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Towards conformal cosmology

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Approximate de Sitter symmetry of inflating Universe is responsible for the approximate flatness of the power spectrum of scalar perturbations. However, this is not the only option. Another symmetry which can explain nearly scale-invariant power spectrum is conformal invariance. We give a short review of models based on conformal symmetry which lead to the scale-invariant spectrum of the scalar perturbations. We discuss also potentially observable features of these models.

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Observational data show that primordial scalar perturbations in the Universe must have been generated at some early cosmological stage, preceding the hot epoch. They are nearly Gaussian and have nearly flat power spectrum [1]. The first property suggests that these perturbations originate from amplified vacuum fluctuations of weakly coupled quantum field(s). Indeed, the defining property of Gaussian random field $\zeta(\mathbf{x})$ is that it obeys the Isserlis–Wick theorem, which holds also for any free quantum field in its vacuum state, while linear evolution in classical background does not induce non-Gaussianity.

The second property is also very suggestive. The power spectrum $\mathcal{P}(k)$ defined as

$$\langle\zeta(\mathbf{k})\zeta(\mathbf{k}')\rangle = \frac{1}{4\pi k^3}\mathcal{P}(k)\delta(\mathbf{k}+\mathbf{k}')$$

gives the fluctuation in logarithmic interval of momenta,

$$\langle(\zeta(\mathbf{x}))^2\rangle = \int\limits_0^\infty \frac{dk}{k}\mathcal{P}(k), \quad \mathcal{P}\propto k^{n_s-1},$$

where n_s is a spectral index. The flat or scale invariant spectrum corresponds to $n_s = 1$. In early 70's Harrison, Zeldovich Peebles and Yu [2] conjectured that the spectrum is flat, to avoid large deviation from homogeneity and isotropy of the observed Universe on large scales and black hole formation on small scales. Current observational data [1] show that the power spectrum is indeed nearly flat and give $n_s - 1 \approx -0.032$.

The flatness of the power spectrum may be due to some symmetry. The best known candidate is the symmetry $SO(4,1)$ of the de Sitter metric $ds^2 = dt^2 - e^{2Ht}d\mathbf{x}^2$, which includes spatial dilatations supplemented by time translations: $\mathbf{x}\to\lambda\mathbf{x}$, $t\to t - (H)^{-1}\log\lambda$. This is the approximate symmetry of inflating Universe [3], and, indeed, the inflationary mechanism of the generation of scalar perturbations [4] produces almost flat power spectrum. Despite this success of the inflationary paradigm, search for alternatives to inflation in general and to de Sitter symmetry in particular is of obvious interest. Alternatives to inflation include ekpyrotic model [5] with negative exponential potential and bounce [6], "starting" [7] Universe, etc, while there are several mechanisms capable of producing flat or almost flat scalar spectrum [8–11]. In some cases, there is no obvious symmetry that guarantees the flatness, i.e., the scalar spectrum is flat accidentally.

In quest for an alternative symmetry behind the nearly flat scalar spectrum one naturally turns to conformal symmetry $SO(4,2)$ [12–15] (see Ref. [16] for related discussion). Conformal group includes dilatations, $x^\mu\to\lambda x^\mu$, which in the end may be responsible for the scale-invariant scalar spectrum. An assumption of conformal invariance at the time the primordial perturbations are generated is in line with the viewpoint that the underlying theory of Nature may have conformal phase, and that the Universe may have started off from,

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or passed through an unstable conformal state and then evolved to much less symmetric state we see today.

In this paper we give a short review of models based on conformal symmetry, which lead to the scale-invariant spectrum of the scalar perturbations. It is worth noting that one does not necessarily have to consider these models as alternatives to inflation, since some of them can work in inflating Universe as well.

1. Two sample ways of getting flat scalar spectrum. *1.1. Conformal and global symmetry instead of de Sitter symmetry.* To begin with, let us consider a scenario proposed in Ref. [12]. In this scenario conformal symmetry is supplemented by a global symmetry. The simplest model of this sort has global symmetry $U(1)$ and involves complex scalar field ϕ, which is conformally coupled to gravity. The action of the model is $S = S_{G+M} + S_\phi$, where S_{G+M} is the action of gravity and dominating matter, while the non-trivial dynamics of the scalar field, which is assumed to be spectator, is governed by

$$S_\phi = \int d^4x\sqrt{-g}\left[g^{\mu\nu}\partial_\mu\phi^*\partial_\nu\phi + \frac{R}{6}\phi^*\phi - (-h^2|\phi|^4)\right].$$

Thus, quartic potential allowed by conformal invariance is assumed to be *negative*. Therefore, $\phi = 0$ is an unstable state with unbroken conformal symmetry. One assumes that the background space-time is homogeneous, isotropic and spatially flat, $ds^2 = a^2(\eta)(d\eta^2 - d\mathbf{x}^2)$. Then in terms of the field $\chi(\eta,\mathbf{x}) = a(\eta)\phi(\eta,\mathbf{x})$ the dynamics is the same as in flat space-time,

$$\eta^{\mu\nu}\partial_\mu\partial_\nu\chi - 2h^2|\chi|^2\chi = 0. \quad (1)$$

Spatially homogeneous background approaches the late-time attractor

$$\chi_c(\eta) = \frac{1}{h(\eta_* - \eta)}, \quad (2)$$

where η_* is an arbitrary real parameter, and we consider real solution, without loss of generality. The background solution (2) breaks conformal group $SO(4,2) \to$ $\to SO(4,1)$. The meaning of the parameter η_* is that the field χ_c would run away to infinity as $\eta \to \eta_*$, if the scalar potential remained negative quartic at arbitrarily large fields. It is worth noting that the particular behaviour $\chi_c \propto (\eta_* - \eta)^{-1}$ is dictated by conformal symmetry.

Phase perturbations. To see how the scale invariant spectrum emerges in the model, let us consider perturbations of the phase $\theta = \mathrm{Arg}\phi$, or, for the real background (2), perturbations of the imaginary part $\chi_2 \equiv \mathrm{Im}\chi/\sqrt{2}$. At the linearized level, perturbations of the phase and modulus decouple and the linearized equation is

$$(\delta\chi_2)'' - \partial_i\partial_i\,\delta\chi_2 - \frac{2}{(\eta_* - \eta)^2}\,\delta\chi_2 = 0. \quad (3)$$

An important assumption of the entire scenario is that the rolling stage begins early enough, so that there is time at which the following inequality holds:

$$k(\eta_* - \eta) \gg 1, \quad (4)$$

where $k = |\mathbf{k}|$ is conformal momentum. Since the momenta k of cosmological significance are as small as the present Hubble parameter, this inequality means that the duration of the rolling stage in conformal time is longer than the conformal time elapsed from, say, the beginning of the hot Big Bang expansion to the present epoch. This is only possible if the hot Big Bang stage was preceded by some other epoch, at which the standard horizon problem is solved; the mechanism we discuss here is meant to operate at that epoch. We note in passing that the latter property is inherent in most, if not all, mechanisms of the generation of cosmological perturbations.

Eq. (3) is exactly the same as equation for minimally coupled massless scalar field in the de Sitter background. Nevertheless, let us briefly discuss its solutions. At early times, when the inequality (4) is satisfied, the third term in (3) is negligible and $\delta\chi_2$ is free massless quantum field,

$$\delta\chi_2(\mathbf{x},\eta) = \int \frac{d^3k}{(2\pi)^{3/2}\sqrt{2k}}\left(\delta\chi_2^{(-)}(\mathbf{k},\mathbf{x},\eta)\hat{A}_\mathbf{k} + \text{h.c.}\right),$$

whose modes are

$$\delta\chi_2^{(-)}(\mathbf{k},\mathbf{x},\eta) = e^{i\mathbf{kx} - ik\eta}. \quad (5)$$

Here $\hat{A}_\mathbf{k}$ and $\hat{A}_\mathbf{k}^\dagger$ are annihilation and creation operators obeying the standard commutational relation, $[\hat{A}_\mathbf{k}, \hat{A}_{\mathbf{k}'}^\dagger] = \delta(\mathbf{k} - \mathbf{k}')$. It is natural to assume that the field $\delta\chi_2$ is initially in its vacuum state.

The rolling background $\chi_c(\eta)$ produces an effective "horizon" for the perturbations $\delta\chi_2$. The oscillations (5) terminate when the mode exits the "horizon", i.e., at $k(\eta_* - \eta) \sim 1$. The solution to Eq. (3) with the initial condition (5) is

$$\delta\chi_2^{(-)}(\mathbf{k},\mathbf{x},\eta) = e^{i\mathbf{kx} - ik\eta_*}F(k,\eta_* - \eta), \quad (6)$$

where

$$F(k,\xi) = -\sqrt{\frac{\pi}{2}k\xi}\;H_{3/2}^{(1)}(k\xi) \quad (7)$$

and $H^{(1)}_{3/2}$ is the Hankel function. In the late-time super-"horizon" regime, when $k(\eta_* - \eta) \ll 1$ and the third term in (3) dominates, one has

$$F(k, \eta_* - \eta) = \frac{i}{k(\eta_* - \eta)}. \qquad (8)$$

Hence, the super-"horizon" perturbations of the phase $\delta\theta \equiv \delta\chi_2/\chi_c$ are time-independent,

$$\delta\theta(\mathbf{x}) = \frac{\delta\chi_2(\mathbf{x},\eta)}{\chi_c(\eta)} = ih \int \frac{d^3k}{4\pi^{3/2}k^{3/2}}\, e^{i\mathbf{kx} - ik\eta_*} \hat{A}_{\mathbf{k}} + \text{h.c.} \qquad (9)$$

This expression describes Gaussian random field whose power spectrum is flat:

$$\mathcal{P}_{\delta\theta} = \frac{h^2}{(2\pi)^2}. \qquad (10)$$

We emphasize that this result is an automatic consequence of the global $U(1)$ and conformal symmetries. To see this, let us consider long wavelength regime. In this regime the second term in (3) is negligible and (3) becomes the equation for spatially homogeneous perturbation. Recall that χ_c is the spatially homogeneous solution to the full field equation (1), hence, due to $U(1)$ symmetry, $e^{i\alpha}\chi_c$ is also a solution, where α is a real constant. For small α the latter solution is $\chi_c + i\alpha\chi_c$, and the imaginary part is a small perturbation, which is precisely $\delta\chi_2$. So, if the perturbation $\delta\chi_2$ oscillates with unit amplitude, it behaves at late times as (cf. (5) and (8)) $\delta\chi_2 = C/[k(\eta_* - \eta)]$, where the factor k^{-1} is evident on dimensional grounds and C is independent of time and k.

Deviation from exact conformal invariance naturally gives rise to the tilt in the power spectrum, which depends both on the way conformal invariance is broken and on the evolution of the scale factor [17].

1.2. Galilean Genesis. The Galileon model has been introduced in Ref. [18]. In Minkowski space-time, the Lagrangian of the simplest conformally-invariant version [13] of the model is

$$L_\pi = -f^2 e^{2\pi}\partial_\mu\pi\partial^\mu\pi + \frac{f^3}{\Lambda^3}\partial_\mu\pi\partial^\mu\pi\Box\pi + \frac{f^3}{2\Lambda^3}(\partial_\mu\pi\partial^\mu\pi)^2\,, \qquad (11)$$

where $\Box = \partial_\mu\partial^\mu$. This Lagrangian is conformally invariant, with π transforming under dilatations as $e^{\pi(x)} \to \to \lambda e^{\pi(\lambda x)}$.

In the Galilean Genesis scenario the Universe begins from Minkowski space-time. The field equation in Minkowski space-time admits a homogeneous attractor solution

$$e^{\pi_c} = \frac{1}{H_{\mathrm{G}}(t_* - t)}, \qquad (12)$$

where $H_{\mathrm{G}}^2 = 2\Lambda^3/(3f)$. The form of the solution is again dictated by conformal invariance.

Initial energy density is zero, while effective pressure is negative. Then the energy density slowly builds up and the Hubble parameter grows in time,

$$H(t) = \frac{1}{3}\frac{f^2}{M_{Pl}^2}\frac{1}{H_{\mathrm{G}}^2(t_* - t)^3},$$

until $(t_* - t) \sim H_{\mathrm{G}}^{-1} f/M_{Pl}$. The growth of the Hubble parameter is due to violation of all energy conditions. Nevertheless, the theory is fully self-consistent: there are no ghosts, tachyons, and other pathologies (there is, however, superluminality issue which has not been quite settled [19]). At some time Galileon is assumed to transmit its energy to conventional matter, and hot epoch begins.

Galileon perturbations per se are not suitable for generating scalar perturbations (see discussion in Sec. 2). For the purpose of generating the scalar perturbations another field θ of conformal weight 0 is introduced. By conformal invariance, its quadratic Lagrangian has the form

$$L_\theta = e^{2\pi}(\partial_\mu\theta)^2 \quad \Rightarrow \quad L_\theta(\pi_c) = \frac{\text{const}}{(t_* - t)^2}(\partial_\mu\theta)^2.$$

That is, the dynamics of perturbations $\delta\theta$ in the background π_c is exactly the same as in the conformal rolling model discussed above.

The similarity between Galilean Genesis and conformal rolling model is not accidental. In Ref. [20] general arguments are given, which show that the scale invariant power spectrum is inherent in an entire class of models. The general setting is conformally invariant theory of a scalar field ρ of conformal weight $\Delta \neq 0$ in effectively Minkowski space-time. Up to rescaling this field corresponds to $|\phi|$ in the conformal rolling model and to e^π in the Galilean Genesis scenario; in both models $\Delta = 1$. The form of homogeneous classical solution

$$\rho_c(t) = \frac{1}{(t_* - t)^\Delta} \qquad (13)$$

is dictated by conformal invariance (t is conformal time in the case of the conformal rolling scenario). As mentioned above, the perturbations of the field ρ do not have flat power spectrum, so another *spectator* scalar field θ of conformal weight 0 is introduced. Then by conformal invariance, the kinetic term in the Lagrangian of θ is $L_\theta \propto \rho^{2/\Delta}(\partial_\mu\theta)^2$. Assuming that possible potential terms are negligible the Lagrangian in the rolling background (13) takes the form

$$L_\theta = \frac{\text{const}}{(t_* - t)^2}(\partial_\mu\theta)^2,$$

which is exactly the same as the Lagrangian of a scalar field minimally coupled to gravity in de Sitter space with conformal time t and scale factor $a(t) \propto 1/(t_* - t)$. As a result, θ develops perturbations with the flat power spectrum.

1.3. Further aspects. Obviously, generating the field perturbations $\delta\theta$ is not the whole story. There are several other ingredients of the conformal scenario. Some of them have not yet been worked out in detail.

Beginning of rolling. The rolling stage (13) has to start in one or another way. One possibility is spontaneous decay of the unstable conformally invariant vacuum $\rho = 0$, which proceeds through bubble nucleation. Such a decay has been discussed in Ref. [21] within the holographic approach (holographic picture of the conformal scenario has been suggested earlier in Refs. [22, 23]). Even though the rolling field is not spatially homogeneous in the false vacuum decay process, the field perturbations have the properties we discuss in Secs. 1.1, (2).

End of rolling. The rolling stage (13) should terminate at some late time. This implies that conformal invariance is broken at large field values. As an example, in the conformal rolling scenario of Sec. 1.1 one assumes that the scalar potential has a minimum or nearly flat valley at large $|\phi|$ and that $|\phi|$ eventually settles there. Furthermore, in the Galilean Genesis scenario there must be a stage of defrosting, i.e., transmission of energy from the rolling field ρ to heat [24], after which the usual hot Big Bang epoch begins.

Reprocessing perturbations $\delta\theta$ into adiabatic perturbations. The field perturbations $\delta\theta$ are to be converted into adiabatic perturbations. This can happen at the hot Big Bang epoch. One possibility is to make use of the curvaton mechanism [25]. As an example, in the scenario of Sec. 1.1 the phase θ may actually be a pseudo-Nambu–Goldstone field. Generically, conformal rolling ends up at a slope of its potential, see Fig. 1. The perturbations $\delta\theta$ are reprocessed into adiabatic perturbations at later time at the hot Big Bang epoch when the field θ oscillates and decays into conventional particles, cf. Ref. [26].

Another possibility is the modulated decay mechanism suggested in the inflationary context in Refs. [27–29].

In both cases the conversion of the field perturbations $\delta\theta$ into adiabatic perturbations induces some degree of non-Gaussianity. This is not a specific property of the conformal scenario, however, as it holds in all models employing the curvaton or modulated decay mechanism, including versions of inflation.

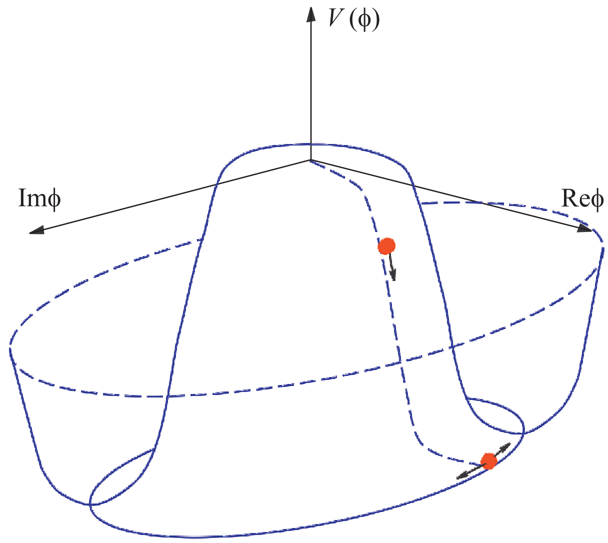

Fig. 1. (Color online) The scalar potential in the pseudo-Nambu–Goldstone scenario

2. Perturbations of modulus. Let us now come back to the conformal rolling stage. From now on we use the nomenclature of the negative quartic model of Sec. 1.1 for definiteness. We are interested in the radial perturbations [12, 30, 31], i.e., perturbations of the modulus of the field χ, or, with our convention of real background χ_c, perturbations of the real part $\chi_1 \equiv \mathrm{Re}\,\chi/\sqrt{2}$. At the linearized level, they obey the following equation,

$$(\delta\chi_1)'' - \partial_i\partial_i\,\delta\chi_1 - 6h^2\chi_c^2\delta\chi_1 = 0.$$

Its solution that tends to properly normalized mode of free quantum field at early times, $k(\eta_* - \eta) \to \infty$, is

$$\delta\chi_1 = -e^{i\mathbf{kx} - ik\eta_*}\frac{i}{4\pi}\sqrt{\frac{\eta_* - \eta}{2}}H^{(1)}_{5/2}\left[k(\eta_* - \eta)\right]\hat{B}_{\mathbf{k}} + \text{h.c.},$$

where $\hat{B}_{\mathbf{k}}$, $\hat{B}^\dagger_{\mathbf{k}}$ is another set of annihilation and creation operators. At late times, when $k(\eta_* - \eta) \ll 1$ (super-"horizon" regime), one has

$$\delta\chi_1 = -e^{i\mathbf{kx} - ik\eta_*}\frac{3}{4\pi^{3/2}}\frac{1}{k^{5/2}(\eta_* - \eta)^2}\hat{B}_{\mathbf{k}} + \text{h.c.}$$

Hence, the super-"horizon" perturbations of the modulus have red power spectrum

$$\mathcal{P}_{|\phi|}(k) \propto k^{-2}. \quad (14)$$

The dependence $\delta\chi_1 \propto (\eta_* - \eta)^{-2}$ is interpreted in terms of the local shift of the "end time" parameter η_*. Indeed, with the background field given by (2), the sum $\chi_c + \delta\chi_1$, i.e., the radial field including perturbations, is the linearized form of

$$\chi_c[\eta_*(\mathbf{x}) - \eta] = \frac{1}{h[\eta_*(\mathbf{x}) - \eta]}, \quad (15)$$

where $\eta_*(\mathbf{x}) = \eta_* + \delta\eta_*(\mathbf{x})$ and

$$\delta\eta_*(\mathbf{x}) = -\frac{3h}{4\pi^{3/2}} \int \frac{d^3k}{k^{5/2}} \left(e^{i\mathbf{kx} - ik\eta_*} \hat{B}_{\mathbf{k}} + \text{h.c.} \right). \quad (16)$$

So, the infrared radial modes modify the effective background by transforming the "end time" parameter η_* into time-independent random field that slowly varies in space, with red power spectrum clearly seen from (16). This observation is valid beyond the linear approximation: once the spatial scale of variation of $\chi_1(\mathbf{x}, \eta)$ exceeds the "horizon" size, spatial gradients in Eq. (1) are negligible, and the late-time solutions to the full nonlinear field equation have locally one and the same form (2), modulo slow variation of η_* in space.

A few remarks are in order. First, the infrared modes contribute both to the field $\delta\eta_*(\mathbf{x})$ itself and to its spatial derivatives. The contribution of the modes which are superhorizon today, i.e., have momenta $k \lesssim H_0$, to the fluctuation of $\partial_i \eta_*$ is given by

$$\langle \partial_i \eta_*(\mathbf{x}) \partial_j \eta_*(\mathbf{x}) \rangle_{k \lesssim H_0} = \delta_{ij} \frac{3h^2}{4\pi} \int_{k \lesssim H_0} \frac{dk}{k} = \\ = \delta_{ij} \frac{3h^2}{4\pi} \log \frac{H_0}{\Lambda}, \quad (17)$$

where Λ is the infrared cutoff which parametrizes our ignorance of the dynamics at the beginning of the conformal rolling stage.

Second, modulo field redefinition and notations, the properties of Galileon perturbations are exactly the same as the properties of radial perturbations $\delta\chi_1$ in conformal rolling scenario [32]. Furthermore, these properties are unambiguously determined by conformal invariance [32, 20]. The same properties – flat and red spectra of zero conformal weight and rolling fields, respectively – are inherent in the false vacuum decay setup mentioned in Sec. 1.3. Thus, we are dealing with the whole class of models.

Finally, so far we have considered spectator fields. That is, we have ignored the backreaction of the scalar fields on gravity. In particular, we have considered a scalar field *conformally* coupled to gravity (in the conformal rolling scenario of Sec. 1.1) and assumed negligible energy density (in both Galilean Genesis and conformal rolling cases). However, it is of interest to consider also dynamical versions with scalar fields *minimally* coupled to gravity and dominating the cosmological evolution. In that case there is a potential danger that the strong-coupling regime arises at too low energy scales [14]. This option has been studied in Ref. [33]. It has been shown that mixing of the scalar field(s) with the metric in dynamical pseudo-conformal models does not introduce new strong-coupling UV scales. Furthermore, the spectator approximation gives correct results in dynamical models provided that the background space-time is sufficiently flat. This applies, in particular, to potentially observable effects discussed in Sec. 1.3. These effects are inherent in the entire class of both spectator and dynamical (pseudo-)conformal models.

3. Effect of infrared radial modes on perturbations of phase. Let us discuss how the interaction with the infrared radial modes affects the properties of the phase perturbations $\delta\theta$ [30, 31, 34]. To this end, we consider perturbations of the imaginary part $\delta\chi_2$, whose wavelengths are much smaller than the scale of the spatial variation of the modulus. Because of the separation of scales, perturbations $\delta\chi_2$ can still be treated in the linear approximation, but now in the background (15), see Fig. 2.

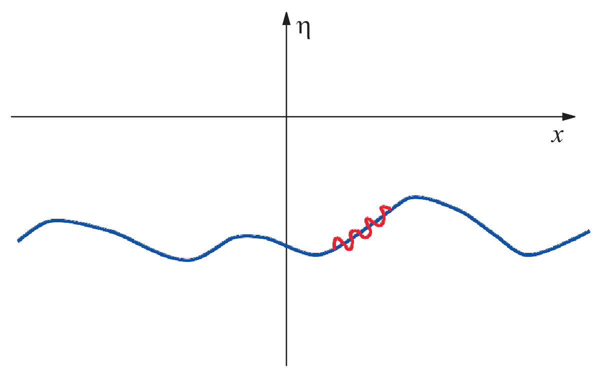

Fig. 2. (Color online) Due to the perturbations of the radial field, the evolution of phase perturbations proceeds in inhomogeneous background that slowly varies in space

Since our concern is the infrared part of $\eta_*(\mathbf{x})$, we make use of the spatial gradient expansion, consider a region near the origin and write

$$\eta_*(\mathbf{x}) = \eta_*(0) - v_i x_i + \ldots, \quad v_i = -\delta_i \eta_*(\mathbf{x})|_{\mathbf{x}=0}, \quad (18)$$

where dots denote higher order terms in $\mathbf{x}$. Importantly, the field $\delta_i \delta_j \eta_*(\mathbf{x})$ has blue power spectrum, unlike $\eta_*(\mathbf{x})$ and $\delta_i \eta_*(\mathbf{x})$, so the major effect of the infrared modes is accounted for by considering the two terms of the gradient expansion written explicitly in (18). Furthermore, we assume in what follows that $|\mathbf{v}| \ll 1$. The expansion in $|\mathbf{v}|$ is legitimate, since the field $\mathbf{v}(\mathbf{x})$ has flat power spectrum (cf. (17)), so the fluctuation of $\mathbf{v}$ is of order $h^2 |\log \Lambda|$, where Λ is the infrared cutoff, and it is small for small h and not too large $|\log \Lambda|$.

Keeping the two terms in (18) only, we have, instead of (3),

$$(\delta\chi_2)'' - \delta_i \delta_i \, \delta\chi_2 - \frac{2}{[\eta_*(0) - \mathbf{vx} - \eta]^2} \delta\chi_2 = 0\,. \quad (19)$$

We observe that the denominator in the expression for the background field

$$\chi_c = \frac{1}{h[\eta_*(0) - \eta - \mathbf{vx}]} \tag{20}$$

contains the combination $\eta_*(0) - (\eta + \mathbf{vx})$. We interpret this as the local time shift and Lorentz boost of the original background (2). Note that the field (20) is a solution to the field equation (1) in our approximation. Our interpretation makes it clear that the solutions to Eq. (19) can be obtained by time translation and Lorentz boost of the original solution (6), (7). In particular, instead of (9) the phase field freezes out at

$$\delta\theta(\mathbf{x}) = ih \int \frac{d^3k}{4\pi^{3/2}\sqrt{k}(k + \mathbf{kv})}\, e^{i\mathbf{kx} - ik\eta_*(\mathbf{x})} \hat{A}_{\mathbf{k}} + \text{h.c.} \tag{21}$$

So far we discussed the dynamics of the phase perturbations $\delta\theta$ at the conformal rolling stage which is governed solely be their interaction with the background field (20) as well as with the radial perturbations $\delta\chi_1$; the evolution of the scale factor $a(\eta)$ is irrelevant. After the end of conformal rolling, the situation is reversed. Once the radial field $|\phi|$ has relaxed to the minimum of the scalar potential, the phase θ is a massless scalar field minimally coupled to gravity (this is true for any Nambu–Goldstone field [35]). Since we are talking about a yet unknown pre-hot epoch, it is legitimate to ask what happens to the perturbations of the phase right after the end of conformal rolling. Barring fine tuning, there are two possibilities for the perturbations $\delta\theta$:

(i) they are already superhorizon in the conventional sense at that time, or

(ii) they are still subhorizon.

3.1. Superhorizon phase perturbations. Let us consider the first sub-scenario [30, 31]: phase perturbations do not evolve after the end of the conformal rolling stage, and the properties of the adiabatic perturbations are determined entirely by the dynamics at conformal rolling (modulo possible non-Gaussianity generated at the conversion epoch, see Sec. 1.3). This option is particularly natural in the Galilean Genesis, but it is not contrived in the conformal rolling scenario of Sec. 1.1 either.

In that case, there are no potentially observable effects to the linear order in $\mathbf{v}$. Indeed, using (21) one finds for, e.g., two-point correlation function

$$\langle\delta\theta(\mathbf{x})\delta\theta(\mathbf{x}')\rangle \propto$$
$$\propto \int \frac{d^3k}{k}\frac{1}{(k + \mathbf{kv})^2} e^{i\mathbf{k}(\mathbf{x}-\mathbf{x}') - ik[\eta_*(\mathbf{x}) - \eta_*(\mathbf{x}')]} =$$
$$= \int \frac{d^3q}{q}\frac{1}{q^2} e^{i\mathbf{q}(\mathbf{x}-\mathbf{x}')}, \tag{22}$$

where we have used (18) in the exponent and changed the integration variable from $\mathbf{k}$ to

$$\mathbf{q} = \mathbf{k} + k\mathbf{v}, \quad q = |\mathbf{q}| = k + \mathbf{kv}, \tag{23}$$

which is nothing but Lorentz boost. The result (22) is precisely the two-point correlation function of the linear field (9). The latter argument is straightforwardly generalized to multiple correlators: for a given realization of the random field $\eta_*(\mathbf{x})$, they are all expressed in terms of the two-point correlation function (22). The reason for the disappearance of the linear order effect is obviously Lorentz invariance.

The non-trivial effect of the large wavelength perturbations $\delta\eta_*(\mathbf{x})$ on the perturbations of the phase, and hence on the resulting adiabatic perturbations, occurs for the first time at the second order in the gradient expansion, i.e., at the order $\partial_i\partial_j\eta_*$ [30]. Let us concentrate for the moment on the effect of the modes of $\delta\eta_*$ whose present wavelengths exceed the present Hubble size. We are dealing with one realization of the random field $\delta\eta_*$, hence at the second order of the gradient expansion, $\partial_i\partial_j\eta_*$ is merely a tensor, constant throughout the visible Universe. In this long wavelength regime the perturbation of the phase has the following form

$$\delta\theta(\mathbf{k}) = \frac{ihe^{i\mathbf{kx} - ik\eta_*(\mathbf{x})}}{4\pi^{3/2}\sqrt{k}q}\left(1 - \frac{\pi}{2k}\frac{k_ik_j}{k^2}\partial_i\partial_j\eta_*\right)\hat{A}_{\mathbf{k}} + \text{h.c.}, \tag{24}$$

where q is given by (23). It results in the power spectrum of the adiabatic perturbation which depends on *directionality* of momentum (see [30] for details)

$$\mathcal{P}_\zeta(\mathbf{k}) = \mathcal{P}^0(k)\left[1 + c_1 h\frac{H_0}{k}(n_k)_i(n_k)_j w_{ij} - \right.$$
$$\left. - c_2 h^2(\mathbf{n_k u})^2\right]. \tag{25}$$

In the first non-trivial term, w_{ij} is a traceless symmetric tensor of a general form with unit normalization, $w_{ij}w_{ij} = 1$, $\mathbf{n_k}$ is a unit vector, $\mathbf{n_k} = \mathbf{k}/k$, and c_1 is a constant of order 1 whose actual value is undetermined because of the cosmic variance. The last term is the result of the expansion of $\delta\theta$ in h; the corresponding term is not present in (24). In this term, $\mathbf{u}$ is a unit vector independent of w_{ij}, and the positive parameter c_2 is logarithmically enhanced due to the infrared effects, $c_2 \propto \log\frac{H_0}{\Lambda}$. This is the first place where the deep infrared modes show up. Clearly, their effect is subdominant for small h.

We see that the large wavelength modes induce statistical anisotropy in the adiabatic perturbations. The statistical anisotropy encoded in the second term in

(25) is similar to that commonly discussed in inflationary context [36] (see Ref. [37] for earlier analysis), and, indeed, generated in some concrete inflationary models [38]: it does not decay as momentum increases and has special tensorial form $(\mathbf{n_k u})^2$ with constant $\mathbf{u}$. On the other hand, the first non-trivial term in (25) has the general tensorial structure and decreases with momentum. The latter property is somewhat similar to the situation that occurs in cosmological models with the anisotropic expansion before inflation [39].

Surprisingly, the statistical anisotropy in the form of the special type quadrupole was found in the WMAP 5 and 7 years data [40–43]. It was argued, however, that the anomaly may result from the detector beam asymmetry not accounted for in the WMAP analysis [44]. In the final 9 years data release, WMAP collaboration provided a set of maps deconvolved with the instrument response function corresponding to the beam asymmetry effect [45]. Deconvolved maps do not indicate the statistical anisotropy allowing to constrain the coupling constant in the first sub-scenario: $h^2 \log \frac{H_0}{\Lambda} < 1.2$ [46]. Later, the Planck data led to stronger constraints on the statistical anisotropy [47, 48] and correspondingly to tighter limits on the coupling constant, $h^2 \log \frac{H_0}{\Lambda} < < 0.30$ [49].

Another effect that emerges at order h^2 is the non-Gaussianity of the perturbations $\delta\theta$, and hence the adiabatic perturbations [31, 32], over and beyond the non-Gaussianity that may be generated at the time when the phase perturbations get reprocessed into the adiabatic perturbations. In the absence of the cubic self-interaction of the field θ, the intrinsic bispectrum vanishes, so we have to consider the trispectrum. It is fully calculated [32], the most striking feature being the singularity in the limit where two momenta are equal in absolute value and have opposite directions (folded limit, in nomenclature of Ref. [50]):

$$\langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\zeta_{\mathbf{k}_4}\rangle = \text{const}\delta\left(\sum_{i=1}^{n}\mathbf{k}_i\right)\frac{1}{k_{12}k_1^4k_3^4}\times \\ \times\left[1-3\left(\frac{\mathbf{k}_{12}\mathbf{k}_1}{k_{12}k_1}\right)^2\right]\left[1-3\left(\frac{\mathbf{k}_{12}\mathbf{k}_3}{k_{12}k_3}\right)^2\right], \qquad (26) \\ \mathbf{k}_{12} = \mathbf{k}_1+\mathbf{k}_2 \to 0,$$

i.e., the trispectrum blows up as k_{12}^{-1}. This is in contrast to trispectra obtained in single-field inflationary models; indeed, there are general arguments [51] showing that in these models, the four-point function is finite in the limit $k_{12} \to 0$. The singularity in the four-point function (26) is due to the enhancement of the radial perturbations $\delta|\phi|$ at low momenta.

Even though the results (25), (26) were first derived in the concrete model with negative quartic potential (Sec. 1.1), they are actually consequences of consistency relations [52] valid in the whole class of conformal models. Furthermore, these consistency relations enable one to calculate the one-loop non-Gaussianity in the folded limit $k_{12} \to 0$. Interestingly the one-loop contribution to the trispectrum is even more singular in the folded limit than the tree-level result (26): one finds [52]

$$\langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\zeta_{\mathbf{k}_4}\rangle^{(1-\text{loop})} = \\ = \text{const}\,\delta\left(\sum_{i=1}^{n}\mathbf{k}_i\right)\frac{1}{k_{12}^3k_1^3k_3^3}\log\frac{k_{12}}{\Lambda}$$

with suppressed coefficient (the suppression factor is h^2 in the model of Sec. 1.1). The k_{12}^{-3}-enhancement of the one-loop trispectrum in the folded limit makes the non-Gaussianity even more promising from the observational viewpoint.

3.2. Subhorizon phase perturbations. Let us consider another option: assume that there is a long enough period of time after the end of conformal rolling, at which the phase perturbations remain subhorizon in the conventional sense [34]. This option is fairly natural in the conformal rolling model of Sec. 1.1 and more contrived in Galilean Genesis.

The behavior of $\delta\theta$ between the end of conformal rolling and horizon exit depends strongly on the evolution of the scale factor at this intermediate stage. In order that the flat power spectrum (10) be not grossly modified at this epoch, the scale factor should evolve in such a way that the dynamics of $\delta\theta$ is effectively nearly Minkowskian. Although this requirement sounds prohibitively restrictive, it is obeyed in the bouncing Universe, with matter at the contracting stage having super-stiff equation of state, $p \gg \rho$. It is worth noting in this regard that stiff equation of state is preferred at the contracting stage for other reasons [53, 54] and is inherent, e.g., in a scalar field theory with negative exponential potential, like in the ekpyrotic model [5]. It is known [55] that in models with super-stiff matter at contracting stage, the resulting power spectrum of scalar perturbations is almost the same as that of massless scalar field in Minkowski space, $\mathcal{P}(k) \propto k^2$. In tractable bouncing models like those of Ref. [6], the phase perturbations evolve almost like in Minkowski space, exit the horizon at the contracting stage, pass through the bounce unaffected (cf. Ref. [9]), remain superhorizon early at the hot expansion epoch and get reprocessed into adiabatic perturbations, as discussed in Sec. 1.3.

Let the effectively Minkowskian stage ends at some time η_1 (see Fig. 3). The field $\delta\theta(\mathbf{x}, \eta_*)$, determined by

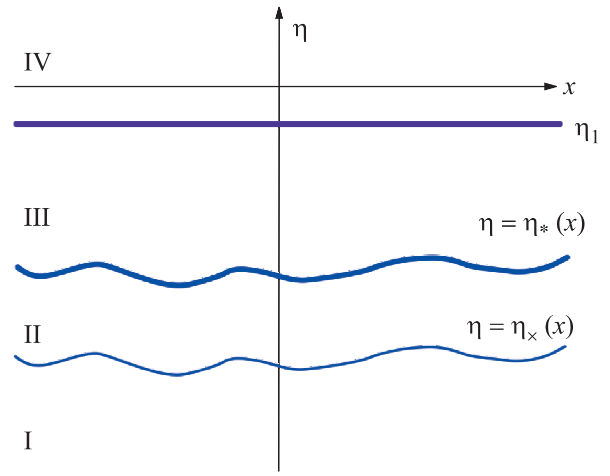

Fig. 3. (Color online) Due to the perturbations of the radial field, the evolution of phase perturbations proceeds in inhomogeneous background. Perturbations $\delta\theta$ oscillate in time at early stage (region I), freeze out at time $\eta = \eta_\times(\mathbf{x})$ and temporarily stay constant (region II) until the end of conformal rolling that occurs at $\eta = \eta_*(\mathbf{x})$. Then they evolve again, now in nearly Minkowskian regime (region III), until the horizon exit time η_1. Later on (region IV), perturbations $\delta\theta$ are superhorizon and stay constant

the dynamics at the conformal rolling stage, serves as the initial condition for further Minkowskian evolution from η_* to η_1. We are interested in the properties of the phase perturbations at $\eta = \eta_1$, as these properties are inherited by the adiabatic perturbations.

To the leading order in h, we find nothing new: the phase perturbations at $\eta = \eta_1$ are Gaussian and have flat power spectrum. Subleading orders in h are more interesting. The effect of the perturbations $\delta\eta_*(\mathbf{x})$ on the phase perturbations $\delta\theta$ is twofold. First, the perturbations $\delta\eta_*$ modify the dynamics of $\delta\theta$ at the conformal rolling stage. This property is the same as in Sec. 3.1. Second, the initial condition for the Minkwskian evolution is now imposed at the non-trivial hypersurface $\eta = \eta_*(\mathbf{x})$. This is illustrated in Fig. 3. The net result is that the perturbation $\delta\theta(\mathbf{x})$ at the time η_1 is a combination of two Gaussian random fields originating from vacuum fluctuations of the phase θ and radial field $|\phi|$, respectively. This leads to potentially observable effects.

Let us consider the phase perturbation of given momentum $\mathbf{k}$. At the end of conformal rolling it is given by (21) with $\partial\delta\theta/\partial\eta = 0$. This gives the initial condition for the evolution at the intermediate stage, when the perturbation is a linear combination of $\exp(i\mathbf{kx} \pm ik\eta)$. So, after the second freeze-out it is a linear combination of waves coming from the direction $\mathbf{n_k} = \mathbf{k}/k$ and from the opposite direction and traveling distance $r = \eta_1 - \eta_*$. This leaves an imprint on $\delta\theta(\mathbf{k})$ of the random field $\mathbf{v}$ existing at points $\mathbf{x} = \pm\mathbf{n_k} r$. In particular, one finds the power spectrum with the non-trivial dependence on $\mathbf{n_k}$:

$$\mathcal{P}_{\delta\theta}(\mathbf{k}) = \mathcal{P}_0 \left\{1 + \mathbf{n_k} \cdot [\mathbf{v}(\mathbf{x} = +\mathbf{n_k} r) - \mathbf{v}(\mathbf{x} = -\mathbf{n_k} r)]\right\}.$$

As a result, the statistical anisotropy of the adiabatic perturbations has the form

$$\mathcal{P}_\zeta(\mathbf{k}) = \mathcal{P}_\zeta^{(0)}(k)\left[1 + Q(\mathbf{n_k})\right], \quad (27)$$

where $\mathcal{P}_\zeta^{(0)}$ is independent of the directionality of momentum (nearly flat spectrum with small tilt) and $Q(\mathbf{n_k})$ is itself a random field, which depends on the direction of $\mathbf{k}$ only. Unlike the statistical anisotropy discussed in the inflationary context [36, 38, 39] and also in Sec. 3.1, the function $Q(\mathbf{n_k})$ contains all even angular harmonics, starting from quadrupole. We give here the expression for $Q(\mathbf{n_k})$ which accounts for the quadrupole component only (see Ref. [34] for all multipoles), $Q(\mathbf{n_k}) = \mathcal{Q} w_{ij} n^i_{\mathbf{k}} n^j_{\mathbf{k}}$ where w_{ij} is a general symmetric traceless tensor normalized to unity, $w_{ij}w_{ij} = 1$, and the variance of the quadrupole component (in the sense of an ensemble of universes) is

$$\langle \mathcal{Q}^2 \rangle = \frac{225 h^2}{32\pi^2}. \quad (28)$$

Of course, the precise values of the multipoles of $Q(\mathbf{n_k})$ in our patch of the Universe are undetermined because of the cosmic variance. It is worth noting also that all multipoles are independent of k and, hence, unlike in the version of Sec. 3.1, there is no suppression of the leading order effect on cosmic microwave background (CMB) power spectrum at large l. This property enables one to utilize the high statistics of the Planck data up to $l = 1600$ and place a strong limit on the coupling constant in the sub-scenario with intermediate stage [49]:

$$h^2 < 0.0011. \quad (29)$$

The statistical anisotropy is probably the most promising signature of this sub-scenario.

The second effect is non-Gaussianity. While, as before, the bispectrum vanishes, the four-point correlation function has a peculiar form

$$\langle \zeta_{\mathbf{k}} \zeta_{\tilde{\mathbf{k}}} \zeta_{\mathbf{k}'} \zeta_{\tilde{\mathbf{k}}'} \rangle = \frac{\mathcal{P}_\zeta^{(0)}(k)}{4\pi k^3} \frac{\mathcal{P}_\zeta^{(0)}(k')}{4\pi k'^3} \delta(\mathbf{k} + \tilde{\mathbf{k}}) \delta(\mathbf{k}' + \tilde{\mathbf{k}}') \times$$
$$\times \left[1 + F_{NG}(\mathbf{n_k} \cdot \mathbf{n_{k'}})\right] + (\mathbf{k} \leftrightarrow \mathbf{k}') + (\tilde{\mathbf{k}} \leftrightarrow \mathbf{k}'). \quad (30)$$

The leading term in (30) (unity in square brackets) is the Gaussian part, while the non-Gaussianity is encoded in $F_{NG} = O(h^2)$. Note that the structure of the non-Gaussian part is fairly similar to that of the disconnected four-point function. Note also that F_{NG} depends on the angle between $\mathbf{k}$ and $\mathbf{k}'$ only. If the angle between $\mathbf{k}'$ and $\mathbf{k}$ is small, i.e., $|\mathbf{n_k} - \mathbf{n_{k'}}| \ll 1$, the leading behavior of F_{NG} is

$$F_{NG} = \frac{3h^2}{\pi^2} \log \frac{\text{const}}{|\mathbf{n_k} - \mathbf{n_{k'}}|},$$

where constant in the argument of logarithm cannot be reliably calculated because of the cosmic variance. The logarithmic behavior does not hold for arbitrarily small $|\mathbf{n_k} - \mathbf{n_{k'}}|$: the function $F_{NG}(\mathbf{n_k} - \mathbf{n_{k'}})$ flattens out most likely at $|\mathbf{n_k} - \mathbf{n_{k'}}| \sim [k(\eta_1 - \eta_*)]^{-1/2}$, and certainly at $|\mathbf{n_k} - \mathbf{n_{k'}}| \sim [k(\eta_1 - \eta_*)]^{-1}$. So, the parameter $\eta_1 - \eta_*$ is detectable in principle.

The third effect is negative scalar tilt

$$n_s - 1 = -\frac{3h^2}{4\pi^2} \; .$$

However, this is not a particularly strong result, as small scalar tilt in our scenario may also originate from explicit violation of conformal invariance at the conformal rolling stage [17] and/or not exactly Minkowskian evolution of $\delta\theta$ at the intermediate stage. Moreover, to account for the whole scalar tilt detected by WMAP and Planck, one needs $h \simeq 0.6$, in conflict with the constraint (29).

4. Conclusions. Flat or nearly flat power spectrum of the adiabatic perturbations may be a consequence of conformal symmetry rather than de Sitter symmetry. Models of this sort include conformally coupled complex scalar field with negative quartic potential, Galilean Genesis and decay of conformally invariant metastable vacuum. Properties of the perturbations in these models are to large extent dictated by conformal invariance and the predictions are mostly model-independent, at least at the leading non-linear level (modulo effects due to conversion of field fluctuations into adiabatic perturbations). A peculiar property which has potentially observable consequences is fluctuations along rolling direction. These fluctuations have red power spectrum and can be interpreted in terms of the local time shift. Interplay between phase perturbations, responsible for density perturbations in the end, and local time shift yields the non-trivial correlation properties of the density perturbations such as statistical anisotropy and the intrinsic non-Gaussianity of special forms. The latter properties are potentially observable with CMB data.

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