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On a search for the $\eta \to e^+e^-$ decay at the VEPP-2000 e^+e^- collider

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A sensitivity of the VEPP-2000 e^+e^- collider in a search for the rare decay $\eta \to e^+e^-$ has been studied. The inverse reaction $e^+e^- \to \eta$ is proposed for this search. We have analyzed a data sample with an integrated luminosity of 108 nb^{-1} collected with the SND detector in the center-of-mass energy range 520−580 MeV and found no background events for the reaction $e^+e^- \to \eta$ in the decay mode $\eta \to \pi^0\pi^0\pi^0$. In the absence of background, a sensitivity to $\mathcal{B}(\eta \to e^+e^-)$ of 10^{-6} can be reached during two weeks of VEPP-2000 operation. Such a sensitivity is better than the current upper limit on $\mathcal{B}(\eta \to e^+e^-)$ by a factor of 2.3.

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Decays of pseudoscalar mesons to the pair of leptons $P \to l^+l^-$ are rare. In the Standard Model (SM) these decays proceed through the two-photon intermediate state as shown in Fig. 1 and therefore are suppressed

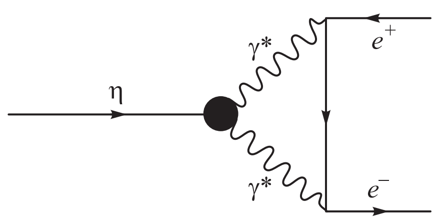

Fig. 1. The Feynman diagram for $\eta \to e^+e^-$

by a factor of $\sim \alpha^2$ comparing with the $P \to \gamma\gamma$ decays, where α is the fine structure constant. An additional suppression of $(m_l/m_P)^2$ arises from the approximate helicity conservation, where m_l and m_P are the lepton and meson masses, respectively. So, the $P \to l^+l^-$ partial width is less than the two-photon width $\Gamma(P \to \gamma\gamma)$ by a factor of $\sim \alpha^2(m_l/m_P)^2$. The low probability makes these decays sensitive to possible contributions of new physics beyond the SM [1, 2]. In the SM, the knowledge of the transition meson-photon form factor $F(q_1^2, q_2^2)$ for the $\gamma^*\gamma^* \to P$ vertex is needed for a $\Gamma(P \to l^+l^-)$ calculation, where q_1^2 and q_2^2 are four-momenta squared of virtual photons in Fig. 1. The real and imaginary parts of the decay amplitude are usually calculated separately. The imaginary part $\Im(A)^2$ is proportional to the form-factor value $|F(0,0)|^2$ at $q_1^2 = q_2^2 = 0$ and, consequently, can be calculated using the known width of the $P \to \gamma\gamma$ decay. Since $|A|^2 > \Im(A)^2$, a model-independent lower boundary (unitary bound) on the $P \to l^+l^-$ width can be obtained from $\Gamma(P \to \gamma\gamma)$ [3]. The real part of amplitude cannot be calculated in a model-independent way. In Ref. [4] it is shown that the real part depends on the integral over $q^2 < 0$ of the form factor in symmetric kinematics, $F(q^2, q^2)$. The ranges of predictions for the $P \to l^+l^-$ branching fractions obtained in different form-factors models [5, 6] are listed in Table 1. For comparison, the last column of Table 1 contains the current experimental values of the branching fractions. Currently, only two of the five decays are measured and for two more upper limits on the width exist. The value of $\mathcal{B}(\eta \to \mu^+\mu^-)$ agrees with the prediction but the measurement accuracy is low. The more precise value of $\mathcal{B}(\pi^0 \to e^+e^-)$ differs from the prediction by about three standard deviations.

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Table 1. The unitary bounds, theoretical predictions in units of the unitary bound [5, 6], and experimental values for the $P \to l^+l^-$ branching fractions

$\mathcal{B}(P \to l^+l^-)$	Unitary bound (U)	Theory	Experiment
$\mathcal{B}(\pi^0 \to e^+e^-) \cdot 10^8$	4.69	6.23−6.38	7.49 ± 0.38 [7]
$\mathcal{B}(\eta \to e^+e^-) \cdot 10^9$	1.78	4.60−5.24	< 2300 [8]
$\mathcal{B}(\eta \to \mu^+\mu^-) \cdot 10^7$	4.36	4.64−5.12	5.8 ± 0.8 [9]
$\mathcal{B}(\eta' \to e^+e^-) \cdot 10^{10}$	0.36	1.15−1.86	< 56 [10, 11]
$\mathcal{B}(\eta' \to \mu^+\mu^-) \cdot 10^7$	1.35	1.14−1.36	–

It is clear that the current experimental situation requires improvements in accuracy for the $\pi^0 \to e^+e^-$ and $\eta \to \mu^+\mu^-$ decays as well as measurements of the other three decays. Such measurements are planned at the BES-III [12], KLOE-2 [13], Crystal Ball [14], and WASA [15, 16] detectors in the near future.

The upper limit on $\mathcal{B}(\eta' \to e^+e^-)$ has been recently set in experiments with the CMD-3 [11] and SND [10] at VEPP-2000, where the inverse reaction $e^+e^- \to \eta'$ was used. In this paper we propose the same method for a search for the $\eta \to e^+e^-$ decay and perform its feasibility study.

Data used in this paper were collected with the SND detector at the VEPP-2000 [17] collider in 2013. VEPP-2000 is designed for a study of e^+e^- annihilation at the center-of-mass (c.m.) energy E from 160 MeV up to 2 GeV. There are two detectors at the collider, SND and CMD-3, which collect data simultaneously. At the moment of data taking the VEPP-2000 accelerator complex consisted of the 3 MeV electron linac ILU, the 250 MeV pulsed synchrotron B-3M, the 900 MeV booster storage ring BEP, and the collider storage ring VEPP-2000.

Currently the complex VEPP-2000 is being upgraded. The maximum energy of BEP will be increased up to 1 GeV, and the injection system will be changed. Electrons and positrons will be transported to BEP from the VEPP-5 injection complex [18] through a 250 m beamline. The upgrade allows to increase the VEPP-2000 luminosity at maximum energy up to $10^{32}\,\mathrm{cm^{-2}\,s^{-1}}$ and should result in a more stable operation of the accelerator complex.

For the current study, VEPP-2000 parameters at c.m. energy close to $m_\eta c^2 = 548.862 \pm 0.018$ MeV [9] such as luminosity, accuracy of the energy setting, energy spread, are important. In 2013 SND did not record data exactly at this energy. Therefore, we analyze data from four energy points near $m_\eta c^2$, with c.m. energies of 520, 540, 560, and 580 MeV. The integrated luminosity collected at these energy points is measured using the reaction $e^+e^- \to \gamma\gamma$ to be 108.1 ± 2.0 nb^{-1}. The average luminosity during data taking varied from $0.26 \cdot 10^{30}\,\mathrm{cm^{-2}\,s^{-1}}$ at 520 MeV to $0.73 \cdot 10^{30}\,\mathrm{cm^{-2}\,s^{-1}}$ at 580 MeV. Interpolating the energy dependence we estimate that the average luminosity expected at $E = m_\eta c^2$ is $0.34 \cdot 10^{30}\,\mathrm{cm^{-2}\,s^{-1}}$.

The width of the η resonance, $\Gamma_\eta = 1.31 \pm \pm 0.05$ keV [9], is much less than the collider c.m. energy spread σ_E. This means that the value of σ_E is crucial for the proposed search for the $e^+e^- \to \eta$ reaction. At $E = m_\eta c^2$ the energy spread is determined by the Touschek effect. It is uniquely related to the RMS of the longitudinal distribution of the e^+e^- interaction point σ_Z [11]:

$$\sigma_E = 4.05\sigma_Z\sqrt{V_{\mathrm{cav}} E_b \sin[\arccos(63.2E_b^4/V_{\mathrm{cav}})]}, \quad (1)$$

where σ_E is measured in keV, σ_Z in mm, the RF cavity voltage V_{cav} in kV, and the beam energy $E_b = E/2$ in GeV. The value of σ_Z is measured using detected events of the elastic scattering $e^+e^- \to e^+e^-$. At $E = 540$ MeV data taking was carried out with several values of the RF cavity voltage ranged from 18 to 36 kV. The values of σ_Z varied from 16 to 12 mm, while the values of σ_E from 120 to 160 keV. Below we will use the value of $\sigma_E = 150$ keV as an estimate of the energy spread at $E = m_\eta c^2$.

In the proposed experiment the collider energy must be set and monitored with an accuracy better than σ_E. This is provided by the beam-energy-measurement system based on measuring energies of Compton backscattered laser photons [19]. The accuracy of this method is about 60 keV. The system monitors beam energy continuously during data taking; the single measurement duration at $E \approx 550$ MeV is about one hour.

The search for $e^+e^- \to \eta$ events will be performed with the SND detector [20–23]. It is a nonmagnetic detector. The main part of the detector is a three-layer spherical electromagnetic calorimeter consisting of 1640 NaI(Tl) crystals. The calorimeter covers a solid angle of about 90 % of 4π. The energy and angular resolutions for photons with energy E_γ are described by the following formulae

$$\sigma_{E_\gamma}/E_\gamma = 4.2\,\%/\sqrt[4]{E_\gamma(\mathrm{GeV})}, \quad (2)$$

$$\sigma_{\theta,\phi} = 0.82^\circ/\sqrt{E(\mathrm{GeV})}. \quad (3)$$

Directions of charged particles are measured in a nine-layer drift chamber. The calorimeter is surrounded by an iron absorber and a muon detector. In the proposed search the muon detector is used to veto cosmic rays.

The energy dependence of the Born cross section for the reaction $e^+e^- \to \eta$ is described by the Breit–Wigner formula:

$$\sigma_0 = \frac{4\pi}{E^2}\mathcal{B}(\eta \to e^+e^-)\frac{m_\eta^2\Gamma_\eta^2}{(m_\eta^2 - E^2)^2 + m_\eta^2\Gamma_\eta^2}. \quad (4)$$

In experiment, it is necessary to take into account radiative corrections arising, for example, from radiation of extra photons from the initial state. To do this, the cross section (4) is convolved with the so-called radiator function $W(s,x)$ [24, 25]

$$\sigma(s) = \int_0^{x_{\max}} W(x,s)\sigma_0[s(1-x)]dx, \quad (5)$$

where $s = E^2$. The upper limit of integration in Eq. (5) depends on the decay mode and equals unity for the η decay into two photons and $1-(3m_{\pi^0})^2/s$ for the decay into $3\pi^0$. The theoretical accuracy of the corrected cross section (5) is better than 1 % [24, 25]. For the unitary bound $\mathcal{B}(\eta \to e^+e^-) = 1.78\cdot10^{-9}$ the Born cross section in the resonance maximum is $\sigma_0 = 29$ pb. The radiative corrections lead to decrease of this cross section down to $\sigma = 14$ pb.

To take into account the collider energy spread, the cross section (5) should be convolved with the Gaussian function describing the distribution of the integrated luminosity over energy

$$\sigma_{\exp}(E_0) = \frac{1}{\sqrt{2\pi}\sigma_E}\int_{-\infty}^{+\infty} \exp\left[-\frac{(E-E_0)^2}{2\sigma_E^2}\right]\sigma(E)dE, \quad (6)$$

where E_0 is the average beam energy. For $\sigma_E = 150$ keV, $E_0 = m_\eta c^2$, and $\mathcal{B}(\eta \to e^+e^-) = 1.78\cdot10^{-9}$ the visible cross section is

$$\sigma^U_{\exp}(m_\eta c^2) = 105 \pm 11 \text{ fb}. \quad (7)$$

Thus, the energy spread allows to use only $\sigma_{\exp}/\sigma \approx \approx 1/140$ of collected integrated luminosity. The uncertainty on $\sigma^U_{\exp}(m_\eta c^2)$ is due to the uncertainty of the beam-energy measurement (60 keV).

The main η decay modes are $\eta \to \gamma\gamma$ (39.4 %), $\eta \to \pi^0\pi^0\pi^0$ (32.7 %), and $\eta \to \pi^+\pi^-\pi^0$ (22.9 %). The numbers in parentheses are the branching fractions [9]. In the $\eta \to \gamma\gamma$ mode the process $e^+e^- \to \eta$ cannot be separated from the QED process $e^+e^- \to \gamma\gamma$.

The most suitable η decay mode for the search for the $e^+e^- \to \eta$ reaction at SND is $\eta \to \pi^0\pi^0\pi^0 \to 6\gamma$, for which physical background is small. The main source of background is cosmic-ray events. For the search for $e^+e^- \to \eta$, events with six or more detected photons and with the energy deposition in the calorimeter larger than $0.6E$ are selected. Background from events with charged particles is rejected by the selection condition that the number of fired wires in the drift chamber is less than four. Cosmic-ray background is suppressed by the veto from the muon detector.

For events passing the preliminary selection, a kinematic fit to the $e^+e^- \to \pi^0\pi^0\pi^0 \to 6\gamma$ hypothesis is performed with a requirement of total energy and momentum conservations and a condition that invariant masses of three photon pairs are equal to π^0 mass. The quality of the kinematic fit is characterized by the parameter χ^2. During the fit all possible combinations of photons are checked and a combination with the smallest χ^2 value is selected. The χ^2 distribution for simulated events of the process $e^+e^- \to \eta$ passed the selection criteria for six-photon events is shown in Fig. 2. The condition $\chi^2 < 100$ is used to select η candidates.

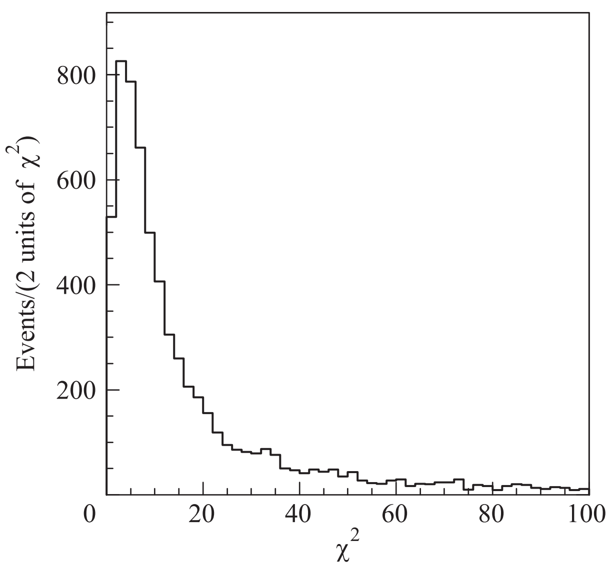

Fig. 2. The distribution of χ^2 of the kinematic fit for simulated $e^+e^- \to \eta$ events passing the selection criteria for six-photon events

The detection efficiency for $e^+e^- \to \eta$ events is obtained using simulation to be $\varepsilon = 12.5 \pm 0.6$ %. The quoted error is systematic. It is estimated using results of Ref. [26], where comparison of data and simulated χ^2 distributions was performed for five-photon events from the process $e^+e^- \to \omega\pi^0 \to \pi^0\pi^0\gamma$.

No events satisfying the six-photon selection criteria are found in the data sample with an integrated luminosity of $108.1 \pm 2.0\,\mathrm{nb}^{-1}$ recorded in the energy points with $E = 520$, 540, 560, and 580 MeV.

Main sources of physical background for the decay mode $\eta \to \pi^0\pi^0\pi^0$ are five-photon events of the processes $e^+e^- \to \pi^0\pi^0\gamma$ and $e^+e^- \to 5\gamma$ with a spurious photon from beam-generated background or splitting of electromagnetic showers in the calorimeter. The probability to find a spurious photon in an event may reach up to 5 %. This number is used below for background estimation. The cross section for $e^+e^- \to \pi^0\pi^0\gamma$ at $E \approx 550$ MeV is determined mainly by the transition $e^+e^- \to \rho(770) \to \pi^0\pi^0\gamma$ and is estimated to be 2 ± 1 pb. The quoted uncertainty is due to an unknown mechanism of the the $\rho \to \pi^0\pi^0\gamma$ decay [27]. The detection efficiency for $e^+e^- \to \pi^0\pi^0\gamma$ events with the six-photon selection criteria is about 1 %. The cross section for $e^+e^- \to 5\gamma$ with detection of all five photons is calculated using the CompHep program [28] to be about 4 pb. The requirement of an extra photon and condition $\chi^2 < 100$ decrease the cross section by a factor of about 1/200. Thus, the total cross section of background processes with the selection criteria for the decay mode $\eta \to \pi^0\pi^0\pi^0$ is about 0.04 pb and negligible at integrated luminosities, which can be collected at VEPP-2000.

The second decay mode suitable to search for $e^+e^- \to \eta$ events is $\eta \to \pi^+\pi^-\pi^0$. For this mode there is a background from the nonresonant process $e^+e^- \to \pi^+\pi^-\pi^0$. There are no data on the $e^+e^- \to \to \pi^+\pi^-\pi^0$ cross section at $E \approx 550$ MeV. To estimate it, we use the results of Ref. [29], where the cross section $e^+e^- \to \pi^+\pi^-\pi^0$ was measured by the SND detector at the VEPP-2M collider in the energy region of the $\omega(782)$ and $\rho(770)$ resonances. In Ref. [29] the data were fitted in several models, which at $E = 550$ MeV give the cross section value ranged from 6 to 16 pb. Such cross section values correspond to branching fractions $\mathcal{B}(\eta \to e^+e^-) = (0.4–1.2) \cdot 10^{-6}$. Thus, at the level of sensitivity to $\mathcal{B}(\eta \to e^+e^-)$ of 10^{-6} the measurement of the $e^+e^- \to \pi^+\pi^-\pi^0$ out of the η-meson resonance may be required. Other background sources in the mode $\eta \to \pi^+\pi^-\pi^0$ are the QED processes $e^+e^- \to e^+e^-\gamma(\gamma)$ and $e^+e^- \to \mu^+\mu^-\gamma(\gamma)$. In a data sample collected with the SND detector at $E = 520$, 540, 560, and 580 MeV, we cannot suppress background from these processes to the level reached in the mode $e^+e^- \to \pi^0\pi^0\pi^0$.

The measured cross section $e^+e^- \to \eta$ is determined as

$$\sigma_{\mathrm{exp}} = \frac{N_s}{\varepsilon L}, \tag{8}$$

where N_s is the number of selected $e^+e^- \to \eta$ events, ε is the detection efficiency, and L is the integrated luminosity.

From our result that no background events for the decay mode $\eta \to \pi^0\pi^0\pi^0$ are found in the data sample with an integrated luminosity of $108.1\,\mathrm{nb}^{-1}$ we estimate the sensitivity of the $e^+e^- \to \eta$ cross section measurement. The upper limit on the cross section at the 90 % confidence level (CL) [30] corresponding to $N_s = 0$ is

$$\sigma_{\mathrm{exp}} < \frac{2.3}{0.125 \cdot 108} = 170\ \mathrm{pb}. \tag{9}$$

Comparing this limit with the cross section (7) calculated for the unitary bound $\mathcal{B}(\eta \to e^+e^-) = 1.78 \cdot 10^{-9}$, we estimate the sensitivity to the search for the decay $\eta \to e^+e^-$ with an integrated luminosity of 108 nb^{-1} to be

$$\mathcal{B}(\eta \to e^+e^-) < 2.9 \cdot 10^{-6} \tag{10}$$

at 90 % CL. This result is close to the upper limit $\mathcal{B}(\eta \to e^+e^-) < 2.3 \cdot 10^{-6}$ set recently in the HADES experiment [8], in which η mesons were produced in proton-nucleon collisions. With a VEPP-2000 luminosity of $0.34 \cdot 10^{30}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$ the current upper limit can be reached in a week of data taking. In two weeks a sensitivity at the level of 10^{-6} can be reached.

In conclusion, we have analyzed the data sample with an integrated luminosity of $108\,\mathrm{nb}^{-1}$ collected with the SND detector at the VEPP-2000 e^+e^- collider at c.m. energies 520, 540, 560, and 580 MeV and found no background for the process $e^+e^- \to \eta$ in the decay mode $\eta \to \pi^0\pi^0\pi^0$. In the absence of background, data with an integrated luminosity of 324 nb^{-1} provide a sensitivity of 10^{-6} for $\mathcal{B}(\eta \to e^+e^-)$. Such data may be accumulated in two weeks of VEPP-2000 operation. The sensitivity of 10^{-6} is 2.3 times better than the current upper limit on $\mathcal{B}(\eta \to e^+e^-)$.

Data at $E = m_\eta c^2$ will be recorded during the experiment on a measurement of hadronic cross sections below 1 GeV planned at VEPP-2000. Two detectors, SND and CMD-3, will collect data simultaneously. Their results may be combined to improve sensitivity to $\mathcal{B}(\eta \to e^+e^-)$ by a factor of about 2.

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1. Y. Kann, M. Schmidt and T. M. P. Tait, Phys. Rev. D **78**, 115002 (2008).
2. Q. Chang and Y. D. Yang, Phys. Lett. B **676**, 88 (2009).
3. S. Berman and D. Geffen, Nuovo Cim. **18**, 1192 (1960).
4. A. E. Dorokhov and M. A. Ivanov, Phys. Rev. D **75**, 114007 (2007).

5. T. Petri, arXiv:1010.2378 [nucl-th].
6. A. E. Dorokhov, Phys. Part. Nucl. Lett. **7**, 229 (2010); arXiv:0905.4577 [hep-ph].
7. E. Abouzaid et al. (KTeV Collaboration), Phys. Rev. D **75**, 012004 (2007).
8. G. Agakishiev et al. (HADES Collaboration), Phys. Lett. B **731**, 265 (2014).
9. K. A. Olive et al. (Particle Data Group), Chin. Phys. C **38**, 090001 (2014).
10. M. N. Achasov et al., (SND Collaboration), Phys. Rev. D **91**, 092010 (2015).
11. R. R. Akhmetshin et al. (CMD-3 Collaboration), Phys. Lett. B **740**, 273 (2015).
12. Q. P. Ji, L. Zhang, S. J. Chen, H. B. Li, and G. R. Lu, Chin. Phys. C **34**, 305 (2010).
13. P. Moskal, Fizika B **20**, 223 (2011).
14. A. Starostin, Eur. Phys. J. ST **198**, 117 (2011).
15. C. O. Gullstrom, PoS STORI **11**, 043 (2011).
16. P. Wurm, *Measurement of the $\eta \to e^+e^-e^+e^-$ double Dalitz decay and the search for new physics beyond the Standard Model in $\eta \to e^+e^-$ with WASA-at-COSY*, PhD Thesis, University of Cologne, Germany (2012).
17. A. L. Romanov, D. E. Berkaev, A. S. Kasaev, I. Koop, A. N. Kyrpotin, A. P. Lysenko, E. Perevedentsev, V. P. Prosvetov, Yu. A. Rogovsky, A. I. Senchenko, P. Yu. Shatunov, Y. M. Shatunov, D. B. Shwartz, A. N. Skrinsky, I. M. Zemlyansky, and Yu. M. Zharinov, in *Proceedings of PAC 2013*, Pasadena, CA USA (2013), p. 14.
18. A. V. Akimov, P. A. Bak, A. M. Barnyakov, M. F. Blinov, Yu. M. Boimelshtein, D. Yu. Bolkhovityanov, F. A. Emanov, A. R. Frolov, E. A. Gusev, N. S. Dikansky, E. S. Kazantseva, S. N. Klyuschev, N. Kh. Kot, A. A. Korepanov, R. M. Lapik, N. N. Lebedev, P. V. Logachev, P. V. Martyshkin, D. A. Nikiforov, V. M. Pavlov, A. V. Petrenko, I. L. Pivovarov, S. L. Samoilov, T. V. Rybitskaya, B. A. Skarbo, A. A. Starostenko, S. V. Shiyankov, and A. S. Tsyganov, in *Proceedings of IPAC 2014*, Dresden, Germany (2014), p. 538.
19. E. V. Abakumova, M. N. Achasov, D. E. Berkaev, V. V. Kaminsky, I. A. Koop, A. A. Korol, S. V. Koshuba, A. A. Krasnov, N. Yu. Muchnoi, E. A. Perevedentsev, E. E. Pyata, P. Yu. Shatunov, Yu. M. Shatunov, and D. B. Shwartz, Nucl. Instrum. Meth. Phys. Res. A **744**, 35 (2014).
20. M. N. Achasov, D. E. Berkaev, A. G. Bogdanchikov, D. A. Bukin, I. A. Koop, A. A. Korol, S. V. Koshuba, D. P. Kovrizhin, A. V. Otboev, E. A. Perevedentsev, Yu. A. Rogovsky, A. L. Romanov, P. Yu. Shatunov, Yu. M. Shatunov, D. B. Shwartz, A. A. Valkovich, and I. M. Zemlyansky, Nucl. Instrum. Meth. Phys. Res. A **598**, 31 (2009).
21. V. M. Aulchenko, A. G. Bogdanchikov, A. A. Botov, A. D. Bukin, D. A. Bukin, T. V. Dimova, V. P. Druzhinin, P. V. Filatov, V. B. Golubev, A. G. Kharlamov, A. A. Korol, S. V. Koshuba, A. E. Obrazovsky, E. V. Pakhtusova, V. M. Popov, S. I. Serednyakov, A. A. Sirotkin, I. K. Surin, Yu. A. Tikhonov, E. G. Tsukanova, Yu. V. Usov, A. V. Vasiljev, and V. M. Vesenev, Nucl. Instrum. Meth. Phys. Res. A **598**, 102 (2009).
22. A. Yu. Barnyakov, M. Yu. Barnyakov, K. I. Beloborodov, V. S. Bobrovnikov, A. R. Buzykaev, A. F. Danilyuk, V. B. Golubev, V. L. Kirillov, S. A. Kononov, E. A. Kravchenko, A. P. Onuchin, K. A. Martin, S. I. Serednyakov, and V. M. Vesenev, Nucl. Instrum. Meth. Phys. Res. A **598**, 163 (2009).
23. V. M. Aulchenko, A. G. Bogdanchikov, A. A. Botov, D. A. Bukin, M. A. Bukin, E. A. Chekushkin, T. V. Dimova, V. P. Druzhinin, A. A. Korol, S. V. Koshuba, A. I. Tekutiev, and Yu. V. Usov, Nucl. Instrum. Meth. Phys. Res. A **598**, 340 (2009).
24. O. Nicrosini and L. Trentadue, Phys. Lett. B **196**, 551 (1987).
25. F. J. Berends, W. L. Van Neerven, and G. J. H. Burges, Nucl. Phys. B **297**, 429 (1988); Nucl. Phys. B **304**, 921 (1988).
26. M. N. Achasov et al. (SND Collaboration), Phys. Rev. D **88**, 054013 (2013).
27. M. N. Achasov et al. (SND Collaboration), Phys. Lett. B **537**, 201 (2002).
28. E. Boos et al. (CompHEP Collaboration), Nucl. Instrum. Meth. Phys. Res. A **534**, 250 (2004).
29. M. N. Achasov et al. (SND Collaboration), Phys. Rev. D **68**, 052006 (2003).
30. R. D. Cousins and V. L. Highland, Nucl. Instrum. Meth. Phys. Res. A **320**, 331 (1992).