length), i.e. one has a BKT phase as well. The Bose–Einstein principle implies that the chiral condensate increases the probability of reflection, i.e. the effective reflection coefficient $|\mathcal{R}_\omega|^2$ at small frequencies, while the charged condensate increases the probability of transition. As a result, $|\mathcal{R}_\omega|^2 = 1$ for repulsion and $|\mathcal{K}_\omega|^2 = 1$ for attraction. As we mentioned above, this does not happen if $V(k \to 0) = 0$. We will see that in this case the long- range order in the Luttinger model also disappears. This leads to finite conductance of the channel.

Effective transition/reflection coefficients and conductance. The Fermi surface in one dimension reduces to two isolated points $\pm p_\mathrm{F}$. The electrons with momenta close to the Fermi surface can be divided in right (R) and (L) movers,

$$\Psi = e^{ip_\mathrm{F}x - i\varepsilon_\mathrm{F}t}\psi_\mathrm{R} + e^{-ip_\mathrm{F}x - i\varepsilon_\mathrm{F}t}\psi_\mathrm{L},$$

where $\psi_\mathrm{R,L}$ are slowly varying on the scale $1/p_\mathrm{F}$.

By means of the Hubbard [9] trick, the Luttinger model can be reduced to a system of noninteracting electrons in a random external field $U(x,t)$ with a simple Gaussian weight and subsequent integration in all possible realizations of the field. The Schrödinger equation for the non-interaction R, L electrons in the external field reduces to the Dirac equation in $d = 1$ (in our units $\hbar = v_\mathrm{F} = 1$; we will also omit electron charge e to restore it in the final expression for conductivity),

$$[i\partial_t \pm i\partial_x - U]\psi_\mathrm{R,L} = 0. \tag{1}$$

The Luttinger liquid is a system which can be solved exactly. The ultimate reason for this is that a one-dimensional fermion Green function in the external field can be found

$$\begin{gathered} G_\mathrm{R,L}(x,x') = G^{(0)}_\mathrm{R,L}(x-x')e^{i\gamma_\mathrm{R,L}(x) - i\gamma_\mathrm{R,L}(x')}, \\ \gamma_\mathrm{R,L}(x) = -\int d^2x' G^{(0)}_\mathrm{R,L}(x,x')U(x'). \end{gathered} \tag{2}$$

The Green functions $G_\mathrm{R,L}$ only by phase differs from the free Green functions

$$G^{(0)}_\mathrm{R,L}(x,t) = \frac{1}{2\pi i(t \mp x - i\delta t)} \tag{3}$$

($\delta > 0$ is infinitesimal).

A point-like impurity located at $x = 0$ mixes left and right electrons. Impurity plays the role of a boundary condition, solutions of Eq. (1) should be matched at $x = 0$. Nevertheless, the general solution in the external field can be found [8]. Solution depends on a new functional variable $\alpha(t)$ which is the difference of phases for R- and L-electrons at the point of impurity

$$\alpha(t) = \gamma_\mathrm{R}(0,t) - \gamma_\mathrm{L}(0,t). \tag{4}$$

Construction of the Green function with an impurity is impeded by the Feynman boundary conditions which lead to some integral equation. This equation can be solved perturbatively either in bare reflection or in transition coefficient (for details see [8]).

The Luttinger model has high symmetry: it is invariant both under gauge (vector) and chiral transformations (the latter symmetry is broken by the anomaly). The charge density ($\rho = \rho_\mathrm{R} + \rho_\mathrm{L}$) and current ($j = \rho_\mathrm{R} - \rho_\mathrm{L}$) can be completely determined from the conservation of the vector and axial currents:

$$\partial_t\rho + \partial_x j = 0, \quad \partial_t j + \partial_x\rho = -\frac{1}{\pi}\partial_x U + \mathfrak{D}(t)\delta(x). \tag{5}$$

Here, the first term on the right-hand side is the Adler anomaly [11]. The second term describes the influence

of the impurity, and $\mathfrak{D}$ is the charge jump at $x = 0$ which depends only on phase $\alpha(t)$. It can be calculated if the Green function is known.

Integrating in fermion degrees of freedom allows to present any quantity as a product of Green functions in the external field and fermion determinant describing the sum of the loop diagrams. As it was shown in [8] the effect of impurity is completely determined by the phase $\alpha(t)$: non-trivial part of the Green functions and determinant depends only on $\alpha(t)$. Introducing α as a new variable one can integrate also in $U(x,t)$ and reduce original $1+1$-dimensional model with impurity to the effective $0+1$ field theory (non-local quantum mechanics of the phase $\alpha(t)$).

The conductance of the channel $\mathcal{C}(\omega)$ is related to the exact transition coefficient

$$\mathcal{C}(\omega) = \frac{e^2|\mathcal{K}_\omega|^2}{2\pi v_r(\omega)}. \tag{6}$$

Here, $v_r(\omega)$ is the renormalized speed of an electron

$$v_r(\omega) = \sqrt{1+\frac{V(\omega)}{\pi}}, \quad v_c = v_r(0), \tag{7}$$

$$|\mathcal{R}_\omega|^2 \cdot 2\pi\delta(\omega-\omega') = \frac{i\pi}{|\omega|W(\omega)v_r(\omega)}\langle\langle\alpha(-\omega')\mathfrak{D}(\omega)\rangle\rangle. \tag{8}$$

Here the average is understood as an integral with the effective action:

$$\langle\langle\ldots\rangle\rangle = \frac{1}{\mathcal{Z}}\int D\alpha\ldots\mathfrak{D}et_{\rm imp}\exp\left[-\int\frac{d\omega}{2\pi}\frac{\alpha(-\omega)\alpha(\omega)}{2W(\omega)}\right], \tag{9}$$

and $W(\omega)$ is

$$W(\omega) = -\int\frac{dk}{2\pi i}\frac{4k^2V(k)}{(\omega^2-k^2+i\delta)[\omega^2-v_r^2(k)k^2+i\delta]}. \tag{10}$$

The "kinetic energy" in Eq. (9) is a well-known contribution of the Adler anomaly [4] to the effective action rewritten in terms of phase $\alpha(t)$ (see [8] for details).

At last, $\mathfrak{D}et_{\rm imp}$ takes care about the loop diagrams describing the multiple rescattering on the impurity. It can be expressed in terms of the charge jump $\mathfrak{D}$

$$\log\mathfrak{D}et_{\rm imp} = -\frac{i}{2}\int_0^1 d\lambda\int\frac{d\omega}{2\pi}\alpha(-\omega)\mathfrak{D}[\lambda\alpha](\omega),$$
$$\mathfrak{D}(\omega) = 2i\frac{\delta}{\delta\alpha(-\omega)}\log\mathfrak{D}et_{\rm imp}[\alpha]. \tag{11}$$

Expression (8) is only one of the possible representations for the effective reflection coefficient. Another useful representation

$$|\mathcal{R}_\omega|^2 = \frac{2\pi}{|\omega|W^2(\omega)v_r(\omega)}[\mathfrak{g}_0(\omega)-\mathfrak{g}(\omega)], \tag{12}$$

relates $\mathcal{R}_\omega$ to the Green function of the electron phase $\mathfrak{g}(\tau-\tau') = \langle\langle\alpha(\tau)\alpha(\tau')\rangle\rangle$ ($\mathfrak{g}_0$ is a Green function without impurity determinant). Expression (12) can be obtained from Eq. (8) taking functional integral by parts, it is one of the Ward identities in the effective theory.

The determinant $\mathfrak{D}et_{\rm imp}$ can be built as a series in bare reflection coefficient

$$\log\mathfrak{D}et_{\rm imp}[\alpha] = \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\left(\frac{|R|}{|K|}\right)^{2n}\mathfrak{T}_{2n-1}[\alpha], \tag{13}$$

where

$$\mathfrak{T}_n = \int\frac{d\tau_0\times\ldots\times d\tau_n}{(2\pi i)^{n+1}}\frac{1-\cos[\alpha(\tau_0)-\alpha(\tau_1)+\ldots+\alpha(\tau_n)]}{(\tau_0-\tau_1-i\delta)\ldots(\tau_n-\tau_0-i\delta)}. \tag{14}$$

In fact, we derived in [8] expression not for $\mathfrak{D}et_{\rm imp}$ but for the charge jump $\mathfrak{D}(\omega)$ which is a variational derivative of the determinant in α according to Eq. (11).

Formulae (8)–(14) allow to calculate conductance for attractive $e-e$ interaction as in this case $W(\omega)$ is positive. For the repulsive interaction one should use a different form of $\mathfrak{D}et_{\rm imp}$. As it was proved in [8] Eq. (13) can be also presented in a *dual* form as a series in the inverse parameter

$$\log\mathfrak{D}et_{\rm imp}[\alpha] = \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\left(\frac{|K|}{|R|}\right)^{2n}\mathfrak{T}_{2n-1}[\widetilde{\alpha}] + \int\frac{d\omega}{2\pi}\frac{|\omega|}{4\pi}\widetilde{\alpha}(-\omega)\widetilde{\alpha}(\omega), \tag{15}$$

where $\widetilde{\alpha}(\omega) = \mathrm{sign}(\omega)\alpha(\omega)$. It is natural to call the first term a dual determinant and combine the second one with the "kinetic energy" Eq. (10). Introducing $\widetilde{\alpha}$ as a new variable we obtain a dual theory with the kinetic energy

$$\widetilde{W}^{-1}(\omega) = -W^{-1}(\omega) - \frac{|\omega|}{2\pi}. \tag{16}$$

Effective coefficients $|\mathcal{R}_\omega|^2$ and $|\mathcal{K}_\omega|^2$ can be written either as functional integrals Eq. (9) in the original theory or as the functional integrals in the dual theory after the transformation:

$$|R|^2 \leftrightarrow |K|^2, \quad |\mathcal{R}_\omega|^2 \leftrightarrow |\mathcal{K}_\omega|^2, \quad W \leftrightarrow \widetilde{W}. \tag{17}$$

Conductance reemergence. It is well-known [5] that the coefficient $|\mathcal{R}_\omega|^2 \to 0$ for an attractive potential and $|\mathcal{K}_\omega|^2 \to 0$ for a repulsive one when $\omega \to 0$. This happens only if $V(k{=}0) \neq 0$ due to the infrared divergency. We reproduced this result for our effective $0{+}1$ dimensional theory in [8].

An example of a different behavior (for a repulsive potential with $V(k{=}0) = 0$) is given by a system of

one-dimensional electrons with small concentration. We will see that conductance remains finite in this case.

Indeed, if the concentration is small then the 3D screening radius can be much larger than the width d of the channel (which is considered to be zero in our one-dimensional theory). The bare $e-e$ interaction is screened by the "image charges" that arise on the split gates. The screened interaction is Coulomb one at distances smaller than the distance to the gate (l) and it is dipole-dipole in the opposite limit [12]:

$$V(k) = -\zeta \begin{cases} \log|k|d, & kl \gg 1, \\ (kl)^2 \log|k|d, & kl \ll 1. \end{cases} \qquad (18)$$

Here $\zeta \equiv 2/\pi a_{\rm B} p_{\rm F} \gg 1$, $a_{\rm B}$ is the Bohr radius. We assume that $d \ll l \ll L$ where L is the length of the one-dimensional channel and ζ is a largest parameter of the problem. Moreover, we will consider the case when transition coefficient is small: $|K|^2 \ll 1$. Then one can leave only first term in the expansion (15) and calculate the charge jump $\mathfrak{D}$ according to Eq. (11). Taking Gaussian integral in α we arrive at

$$|\mathcal{K}_\omega|^2 = \frac{|K|^2}{\pi} \int d\tau \frac{1-\cos\omega\tau}{|\omega|\tau^2} e^{-\sigma(\tau)}, \qquad (19)$$

where

$$\sigma(\tau) = \int \frac{d\omega'}{2\pi} \widetilde{W}(\omega') \left(1-\cos\omega'\tau\right). \qquad (20)$$

Behavior of transition coefficient at small ω is related to the asymptotic $\sigma(\tau)$ at large τ. The kinetic energy $\widetilde{W}(\omega)$ in Eq. (16) is nonsingular for repulsive potential (18) at $\omega \to 0$. For this reason $\sigma(\tau)$ has a finite limit σ_∞ at $\tau \to \infty$. This limit determines the conductance

$$\mathcal{C}(\omega=0) = \frac{e^2|K^2|}{2\pi} e^{-\sigma_\infty}, \quad \sigma_\infty = \int \frac{d\omega}{2\pi} \widetilde{W}(\omega). \qquad (21)$$

The renormalized Fermi speed $v_c = 1$ here.

At $l\sqrt{\zeta}/d \gg 1$ the main contribution to the integral in Eq. (21) comes from $kl \gg 1$, where the potential is not screened. The kinetic energy $\widetilde{W}(\omega)$ is determined by the pole $|\omega| = v_r(k_0)|k_0| \approx |k_0|\sqrt{\zeta \log(1/k_0 d)}$ in Eq. (10). It is equal to

$$\widetilde{W}(\omega') = \frac{2\pi}{|\omega'|} \sqrt{\zeta \log\left(\sqrt{\zeta}/|\omega'|d\right)} \qquad (22)$$

at $\omega' l \geq 1$. Finally

$$\sigma_\infty \approx \frac{4}{3}\sqrt{\zeta} \left(\log\sqrt{\zeta}\frac{l}{d}\right)^{3/2} \gg 1. \qquad (23)$$

Hence, conductance of channel in this limit is small. Let us note that conductance is determined not by the $k \to 0$ but by $k \sim 1/l$ where interaction of electrons is important.

Consider now also the opposite limit $\sigma_\infty \ll 1$ while K is not necessarily small. It can be implemented at intermediate concentrations if a 3-dimensional screening radius is of the order of the channel thickness. In this case we are dealing with a weak interaction and the conductance is determined by an expansion in powers of α

$$\mathfrak{T}_{2n-1} = \\ = \frac{n}{2!}\int \frac{d\omega}{(2\pi)^2} |\omega| \alpha(\omega)\alpha(-\omega) - \frac{n^2}{4!}\int \frac{d\omega_1 \times \ldots \times d\omega_4}{(2\pi)^4} \times \\ \times \delta\left(\sum_{i=1}^4 \omega_i\right) \Gamma_4(\omega_i)\alpha(\omega_1) \times \ldots \times \alpha(\omega_4) + \ldots, \qquad (24)$$

where the vertex Γ_4 is

$$\Gamma_4(\omega_i) = \sum_i |\omega_i| - \frac{1}{2}\sum_{i<j} |\omega_i + \omega_j|. \qquad (25)$$

This vertex is equal zero if any frequency ω_i vanishes. Substituting this expression into Eq. (13) and taking an integral over α with the quadratic form $\widetilde{W}$ we obtain the effective transition coefficient

$$|\mathcal{K}_\omega|^2 = |K|^2 - |R|^2|K|^2\sigma_\infty + O(\sigma_\infty^2), \qquad (26)$$

which again means that conductance is non-zero according to Eq. (6). Notice that this expression obeys duality which requires that the term linear in interaction should be symmetrical under $K \leftrightarrow R$ exchange. At small $|K|^2$ we return here to the expression in Eq. (21) expanded at small σ_∞[2)].

The physical reason for restoration of the conductance is, in fact, disappearance of long-range order in the system. In fact, one of two continuum symmetries present in the Luttinger model (chiral and gauge invariance) is always broken. To prove this let us consider first the case of repulsive potential. To check, whether the chiral order is present in the system we consider the following correlator

$$\mathcal{G}(R) = \langle \psi_{\rm R}(R,0)\psi^+_{\rm L}(R,0)\psi^+_{\rm R}(0,0)\psi_{\rm L}(0,0)\rangle. \qquad (27)$$

There are a number of methods to calculate (27) but the simplest is to use again the Hubbard trick [9]. Then one

[2)] For a point-like potential one has to substitute here $\sigma_\infty = 2\tilde{\nu}\log(M/\omega)$. Then Eq. (26) will coincide with the expression for the first logarithmic correction to conductance obtained in [6]. This expression was the base for the renorm-group approach developed in Ref. [6]. Comparison of the results obtained in our effective field theory with the renorm-group considerations will be published elsewhere.

can present $\mathcal{G}(R)$ as a product of two Green functions $G_{\rm R,L}$ of electron in the external field (2)

$$\mathfrak{G}(x,\bar{x},y,\bar{y}) = \overline{G_{\rm R}(x,\bar{x})G_{\rm L}(y,\bar{y})}, \tag{28}$$

where averaging is a functional integral

$$\frac{1}{\mathcal{Z}}\int DU\ldots\exp\left\{\frac{i}{2}\int\frac{d^2k}{(2\pi)^2}\frac{U(-k,-\omega)U(k,\omega)}{V(k)}\times\right.$$
$$\left.\times\frac{\omega^2-v_r^2(k)k^2+i\delta}{\omega^2-k^2+i\delta}\right\}, \tag{29}$$

and $x=(x,t)$ is two-dimensional coordinate (for details see Appendix of Ref. [13]).

According to Eq. (2) R- and L-electrons acquire a phase in the external field which is linear in U. Therefore integral in U is gaussian

$$\mathfrak{G}_{\rm RL}(x,\bar{x},y,\bar{y}) = G_{\rm R}^{(0)}(x,\bar{x})G_{\rm L}^{(0)}(y,\bar{y})\times \tag{30}$$
$$\times\exp\left[-\frac{i}{2}\int\frac{d^2k}{(2\pi)^2}\frac{V(k)}{[\omega^2-v_r^2(k)k^2+i\delta](\omega^2-k^2+i\delta)}\right]\times$$
$$\times\left[\left|(\omega+k)\left(e^{-ikx}-e^{-ik\bar{x}}\right)+(\omega-k)\left(e^{-iky}-e^{-ik\bar{y}}\right)\right|^2\right].$$

Now, to investigate chiral properties of the system we put here $x=\bar{y}=R$ and $\bar{x}=y=0$ and take the limit $R\to\infty$,

$$\mathcal{G}(R) = \frac{1}{4\pi^2R^2}\exp\left\{\int\frac{d^2k}{(2\pi)^2i}|1-e^{ikR}|^2\times\right.$$
$$\left.\times\frac{2k^2V(k)}{[\omega^2-(v_r)^2k^2+i\delta](\omega^2-k^2+i\delta)}\right\}. \tag{31}$$

The asymptotic at large R of this expression is given by:

$$\mathcal{G}(R) = \frac{1}{4\pi^2R^2}\exp\left\{-2\int_{1/R}^{\infty}\frac{dk}{|k|}\left[v_r(k)^{-1}-1\right]\right\}. \tag{32}$$

(Convergence of the integral in the ultraviolet is provided by the condition $v_r(k)\to 1$ at $k\to\infty$.) The renormalized Fermi speed $v_r(k)>1$ for all k. For the point-like interaction Eq. (32) turns into

$$\mathcal{G}(R)\sim 1/R^{2-2\eta},\quad \eta=1-\frac{1}{v_c}, \tag{33}$$

where v_c is the speed on the Fermi surface. In the limit $v_c\to\infty$ (an infinitely strong interaction) $\mathcal{G}(R)$ goes to some constant at large R. It means that a chiral condensate $\langle\psi_{\rm R}\psi_{\rm L}^+\rangle\neq 0$ is formed in the system. This phenomenon is known to happen also in the Schwinger model (see, e.g., [14]). For a finite interaction there is no condensate but the correlator $\mathcal{G}(\mathcal{R})$ decays slower than for free electrons and long-range order still exists. The number of correlated $R\bar{L}$ pairs is macroscopically large $\sim L^\eta$ and the system is in the BKT phase. We have constructed an exact wave function of the ground state of the Luttinger model and investigated the nature of this phase in Ref. [15].

The macroscopic number of $R\bar{L}$ pairs in the vacuum of Luttinger model amplifies the back-scattering of electrons on impurity owing to Bose–Einstein principle. As a result, as it is well-known [5], transition coefficient tends to zero at $\omega\to 0$:

$$|K|^2\sim\omega^{2\eta}. \tag{34}$$

For the dipole-dipole interaction (18) the integral (32) is infrared convergent since $v_r(k)=1$ at $k\le 1/l$. In this case at large distances

$$\mathcal{G}(R)\sim 1/R^2,$$

is the same as for free particles. Only pairs with momenta $k\ge 1/l$ are correlated strongly and the number of such pairs does not increase with system volume L. As a result transition coefficient remains finite.

Similar situation takes place for an attractive interaction but for the *charged* condensate (as for superconductivity). To reveal this condensate one considers the correlator:

$$\widetilde{\mathcal{G}}(R) = \langle\psi_{\rm R}^+(R,0)\psi_{\rm L}^+(R,0)\psi_{\rm R}(0,0)\psi_{\rm L}(0,0)\rangle, \tag{35}$$

i.e. we have to put $x=y=R$ and $\bar{x}=\bar{y}=0$. In the same way one has

$$\widetilde{\mathcal{G}}(R) = \frac{1}{4\pi^2R^2}\exp\left\{\int\frac{d^2k}{(2\pi)^2i}|1-e^{ikR}|^2\times\right.$$
$$\left.\times\frac{2\omega^2V(k)}{[\omega^2-(v_r)^2\omega^2+i\delta](\omega^2-k^2+i\delta)}\right\} \tag{36}$$

or

$$\widetilde{\mathcal{G}}(R) = \frac{1}{4\pi^2R^2}\exp\left\{-2\int_{1/R}\frac{dk}{|k|}\left[v_r(k)-1\right]\right\}. \tag{37}$$

For a point-like potential superconducting condensate arises for $v_c=0$, while for $0<v_c<1$ the system is in the BKT phase:

$$\widetilde{\mathcal{G}}(R)\sim 1/R^{2-2\widetilde{\eta}},\quad \widetilde{\eta}=1-v_c \tag{38}$$

and reflection coefficient $|R|^2\sim\omega^{2\widetilde{\eta}}$. For the screened potential BKT phase disappears and renormalization of conductance by interaction is finite.

We see that in all cases properties of $\mathcal{G}(R)$ at large R and of the conductance for small ω coincide for arbitrary $e-e$ potential. Thus, disappearance or restoration

of the conductance is related to the long-range order in the Luttinger model.

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