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$\pi^{\pm}$- and $\rho^{0,\pm}$-mesons in a strong magnetic field on the lattice

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We calculated the correlators of pseudoscalar and vector currents in external strong abelian magnetic field in $SU(3)$ gluodynamics. From the correlation functions we obtain the ground state energies (masses) of neutral ρ^0-meson and charged $\pi^{\pm}$- and $\rho^{\pm}$-mesons. The energy of the ρ^0-meson with zero spin projection on the axis of the field decreases, while the energies with non-zero spins increase with the field value. The mass of charged $\pi^{\pm}$-mesons increases with the field. We observe the agreement between Landau level picture and behaviour of charged $\rho^{\pm}$-mesons for moderate magnetic fields. There are no evidences in favour of charged vector meson condensation or tachyonic mode existence at large magnetic fields. The g-factor of $\rho^{\pm}$ is estimated in the chiral limit.

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1. Introduction. Researching QCD in the external electromagnetic field plays the important role in understanding the structure of hadrons. Today the strong magnetic fields of hadronic scale can be created in terrestrial laboratories like ALICA, RHIC, NICA, and FAIR. In non-central heavy ion colisions the magnetic field value in the moment of collision can reach up to $15m_{\pi}^2 \sim 0.27\,\mathrm{GeV}^2$ [1]. Such a strong magnetic field can modify the properties of strongly interacting matter. Many interesting effects have been observed in experiment and discovered theoretically, for example inverse magnetic catalysis [2], chiral magnetic effect [3, 4], enhancement of the chiral symmetry breaking [5–9].

The investigations related to QCD phase diagram in strong magnetic field are presented in [10–17]. Numerical simulations in QCD with $N_f = 2$ and $2+1$ show that the strongly interacting matter in strong magnetic field posses paramagnetic properties in the confinement and deconfinement phases [18–20].

In this work we explore the splitting of ground state energy of neutral ρ^0 and charged vector mesons $\rho^{\pm}$ depending on its spin projection on the axis of the external abelian magnetic field. This exploration is important because such splitting can lead to the asymmetry of emitted neutral and charged particles above and under reaction plane and contribute to the chiral magnetic effect. We also give a preliminary estimation of g-factor of charged $\rho^{\pm}$-mesons Articles [21–25] are also devoted to the behaviour of hadron masses in the external abelian magnetic field. The magnetic moments of ρ-mesons have been explored in [26–30]. Our value of g-factor is in agreement with the previous lattice calculations [31].

2. Details of calculations. The technical details of our calculations are presented in [32]. We generate $200-300$ $SU(3)$ statistically independent lattice gauge configurations for lattice volumes 16^4, 18^4 and lattice spacings $a = 0.105$, 0.115, and $0.125\,\mathrm{fm}$. The $U(1)$ external magnetic field is included only into the Dirac operator which is used for the calculation of eigenfunctions ψ_k and eigenvectors λ_k of a test quark in a background gauge field A_μ. This field is a sum of non-abelian $SU(3)$ gluonic field and $U(1)$ abelian constant magnetic field,

$$A_{\mu\,ij} \to A_{\mu\,ij} + A_\mu^B \delta_{ij}, \tag{1}$$

$$A_\mu^B(x) = \frac{B}{2}(x_1\delta_{\mu,2} - x_2\delta_{\mu,1}). \tag{2}$$

To take into account periodic boundary conditions for fermions the twisted boundary conditions are superim-

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posed [33]. Magnetic field is directed along z-axis and its value is quantized

$$qB = \frac{2\pi k}{(aL)^2}, \quad k \in \mathbb{Z}, \tag{3}$$

where $q = -1/3\,e$. Eq. (3) leads to a minimal value of magnetic field $(eB)^{1/2} = 380\,\text{MeV}$ for lattice volume 18^4 and lattice spacing $a = 0.125\,\text{fm}$. For each meson we construct interpolation operators with given quantum numbers. Then we calculate correlation functions of these operators in Euclidean space

$$\langle \psi^\dagger(x) O_1 \psi(x) \psi^\dagger(y) O_2 \psi(y) \rangle_A, \tag{4}$$

where we use $O_1, O_2 = \gamma_\mu, \gamma_\nu$ for the vector particle and γ_5 for pion, $\mu, \nu = 1, \ldots, 4$ are Lorenz indices. The Dirac propagator for the massive quark can be approximated by its eigenvectors and eigenvalues

$$D^{-1}(x,y) = \sum_{k<M} \frac{\psi_k(x)\psi_k^\dagger(y)}{i\lambda_k + m}, \tag{5}$$

where $M = 50$ is the number of the lowest eigenmodes. The correlator (4) is a sum of connected and disconnected contributions. The disconnected parts equal to zero because we consider the isovector states. We make 3-dimensional Fourier transformation of correlators and consider zero momentum $\mathbf{p} = 0$ because we are interested in the ground energy state.

For particles with zero momentum their energy is equal to its mass $E_0 = m_0$ in zero magnetic field. The expansion of correlation function to exponential series has the form

$$\tilde{C}(n_t) = \langle \psi^\dagger(\mathbf{0}, n_t) O_1 \psi(\mathbf{0}, n_t) \psi^\dagger(\mathbf{0}, 0) O_2 \psi(\mathbf{0}, 0) \rangle_A = \\ = \sum_k \langle 0|O_1|k\rangle \langle k|O_2^\dagger|0\rangle e^{-n_t a E_k}, \tag{6}$$

where a is the lattice spacing, n_t is the number of nodes in the time direction, E_k is the energy of the state with quantum number k. At large n_t the main contribution in (6) comes from the ground state. On the lattice due to the periodic boundary conditions the main contribution to the ground state has the following form

$$\tilde{C}_{fit}(n_t) = A_0 e^{-n_t a E_0} + A_0 e^{-(N_T - n_t) a E_0} = \\ = 2A_0 e^{-N_T a E_0/2} \cosh\left[\left(\frac{N_T}{2} - n_t\right) a E_0\right], \tag{7}$$

where A_0 is a constant, E_0 is the energy of the ground state. Therefore we fit our data for the correlators to the (7) function and find the ground state energy as a fit parameter. In order to minimize the errors and exclude the contribution of excited states we take various values of n_t from the interval $5 \le n_t \le N_T - 5$. We also use a smeared gaussian source and point sink for our calculations.

The correlation functions for various spatial directions are given by the following relations

$$C_{xx}^{VV} = \langle \bar\psi(\mathbf{0}, n_t)\gamma_1\psi(\mathbf{0}, n_t)\bar\psi(\mathbf{0}, 0)\gamma_1\psi(\mathbf{0}, 0)\rangle, \tag{8}$$

$$C_{yy}^{VV} = \langle \bar\psi(\mathbf{0}, n_t)\gamma_2\psi(\mathbf{0}, n_t)\bar\psi(\mathbf{0}, 0)\gamma_2\psi(\mathbf{0}, 0)\rangle, \tag{9}$$

$$C_{zz}^{VV} = \langle \bar\psi(\mathbf{0}, n_t)\gamma_3\psi(\mathbf{0}, n_t)\bar\psi(\mathbf{0}, 0)\gamma_3\psi(\mathbf{0}, 0)\rangle. \tag{10}$$

The form of the density matrix for vector particle with spin $s = 1$ gives the formulas for energies of meson with various spin projections on the axis of the external magnetic field.

For the $s_z = 0$ one can obtain the energy of the ground state from the C_{zz}^{VV} correlator. The combinations of correlators

$$C^{VV}(s_z = \pm 1) = C_{xx}^{VV} + C_{yy}^{VV} \pm i(C_{xy}^{VV} - C_{yx}^{VV}) \tag{11}$$

gives the energies of mesons with $s_z = +1$ and -1.

3. Results. In Fig. 1 we depict the mass of the ρ^0-meson with spin projections $s_z = \pm 1$ increasing with the

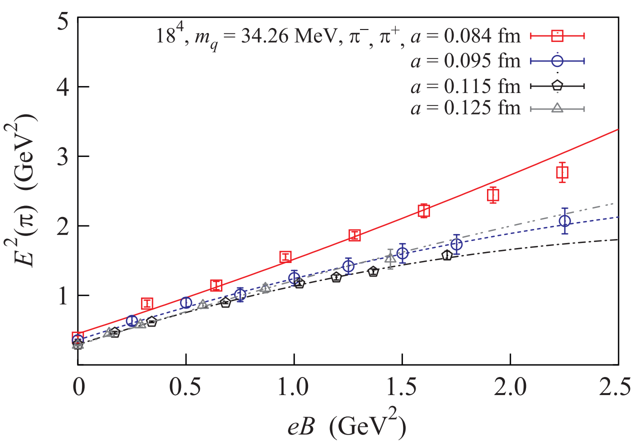

Fig. 1. The ground state energy of the neutral ρ^0-meson with spin $s_z = \pm 1$ as a function of magnetic field for lattice volumes 16^4 and 18^4, lattice spacings $a = 0.115$ fm and 0.125 fm and various bare quark masses

magnetic field value. The masses for $s_z = -1$ and $+1$ are equal which is a manifestation of definite C-parity of ρ^0-meson. Fig. 1 demonstrates small lattice spacing and lattice volume artefacts.

We do not present the neutral pion and ρ^0-meson with zero spin projection on the field axis, because there is a contribution of pion to correlators of vector currents in the external magnetic field due to abelian anomaly.

This problem requires more detailed investigation and will be studied in the future.

The energy levels of free charged pointlike particle in a background magnetic field is described by the formula

$$E^2 = |qB| - gs_z qB + m^2, \quad (12)$$

where g-factor characterizes magnetic properties of the particle, q is the charge of the particle, s_z is the spin projection on the field direction, m is the particle mass at $B = 0$. The Eq. (12) is true only for pointlike particle and doesn't take into account polarizabilities of mesons. If the particle is not pointlike then the magnetic polarizabilty is not zero. In the relativistic case the meson energy levels has the following form

$$E^2 = |qB| - gs_z qB + m^2 - 4\pi m\beta(qB)^2, \quad (13)$$

where β is the magnetic polarizability, the charge of the particle $q = -e$ for π^-, ρ^- or $q = +e$ for π^+, ρ^+ and m is its mass at $B = 0$. We consider ρ^--mesons while ρ^+ corresponds to the reversal of the direction of magnetic field.

In Fig. 2 the energy of charged $\pi^\pm$-meson is depicted.

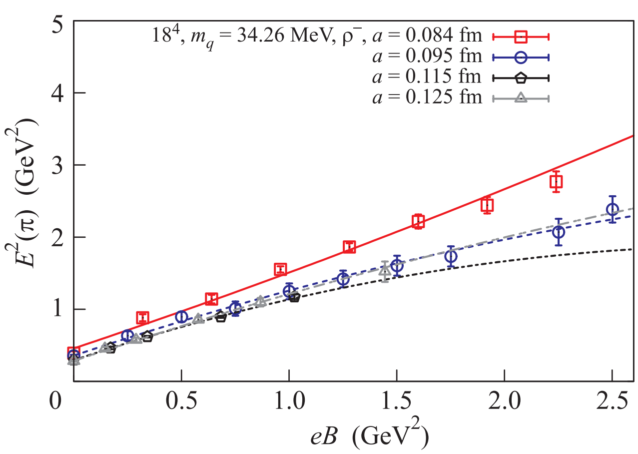

Fig. 2. The squared energy of the ground state of the charged $\pi^\pm$-meson with spin $s = 0$ as a function of the magnetic field for lattice volume 18^4, various lattice spacings $a = 0.084$, 0.095, 0.115, 0.125 fm and the bare quark mass equal to $m_q = 34.26$ MeV. The fit by solid curve corresponds to the data at lattice spacing $a = 0.084$ fm, the dashed curve is for $a = 0.095$ fm, the dashed–dotted curve describes data at $a = 0.115$ fm and dashed–dotted–dotted curve corresponds to $a = 0.125$ fm

The fitting curves correspond to the fit function $E^2 = |qB| + m^2 - 4\pi m\beta(qB)^2$, where m and β are the fit parameters. The masses of the $\pi^\pm$ increase with the field value. The nonzero values of β indicate a not pointlike compound nature of $\pi^\pm$-mesons. In Fig. 3 we represent

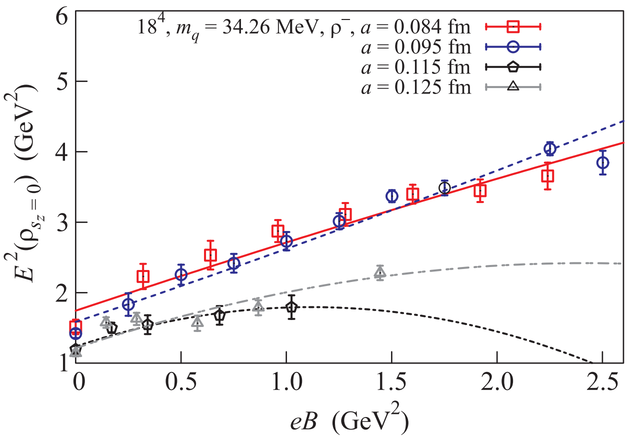

Fig. 3. The squared energy of the ground state of the charged ρ^--meson with spin $s_z = 0$ as a function of the magnetic field for lattice volume 18^4, various lattice spacings $a = 0.084$, 0.095, 0.115, 0.125 fm and the bare quark mass equal to $m_q = 34.26$ MeV. The various types of fitting curves correspond to the same sets of data as in Fig. 2. The data are fitted by Eq. (13) at $s_z = 0$

the energy of the charged vector ρ meson with $s_z = 0$, fits were performed by the formula (13) at $s_z = 0$, m and β are also the fit parameters. The value of magnetic polarizability is also not zero. For the calculation of β value for the ρ^--meson with zero spin and $\pi^\pm$ meson we has to increase statistics significantly, this will be done in the following work.

We show the energy of the ρ^--meson with spin projections $s_z = +1$ in Fig. 4, m, β, and g can be found from the fit. The energy of ρ^- ground state with $s_z = +1$ increases with the field value. The function (13) gives the excellent fits for the all presented data.

The energy of the ρ^- ground state with $s_z = -1$ decreases with the field value. The data are well described by the fitting function which include the term with fourth power in B

$$E^2 = |qB| - gs_z qB + m^2 - 4\pi m\beta(qB)^2 + k(qB)^4, \quad (14)$$

where k is the polarizability, corresponding to higher power in magnetic field B. So there is some effect which prevents the appearance of the tachyonic mode. We also have to note that the fitting curves cross the axis of zero energy for lattice spacings $a = 0.115$ fm and 0.125 fm because of the absence of lattice data for $eB > 1.5\,\text{GeV}^2$.

Let us now discuss statistical errors. From Eq. (11) one can easily see that the absolute errors are equal. This leads to the fact that we have same absolute errors for the correlators with $s_z = +1$ and -1. From the formula of correlator (7) we find error of the energy:

$$\delta E = \frac{\delta C/C}{a\{-N_T/2 + (N_T/2 - n_t)\text{th}[(N_T/2)Ea)]\}}. \quad (15)$$

From this formula for large N_T we have:

$$\frac{\delta E(s_z=+1)}{\delta E(s_z=-1)} \simeq \frac{C(s_z=-1)}{C(s_z=+1)}. \qquad (16)$$

Correlator exponentially decrease with energy (7) that leads to the increase of absolute errors of energy when its value increases. This explains the difference between errors on the Figs. 4 and 5.

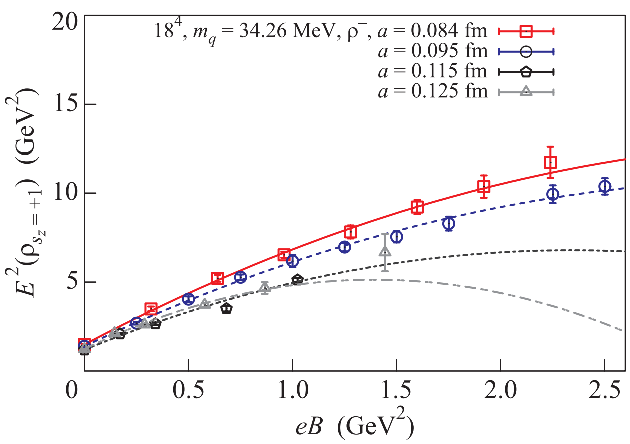

Fig. 4. The squared energy of the ground state of ρ^--meson with spin $s_z = +1$ versus the field value for lattice volume 18^4, various lattice spacings $a = 0.084$, 0.095, 0.115, 0.125 fm and the bare quark mass equal to $m_q = 34.26$ MeV. The data are fitted by Eq. (13)

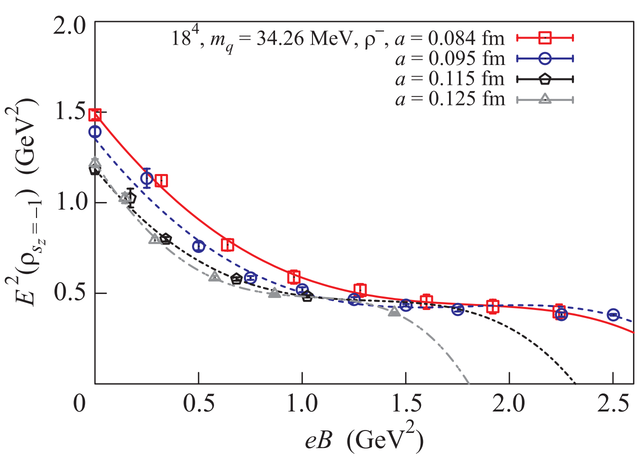

Fig. 5. The squared energy of the ground state of ρ^- meson with spin $s_z = -1$ versus the field value for lattice volume 18^4, various lattice spacings $a = 0.084$, 0.095, 0.115, 0.125 fm and the bare quark mass equal to $m_q = 34.26$ MeV. The data are fitted by Eq. (14)

The presented data allow one to make a preliminary estimation of g-factor of the $\rho^{\pm}$-meson at "small" magnetic field from the following relation

$$g = \frac{E^2(s=+1) - E^2(s=-1)}{2(eB)}, \qquad (17)$$

which is obtained from (12). We use this formula to cancel the contribution of polarizability of the charged ρ-meson, which is the topic for another investigation. After extrapolation to $m_q = 0$ we obtain the value $g = 2.4 \pm 0.2$ for the lattice volume 18^4 and lattice spacing $a = 0.115$ fm. This value is compatible with experimental determination [34]. We will improve the precision of this number in the following work.

Let us stress that Eq. (13) is, generally speaking, valid in the quadratic approximation in B only and may be spoiled by higher powers of B. We may immediately observe that the role of these terms varies dramatically for different spin orientations. While the case of $s = +1$ (Fig. 5) is reasonably well described by (13), the case of $s = +1$ Fig. 4 demands the inclusion of the term $k(eB)^4$ in the fitting formula. We consider this as an evidence for the absence of nullification of energy and related emergence of tachyonic mode. The latter is clearly manifested by the extrapolation of (14) at (Fig. 4) which crosses x-axis at $B \sim 1.75\,\mathrm{GeV}^2$. Contrary to that, our calculations show that nonlinear terms become important already at $B \sim 1\,\mathrm{GeV}^2$ making energy dependence on B rather flat. The tachyonic mode may lead to charged vector mesons condensation, and our data are not supporting such an option. QCD seems to disfavour this exciting opportunity. Of course, such an important conclusion requires both further numerical investigations and search for theoretical explanation of this phenomenon.

4. Conclusions. We have presented the exploring of ground state energies the neutral ρ^0- and charged $\rho^{\pm}$- and π-mesons in $SU(3)$ lattice gauge theory. The energies of the ρ^0-meson with non-zero spin $|s_z| = 1$ increase. The investigation of ρ^0 with zero spin requires additional numerical calculations and will be done in the future.

The energies of charged pions and ρ^--meson with zero spin projection are described by Eq. (13) at $s_z = 0$. While the energy of charged $\rho^{\pm}$-mesons with $s_z = +1$ agrees with the formulae (13), the case of opposite spin $s_z = -1$ is well described by the dependence (14) and demands the inclusion of terms with higher powers in B. There is some mechanism which prevents the appearance of tachyonic mode. We didn't observe any evidence in favour of charged vector meson condensation as presented in [35, 36]. We also estimate g-factor of $\rho^{\pm}$ in the chiral limit, it equals to $g = 2.4 \pm 0.2$ for the lattice volume 18^4 and lattice spacing $a = 0.115$ fm.

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