Pis'ma v ZhETF, vol. 102, iss. 12, pp. 919 – 922

Staircase structure of Shapiro steps[1)]

Yu. M. Shukrinov[+∗ 2)], *I. R. Rahmonov*[+×], *M. Nashaat*[+◦]

[+]*Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141980 Dubna, Russia*

[∗]*Dubna International University for Nature, Society and Man, 141982 Dubna, Russia*

[×]*Umarov Physical and Technical Institute, 734063 Dushanbe, Tajikistan*

[◦]*Department of Physics, Cairo University, 12613 Cairo, Egypt*

Submitted 10 October 2015

We investigate IV-characteristics of coupled Josephson junctions which model the intrinsic Josephson junctions in high temperature superconductors under external electromagnetic radiation. A staircase structure of Shapiro steps is found in the branching region. Its origin is related to the coupling between junctions and their switching from rotating to oscillating states. This conclusion are tested by detailed analysis of the IV-characteristics as for total stack and for each junction in the stack. IV-curves of junctions in the stack are compared with the average of time derivative of phase difference. Experimental observation of this staircase structure would give us a prove of coupling between junctions and a way for precise measurement of its value. Such investigations would be also useful for a diagnostic of Josephson junctions in the stack.

DOI: 10.7868/S0370274X15240042

Coupled Josephson junctions as a system of coupled nonlinear oscillators is used as a model for many physical objects. First of all it's related to the intrinsic Josephson junctions (JJs) in high temperature superconductors [1, 2]. A stack of intrinsic Josephson junctions is a realistic discrete nonlinear system having the intrinsic localized excitation modes which is currently under intensive studies in nonlinear physics [3, 4]. Different types of coupling, such as inductive [5] and capacitive [6], were introduced to describe the experimental results in this system. Capacitive coupling originated from the breakdown of the charge neutrality in thin superconducting layers dominates the phase dynamics of the intrinsic Josephson junctions when no external magnetic field is applied and the charge imbalance effect can be neglected. The main features of a stack of short JJs are captured well by capacitive coupling, but real value of the coupling parameter in intrinsic JJs of different high temperature superconductors is still questionable [7].

Coupled JJs demonstrate a serious of phenomena absent in case of single junction. Particularly, a stack of junctions demonstrates a longitudinal plasma wave and branching of IV-characteristics in the hysteretic region. This branching is related to the transition of the junctions between rotating and oscillating states. A specific feature of the oscillating state of JJ is its voltage even the average of the time derivative of phase difference for this junction is zero. Origin for this voltage is a coupling between JJs in the stack. As it's known, external radiation produces Shapiro steps (SS) in the IV-characteristic of JJ [8]. So, it would be interesting to test experimentally if SS appears in the IV-characteristic of JJ in O-state. Unfortunately, experimental measurement of IV-characteristics of each JJ in the stack is very difficult due to very small thickness of superconducting layers. It should manifest itself in total IV-characteristic of the stack as a difference from canonical value of Shapiro step voltage. To test such possibility we use a simple resistively and capacitively shunted junction (RCSJ) model which have usually a good agreement with the experimental results [9, 10]. Usually the values of coupling parameter are roughly estimated by using material parameters (dielectric constant and lattice constant) and (or) by the analysis of the $I-V$ characteristics [7]. Here we present another possibility for experimental testing of the coupling parameter based on the precise measurement of the Shapiro steps in branching region of IV-characteristics of intrinsic JJs.

An interesting features of Shapiro steps appear at different resonance conditions. Particularly, in LC-shunted Josephson junction the resonance of Josephson oscillations with eigenmode of LC-circuit essentially affect on the dependence of the SS width on amplitude

[1)]See Supplemental material for this paper on JETP Letters suite: www.jetpletters.ac.ru.

[2)]e-mail: shukrinv@theor.jinr.ru

of external radiation [11]. Another phenomenon appears at branching of the IV-characteristics in the parametric resonance region where the Josephson oscillations excite the longitudinal plasma wave. Here we investigate a reaction of the coupled JJ to the external electromagnetic radiation with a frequency close to the parametric resonance one.

Dynamics of intrinsic JJs in high temperature superconductors in the framework of the one-dimensional capacitively coupled JJ with diffusion current (CCJJ+DC) model [12, 13] in the presence of electromagnetic irradiation is described by the system of equations for the gauge-invariant phase differences $\varphi_l(t)$ between S-layers l and $l+1$

$$\begin{cases} \dfrac{\partial \varphi_l}{\partial t} = V_l - \alpha(V_{l+1} + V_{l-1} - 2V_l), \\ \dfrac{\partial V_l}{\partial t} = I - \sin\varphi_l - \beta\dfrac{\partial \varphi_l}{\partial t} + A\sin\omega t + I_{\text{noise}}, \end{cases} \tag{1}$$

where t is dimensionless time, normalized to the inverse plasma frequency ω_p^{-1}, $\omega_p = \sqrt{2eI_c/\hbar C}$, $\beta = 1/\sqrt{\beta_c}$, β_c is McCumber parameter, α gives the coupling between junctions [6], A is the amplitude of the radiation. To find the IV-characteristic of the stack of coupled JJs we solve the system of nonlinear second-order differential equations (1) at periodic boundary conditions using the fourth order Runge–Kutta method. In our simulations we measure the voltage in units of $V_0 = \hbar\omega_p/(2e)$, the frequency in units of ω_p, the bias current I and the amplitude of radiation A in units of I_c. We add a small noise in the bias current which is produced by random number generator (white noise) and its amplitude is normalized to the critical current value.

The main purpose of our paper is to demonstrate a staircase structure of Shapiro step when frequency of external radiation is in parametric resonance region. In the case of a stack with 10 JJs, dissipation parameter $\beta = 0.2$, coupling parameter $\alpha = 0.05$, the existence of the parametric resonance leads to the breakpoint [12] in IV-characteristic at $V = 1.151$, so we put the frequency of the external radiation equal to the same value $\omega = 1.151$. Fig. 1a shows one-loop IV-characteristic of a stack with 10 JJs at this frequency $\omega = 1.151$ and amplitude of radiation $A = 0.05$. The IV-curve is characterized by a large hysteresis region and we see Shapiro step in it. The inset enlarges the part of IV-characteristics in the breakpoint region with Shapiro step structure. We see four steps at $V = 11.510$, 10.335, 9.194, 4.604. Below we discuss the origin of this structure.

Fig. 1b enlarges the parts of IV-characteristics of the same stack at different amplitude of radiation A in the interval 0, 0.08. An increase in A shifts the breakpoint

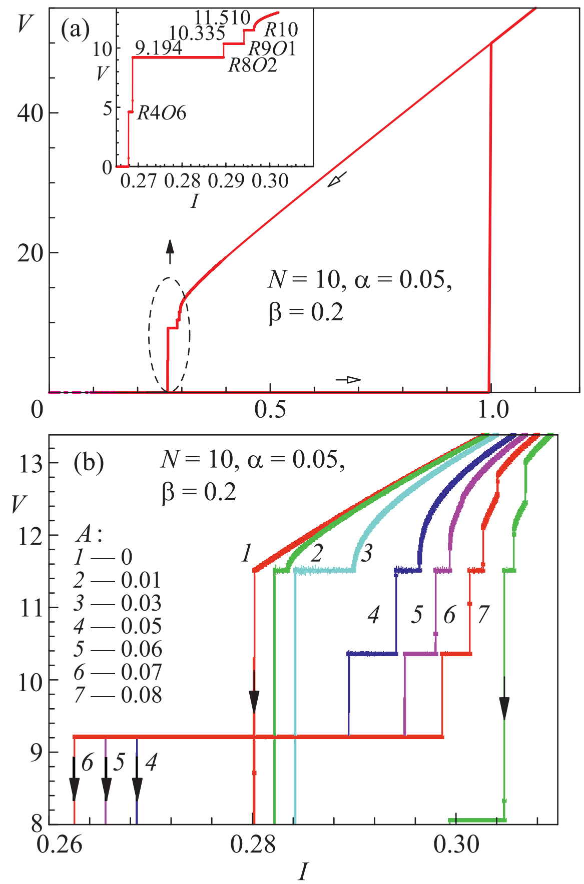

Fig. 1. (Color online) (a) – One-loop IV-characteristics of a stack with 10 JJs at parametric resonance frequency $\omega = 1.151$ and amplitude of radiation $A = 0.05$. Hollow arrows indicate the direction of bias current sweeping. The inset enlarges the part of IV-characteristics in the breakpoint region with staircase structure. (b) – Parts of IV-characteristics of a stack with 10 JJs at parametric resonance frequency $\omega = 1.151$ and different amplitude of radiation in the interval $(0, 0.08)$. Numbers show the values of amplitude

to the larger values of current and we see the formed staircase structure at $A = 0.05$. At large values of A the structure disappears due to the general decrease of the hysteresis region under external radiation [9, 10].

Let us discuss more detailed the steps in staircase structure. In the inset to Fig. 1a the voltage values of the steps in IV-characteristic at $A = 0.05$ were presented. The origin of these steps is related to the coupling between junctions and their switching from rotating to oscillating states. Particularly, in our case the first step corresponds to the state $R10$ with all JJs in the rotating state. The second one corresponds to the state $R9O1$ with nine JJs in rotating state and one JJ in oscillating state. The other two steps correspond to the states $R8O2$ and $R4O6$ with eight and four JJs in rotating state, respectively. To prove it, we demonstrate in Fig. 2a a voltage distribution along the stack in the

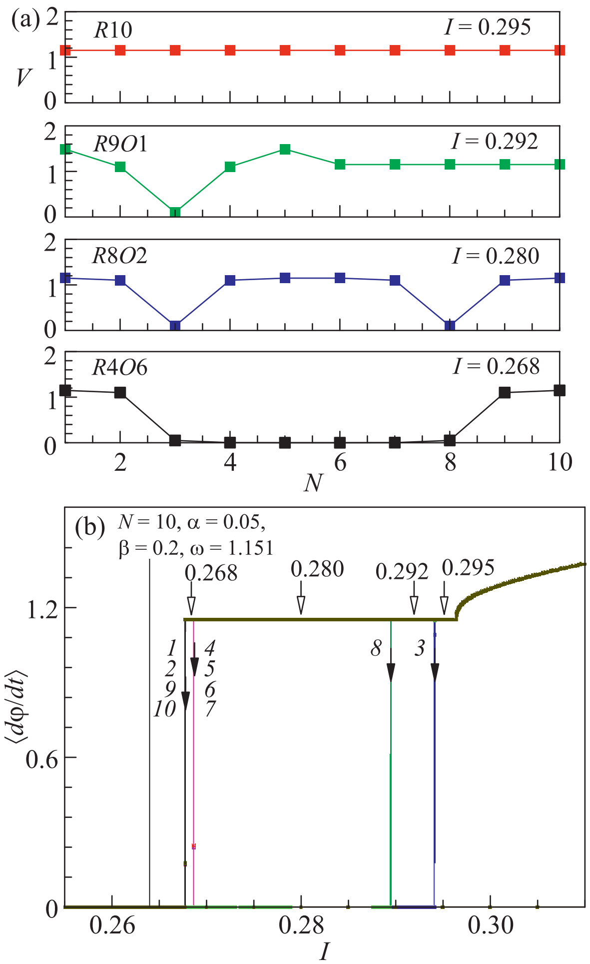

Fig. 2. (Color online) (a) – Voltage distribution along the stack in different states, which are realized at indicated bias current values and marked by hollow arrows in (b). (b) – Average derivative of phase difference in each JJ in the stack as a function of bias current at $A = 0.05$. Numbers indicate the junctions which switches from the rotating to oscillating state at current indicated by arrows

realized states $R10$, $R9O1$, $R8O2$, and $R4O6$. These distributions have taken from the simulation data at the current values presented in the inset and shown by hollow arrows in Fig. 2b. They clearly demonstrate the mentioned states of the stack.

Important fact here is that a position of the steps does not correspond to the canonical value of the Shapiro steps. Particularly, the nine junctions in R-state should produce the step at $V = 10.359$, and position of the step in the stack with eight JJs in R-state should be $V = 9.208$ at given frequency of external radiation $\omega = 1.151$. We explain the difference by the voltage of the JJs in O-states related to the coupling between junctions. Based on this feature it is possible to develop a method for determination of the coupling parameter in these materials.

Of course, the frequency of Josephson oscillations in the junctions at Shapiro step corresponds to the frequency of the external radiation. Fig. 2b shows the average derivative of phase difference of all JJs in the stack as a function of bias current at $A = 0.05$. Numbers indicate the junctions which are switched from the rotating to oscillating state at current value indicated by filled arrows.

Fig. 3a shows the IV-characteristics together with the average derivative of phase difference for the third

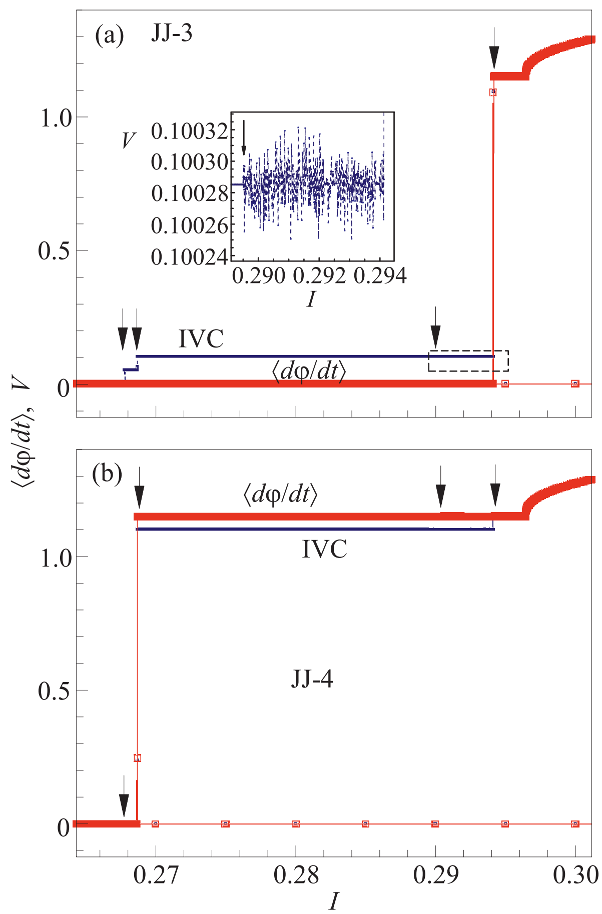

Fig. 3. (Color online) (a) – IV-characteristics (IVC) together with the average derivative of phase difference for the third JJs in the stack at $A = 0.05$. Inset demonstrates that the state $O1$ is probably chaotic. (b) – The same for fourth JJ

JJs in the stack. The IV-curve demonstrates the staircase structure with one part belongs to the R-state and three others belonged to the different oscillating states

$O1$, $O2$, and $O3$, differs by voltage value. Voltage difference between states $O1$ and $O2$ too small to manifest itself in chosen scale of voltage (for details see Supplement). The SS in IV-characteristic of the fourth JJ in the stack has a staircase structure with three steps corresponding to three rotating states $R1$, $R2$, and $R3$ and one – to oscillating state $O3$ (see Fig. 3b). Here the manifestations of transitions $R2 \mapsto R3$ and $O3 \mapsto S$ are also not properly appears due to the chosen scale of voltage (see Supplement). To stress the junction's state we show here also the average derivative of phase difference as a function of bias current. It proves that this JJ has $\langle d\varphi/dt \neq 0\rangle$ in R-states only. In the supplement we show the average derivative of phase difference as a function of bias current and IV-characteristics for each junction in the stack.

Till now the capacitive coupling is roughly estimated by relation $\alpha = \varepsilon\mu/d_s d_I$, where ε, μ, d_s, and d_I are the dielectric constant of the block layers, the charge screening length of the superconducting layers, the thickness of the superconducting, and the insulating layers, respectively [7]. Experimental observation of the staircase structure of Shapiro steps would give us a prove of coupling between junctions and create a novel method for measurement its value. Let us estimate the value of coupling parameter from the results of our numerical experiments. In the coupled system, the Josephson relation for the junction in the O-state ($\langle d\varphi/dt\rangle = 0$) gives

$$\alpha = V_l/(V_{l+1} + V_{l-1} - 2V_l). \quad (2)$$

Particularly, for the case presented in the inset to Fig. 1a, when the third JJ jumps to the O-state (see Fig. 2b), the corresponded voltage values are: $V_3 = 0.10028$ and $V_2 = V_4 = 1.10314$. Then, using (2), we find the value $\alpha = 0.049997$. The difference $\delta V = 0.048$ between canonical value of SS voltage $V = \omega = 1.15100$ and registered value for second and fourth JJ is large enough and can be measured experimentally. The IV-characteristic, presented in Fig. 1a, was simulated at $\alpha = 0.05$, so, the proposed method would have enough high precision. We consider that the detailed investigation of the junction's behavior in the branching region would be useful also for their diagnostic in the stack.

As a summary, we found a staircase structure of Shapiro steps when frequency of external radiation corresponds to the frequency of parametric resonance in coupled Josephson junctions. It is demonstrated that the positions of the steps do not correspond to the frequency of external radiation. We have shown that the steps originate from the different states of Josephson junctions in the stack. The staircase structure features are related to the coupling between junctions and their switching from rotating to oscillating states. Experimental observation of this structure would give us a prove of coupling between junctions and help to measure its value. Based on the obtained results we have proposed a novel method for measurement the coupling parameter value in layered HTSC. Also it would be useful for diagnostic of Josephson junctions in the stack. The presented research might be develop in different directions. Particularly, an investigation of charge imbalance effect which appears at nonequilibrium conditions in layered superconductors might bring to additional opportunities for determination of coupling parameter.

We thank M.Yu. Kupriyanov, L.A. Falkovsky, K.V. Kulikov for discussion of some results of this paper. The reported study was funded by RFBR according to the research project # 15–29–01217 and 15-51-61011. The authors thank T. Hussein, Kh. Hegab, and D. Kamanin and JINR-EGYPT Collaboration for support of this work.

1. U. Welp, K. Kadowaki, and R. Kleiner, Nat. Photon. **7**, 702 (2013).
2. M. Tsujimoto, Y. Maeda, H. Kambara, A. Elarabi, Y. Yoshioka, Y. Nakagawa, Y. Wen, T. Doi, H. Saito, and I. Kakeya, Supercond. Sci. Technol. **28**, 105015 (2015).
3. A. J. Sievers and S. Takeno, Phys. Rev. Lett. **61**, 970 (1988).
4. S. Flach and C. R. Willis, Phys. Rep. **295**, 181 (1998).
5. S. Sakai, P. Bodin, and N. F. Pedersen, J. Appl. Phys. **73**, 2411 (1993).
6. T. Koyama and M. Tachiki, Phys. Rev. B **54**, 16183 (1996).
7. M. Machida and T. Koyama, Phys. Rev. B **70**, 024523 (2004).
8. Y. M. Shukrinov, S. Y. Medvedeva, A. E. Botha, M. R. Kolahchi, and A. Irie, Phys. Rev. B **88**, 214515 (2013).
9. K. K. Likharev, *Dynamics of Josephson Junctions and Circuits*, Gordon and Breach, N.Y. (1986).
10. W. Buckel and R. Kleiner, *Superconductivity: Fundamentals and Applications*, WileyyVCH, Weinheim (2004).
11. Yu. M. Shukrinov, I. R. Rahmonov, K. V. Kulikov, and P. Seidel, EPL **110**, 47001 (2015).
12. Yu. M. Shukrinov and F. Mahfouzi, Phys. Rev. Lett. **98**, 157001 (2007).
13. Yu. M. Shukrinov and I. R. Rahmonov, J. Exp. Theor. Phys. **115**, 289 (2012).