Phonon-particle coupling effects in odd-even double mass differences of magic nuclei

E. E. Saperstein[+*1)], *M. Baldo*[×], *N. V. Gnezdilov*[°], *S. V. Tolokonnikov*[+▽]

[+]*National Research Centre "Kurchatov Institute", 123182 Moscow, Russia*

[*]*National Research Nuclear University MEPhI, 115409 Moscow, Russia*

[×]*INFN, Sezione di Catania, I-95125 Catania, Italy*

[°]*Instituut-Lorentz, Universiteit Leiden, P.O. Box 9506, 2300 RA Leiden, The Netherlands*

[▽]*Moscow Institute of Physics and Technology, 141700 Dolgoprudny, Russia*

Submitted 23 November 2015
Resubmitted 2 December 2015

A method is developed to consider the particle-phonon coupling (PC) effects in the problem of finding odd-even double mass differences (DMD) of magic nuclei within the approach starting from the free NN-potential. Three PC effects are taken into account, the phonon induced interaction, the renormalization of the "ends" due to the Z-factors and the change of the single-particle energies. We use the perturbation theory in g_L^2, where g_L is the vertex of the L-phonon creation. PC corrections to single-particle energies are found self-consistently. In addition to the usual pole diagram, the phonon "tadpole" diagram is taken into account approximately. Results for double-magic ^{132}Sn and ^{208}Pb nuclei show that the PC corrections make agreement with the experimental data better.

DOI: 10.7868/S0370274X1601001X

Recently, the semi-microscopic model developed first for the pairing problem [1–3] was successfully applied to the problem of finding the odd-even double mass differences (DMD) of magic nuclei [4–6]. In the pairing problem, this model starts from the Brueckner theory which results in the BCS gap equation

$$\Delta = \mathcal{V}GG^s, \tag{1}$$

where $\mathcal{V}$ is a "realistic" NN-potential (the Argonne v_{18} in our case), and G (G^s) is the one-particle Green function without (with) pairing. In the case of direct solving this equation in a single-particle basis [7–9] for the ^{120}Sn nucleus, a serious problem of slow convergency exists. To overcome this problem, a two-step renormalization method of solving the gap equation was used in Refs. [1–3]. The complete Hilbert space of the pairing problem S is split in the model subspace S_0, including the single-particle states with energies less than a separation energy E_0, and the complementary one, S'. The gap equation is solved in the model space with the effective pairing interaction (EPI) obeying the Bethe–Goldstone type equation in the subsidiary space:

$$\mathcal{V}_{\rm eff} = \mathcal{V} + \mathcal{V}GG\mathcal{V}_{\rm eff}|_{S'}. \tag{2}$$

In these calculations, the energy density functional (EDF) by Fayans et al. [10–13] was used, which is characterized by the bare mass, $m^*{=}m$. The set DF3 [11, 13] of the EDF parameters and its modification DF3-a [14] were employed.

In contrast, in Refs. [8, 9] an essentially non-bare effective mass of the Skyrme–Hartree–Fock method (the SLy4 EDF [15]) was used with a dramatic suppression of the gap Δ values. To obtain a result close to the experimental value $\Delta_{\rm exp} \simeq 1.3$ MeV, the particle-phonon coupling (PC) corrections to the BCS approximation were introduced. In addition, the contribution of the induced interaction due to exchange of high-lying collective excitations was included in [9]. High uncertainties in a direct finding of all these corrections to the simplest BCS scheme with bare nucleon mass were discussed in detail in [16, 17].

The scale of these uncertainties grow with appearance of the results obtained by Duguet et al. [18, 19] for a number of nuclei with the use of the "low-k" force $\mathcal{V}_{\text{low-k}}$ [20, 21] which is rather soft. The quasipotential $\mathcal{V}_{\text{low-k}}$ is defined in such a way that it describes the NN-scattering phase shifts at momenta $k < \Lambda$, where Λ is a parameter corresponding to the limiting energy $\simeq 300$ MeV, which is much less than the value

[1)]e-mail: saper@mbslab.kiae.ru

of $E_{\max} = 800\,\text{MeV}$ in [8, 9] and helps to carry out systematic calculations. The force $\mathcal{V}_{\text{low-k}}$ vanishes for $k > \Lambda$, so that in the gap equation one can restrict the energy range to $E_{\max} \simeq 300\,\text{MeV}$. Usually the low-k force is found starting from some realistic NN-potential $\mathcal{V}$ with the help of the Renormalization Group method, and the result does not practically depend on the particular choice of $\mathcal{V}$ [20]. In addition, in Ref. [18] $\mathcal{V}_{\text{low-k}}$ was found starting from the Argonne potential v_{18}, that is different only a little from Argonne v_{14}, used in Ref. [9]. Finally, in Ref. [18] the same SLy4 self-consistent basis was used as in Ref. [9]. Thus, the inputs of the two calculations look very similar, but the results turned out to be strongly different. In fact, in Ref. [18] the value $\Delta_{\text{BCS}} \simeq 1.6\,\text{MeV}$ was obtained for the same nucleus ^{120}Sn which is already bigger than the experimental one by $\simeq 0.3\,\text{MeV}$. In Refs. [1, 16, 17] the reasons of these contradictions were analyzed. It turned out that these two calculations differ in the way they take into account the effective mass. It implies that the gap Δ depends not only on the value of the effective mass at the Fermi surface, as it follows from the well-known BCS exponential formula for the gap, but also on the behavior of the function $m^*(k)$ in a wide momentum range. However, this quantity is not known sufficiently well. An additional problem was specified in Ref. [22] where it was found that the inclusion of the 3-body force following from the chiral theory [23] suppresses the gap values much lower than the experimental ones.

To avoid uncertainties under discussion, the semimicroscopic model was suggested [1–3] in which the EPI (2) is supplemented with a phenomenological δ-function addendum:

$$\mathcal{V}_{\text{eff}}(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4) = V_{\text{eff}}^{\text{BCS}}(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4) + \\ + \gamma C_0 \frac{\rho(r_1)}{\bar{\rho}(0)} \prod_{i=2}^{4} \delta(\mathbf{r}_1 - \mathbf{r}_i). \tag{3}$$

Here $\rho(r)$ is the density of nucleons of the kind under consideration, and γ are dimensionless phenomenological parameters. The average central density $\bar{\rho}(0)$ in the denominator of the additional term is obtained with averaging the density $\rho(r)$ over the interval of $r < 2\,\text{fm}$.

The odd-even DMD we deal are defined in terms of nuclear masses $M(N,Z)$ as follows:

$$D_{2n}^{+}(N,Z) = M(N+2,Z) + M(N,Z) - 2M(N+1,Z), \tag{4}$$

$$D_{2n}^{-}(N,Z) = -M(N-2,Z) - M(N,Z) + 2M(N-1,Z), \tag{5}$$

$$D_{2p}^{+}(N,Z) = M(N,Z+2) + M(N,Z) - 2M(N,Z+1), \tag{6}$$

$$D_{2p}^{-}(N,Z) = -M(N,Z-2) - M(N,Z) + 2M(N,Z-1). \tag{7}$$

The "experimental" gap values Δ_{exp} we mentioned above are usually identified with a half of their value.

In magic nuclei which are non-superfluid, these odd-even mass DMD (4)–(7) can be expressed in terms of the same EPI (2) as the pairing gap [4–6]. It can be easily proved starting from the Lehmann expansion for the two-particle Green function K in a non-superfluid system. In the single-particle wave functions $|1\rangle = |n_1, l_1, j_1, m_1\rangle$ representation, it reads [24]:

$$K_{12}^{34}(E) = \sum_s \frac{\chi_{12}^s \chi_{34}^{s+}}{E - E_s^{+,-} \pm i\gamma}, \tag{8}$$

where E is the total energy in the two-particle channel and $E_s^{+,-}$ denote the eigen-energies of nuclei with two particles and two holes, respectively, added to the original nucleus. Instead of the Green function K, it is convenient to use the two-particle interaction amplitude Γ:

$$K = K_0 + K_0 \Gamma K_0, \tag{9}$$

where $K_0 = GG$. Within the Brueckner theory, the amplitude Γ obeys the following equation [24]:

$$\Gamma = \mathcal{V} + \mathcal{V} GG \Gamma, \tag{10}$$

where $\mathcal{V}$ is the same NN-potential as in Eq. (1), which does not depend on the energy. Then the integration over the relative energy can be readily carried out in Eq. (10):

$$A_{12} = \int \frac{d\varepsilon}{2\pi i} G_1\left(\frac{E}{2} + \varepsilon\right) G_2\left(\frac{E}{2} - \varepsilon\right) = \frac{1 - n_1 - n_2}{E - \varepsilon_1 - \varepsilon_2}, \tag{11}$$

where $\varepsilon_{1,2}$ are the single-particle energies and $n_{1,2} = (0;1)$, the corresponding occupation numbers. As a consequence, Eq. (10) reduces to the following form:

$$\Gamma = \mathcal{V} + \mathcal{V} A \Gamma. \tag{12}$$

The two-particle amplitude $\Gamma(E)$ possesses the same poles $E_s^{+,-}$ as the Green function K. After simple manipulations [4], one can obtain the equation for the eigenfunctions χ^s:

$$(E_s - \varepsilon_1 - \varepsilon_2)\chi_{12}^s = (1 - n_1 - n_2) \sum_{34} \mathcal{V}_{12}^{34} \chi_{34}^s. \tag{13}$$

It is different from the Shrödinger equation for two interacting particles in an external field only for the factor $1 - n_1 - n_2$ which reflects the many-body character of the problem, in particular, the Pauli principle. As in the pairing problem, the angular momenta of two-particle

states $|12\rangle$, $|34\rangle$ are coupled to the total angular momentum $I = 0$ ($S = 0, L = 0$).

The relevance of the same interaction $\mathcal{V}_{\rm eff}$ for these two different problems agrees with the well-known theorem by Thouless [25] stating that the gap equation reduces to the in-medium Bethe–Salpeter equation provided the gap Δ vanishes. In our case, the homogeneous counterpart of Eq. (12) is the Bethe–Salpeter equation under discussion, and the Shrödinger-like equation (13) can be obtained from it with the usual procedure. In nuclear physics, this point was evidently first discussed in [26], where the DMD values for double-magic nuclei were analyzed within the theory of finite Fermi systems [24]. In this article, the density dependent EPI was introduced and arguments were found in favor of the surface dominance in this interaction.

The direct solution of this equation is complicated by the same reasons as for the BCS gap equation described above. The same two-step method is used in combination with LPA to overcome this difficulty. As a consequence, Eq. (13) is transformed into the analogous equation in the model space:

$$(E_s - \varepsilon_1 - \varepsilon_2)\chi^s_{12} = (1-n_1-n_2)\sum_{34}{}^{0} (\mathcal{V}_{\rm eff})^{34}_{12}\ \chi^s_{34}, \quad (14)$$

where the effective interaction $\mathcal{V}_{\rm eff}$ coincides with that of the pairing problem, Eq. (2), provided the same value of the separation energy E_0 is used. The next step consists in the use of the ansatz (3) to take into account corrections to the Brueckner theory with a phenomenological addendum ($\sim\gamma$). These corrections are obviously the same as discussed above for the BCS theory. In Refs. [4–6], the semi-microscopic model was successfully applied to non-superfluid components of semi-magic nuclei with the same value of $\gamma = 0.06$ as for the pairing gap.

In this work, we develop a method of direct account for the PC corrections to the DMD values, together with possible change of the optimal value of γ. The introduction of the PC corrections to Eq. (14) consists, first, of the change of ε_λ on the l.h.s. to $\widetilde{\varepsilon}_\lambda{=}\varepsilon_\lambda{+}\delta\varepsilon^{\rm PC}_\lambda$ and, second, a similar change of the $\mathcal{V}_{\rm eff}$ quantity on the r.h.s., to $\widetilde{\mathcal{V}}_{\rm eff}$, with the same meaning of the "tilde" symbol. The explicit form of this PC corrected equation reads:

$$(E_s{-}\widetilde{\varepsilon}_1 - \widetilde{\varepsilon}_2)\chi^s_{12} = (1-n_1-n_2)\sum_{34}{}^{0} \left(\widetilde{\mathcal{V}}_{\rm eff}\right)^{34}_{12}\ \chi^s_{34}. \quad (15)$$

Let us begin with single-particle energies. We follow here the method developed in [27]. Note also that recently PC corrections to the single-particle energies within different self-consistent approaches were studied in Refs. [28–31]. To find the single-particle energies with account for the PC effects, we solve the following equation:

$$\left[\varepsilon - H_0 - \delta\Sigma^{\rm PC}(\varepsilon)\right]\phi = 0, \quad (16)$$

where H_0 is the quasiparticle Hamiltonian with the spectrum $\varepsilon^{(0)}_\lambda$ and $\delta\Sigma^{\rm PC}$ is the PC correction to the quasiparticle mass operator. After expanding this term in the vicinity of $\varepsilon = \varepsilon^{(0)}_\lambda$ one finds

$$\varepsilon_\lambda = \varepsilon^{(0)}_\lambda + Z^{\rm PC}_\lambda \delta\Sigma^{\rm PC}_{\lambda\lambda}(\varepsilon^{(0)}_\lambda), \quad (17)$$

with obvious notation. Here $Z^{\rm PC}$ denotes the Z-factor due to the PC effects,

$$Z^{\rm PC}_\lambda = \left\{1 - \left[\frac{\partial}{\partial\varepsilon}\delta\Sigma^{\rm PC}(\varepsilon)\right]_{\varepsilon=\varepsilon^{(0)}_\lambda}\right\}^{-1}. \quad (18)$$

Expression (17) corresponds to the perturbation theory in the $\delta\Sigma$ operator with respect to H_0. In this article, we limit ourselves to magic nuclei where the so-called g^2_L-approximation, g_L being the L-phonon creation amplitude, is, as a rule, valid. It is worth mentioning that Eq. (17) is more general, including, e.g., g^4_L terms. In the case when several L-phonons are taken into account, the total PC variation of the mass operator in Eqs. (16)–(18) is just the sum:

$$\delta\Sigma^{\rm PC} = \sum_L \Sigma^{\rm PC}_L. \quad (19)$$

The diagrams for the $\delta\Sigma^{\rm PC}_L$ operator within the g^2_L-approximation are displayed in Fig. 1. The first one is

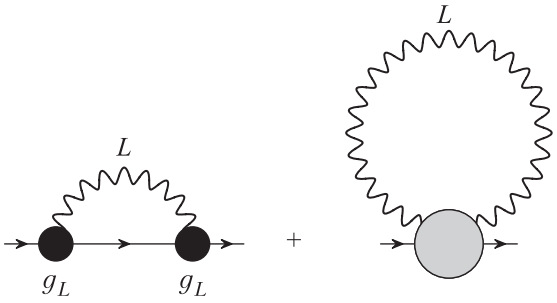

Fig. 1. PC corrections to the mass operator. The gray blob denotes the phonon "tadpole" term

the usual pole diagram, with obvious notation, whereas the second, "tadpole" diagram represents the sum of all non-pole diagrams of the g^2_L order.

In the obvious symbolic notation, the pole diagram corresponds to $\delta\Sigma^{\rm pole} = (g_L, D_L G g_L)$ where $D_L(\omega)$ is the phonon D-function. Explicit expression for the pole term is as follows:

$$\delta\Sigma^{\rm pole}_{\lambda\lambda}(\varepsilon) = \sum_{\lambda_1\, M} |\langle\lambda_1|g_{LM}|\lambda\rangle|^2 \times$$
$$\times \left(\frac{n_{\lambda_1}}{\varepsilon+\omega_L-\varepsilon_{\lambda_1}} + \frac{1-n_{\lambda_1}}{\varepsilon-\omega_L-\varepsilon_{\lambda_1}}\right), \qquad (20)$$

where ω_L is the excitation energy of the L-phonon. In the coordinate form of their creation amplitudes $g_L(\mathbf{r})$ the surface peak dominates,

$$g_L(r) = \alpha_L \frac{dU}{dr} + \chi_L(r), \qquad (21)$$

where $U(r)$ is the nuclear mean-field potential, and the in-volume correction $\chi_L(r)$ being rather small. In Fig. 2, it is illustrated for the 3^-_1-state in ^{208}Pb. If one neglects

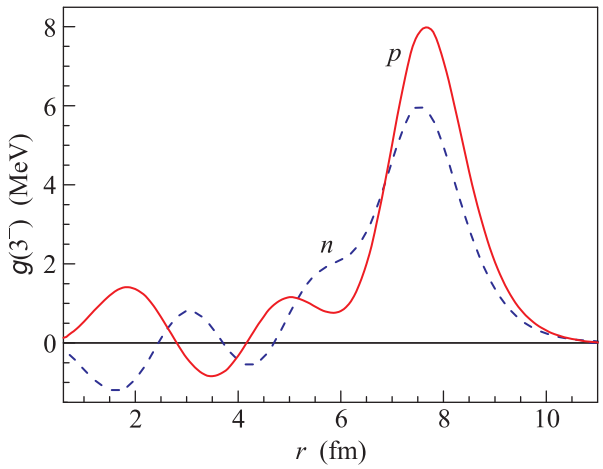

Fig. 2. (Color online) The vertex g_L for the 3^-_1 state in ^{208}Pb

in-volume contributions, the tadpole PC term is reduced [27] to a simple form:

$$\delta\Sigma^{\rm tad}_L = \frac{\alpha_L^2}{2}\frac{2L+1}{3}\triangle U(r). \qquad (22)$$

In this work, following to [27], we use this approximation. It is supported with an old calculation of [32] where it was shown that the in-volume correction to the tadpole term in ^{208}Pb does not exceed 10 %. For a comparison, it can reach 30 % in the light nuclei ^{40,48}Ca.

The tadpole term does not depend on the energy, therefore the $Z^{\rm PC}$-factor (18) is determined with the pole term only and can be found directly in terms of the energy derivative of Eq. (20).

Let us go to PC corrections to the r.h.s. of Eq. (14). They include the phonon induced interaction, Fig. 3, and the "end corrections". An example of them is given in Fig. 4. Partial summation of such diagrams results in the "renormalization" of ends:

$$|\lambda\rangle \to |\widetilde{\lambda}\rangle = \sqrt{Z^{\rm PC}_\lambda}|\lambda\rangle. \qquad (23)$$

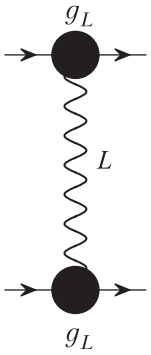

Fig. 3. The phonon induced interaction

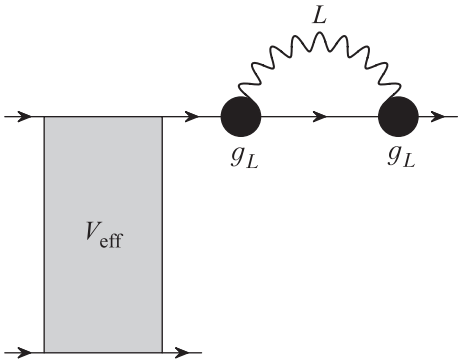

Fig. 4. An example of the PC "end" correction

In the result, we get

$$\langle 11'|\widetilde{\mathcal{V}}_{\rm eff}|22'\rangle = \sqrt{Z^{\rm PC}_1 Z^{\rm PC}_{1'} Z^{\rm PC}_2 Z^{\rm PC}_{2'}} \times$$
$$\times \langle 11'|\mathcal{V}_{\rm eff} + \mathcal{V}_{\rm ind}|22'\rangle. \qquad (24)$$

Remind that we deal with the channel with $I=0$, $S=0$, $L=0$. Hence, the states i, i' in (24) possess the same single-particle angular momenta, $j_1 = j_{1'}$, $l_1 = l_{1'}$; $j_2 = j_{2'}$, $l_2 = l_{2'}$. In this case, the explicit expression of the matrix element of $\mathcal{V}_{\rm ind}$ is as follows:

$$\langle 11'|\mathcal{V}_{\rm ind}|22'\rangle = -\frac{2\omega_L}{\sqrt{(2j_1+1)(2j_2+1)}} \times$$
$$\times \frac{\left[\langle j_1 l_1||Y_L||j_1 l_1\rangle (g_L)_{11'}\right]\left[\langle j_2 l_2||Y_L||j_2 l_2\rangle (g_L)_{22'}\right]^*}{\omega_L^2 - (\varepsilon_2-\varepsilon_1)^2}, \qquad (25)$$

where $\langle\|Y_L\|\rangle$ stands for the reduced matrix element [33], and $(g_L)_{ii'}$ are the radial matrix elements of the vertex $g_L(r)$.

The above formulas (16)–(25) were used to find from Eq. (15) the PC corrections to the odd-even DMD values for double-magic nuclei ^{132}Sn and ^{208}Pb. The Fayans EDF DF3-a [14] was used which reproduces characteristics of the L-phonons in these nuclei sufficiently well [27]. As it is well known, PC corrections are important mainly for single-particle states close to the Fermi

Table 1. Different PC corrections to odd-even double mass differences of magic nuclei

		$D_2^{(0)}$	$\delta D_2(Z^{\rm PC})$	$\delta D_2(\mathcal{V}_{\rm ind}^{\rm PC})$	$\delta D_2(\delta\varepsilon^{\rm PC})$	$\delta D_2^{\rm PC}$	$D_2^{\rm PC}$	$D_2^{\rm exp}$
^{132}Sn-pp	D_2^-	3.184	−1.506	−0.015	−0.982	−1.198	1.986	2.027(160)
	D_2^+	−2.763	1.319	−0.250	1.710	1.494	−1.269	−1.234(6)
^{132}Sn-nn	D_2^-	2.301	−0.396	0.369	−0.009	−0.161	2.140	2.132(9)
	D_2^+	−1.165	0.217	−0.102	−0.045	0.094	−1.071	−1.227(6)
^{208}Pb-pp	D_2^-	1.680	−0.824	−0.083	0.569	−0.745	0.935	0.627(22)
	D_2^-	−2.286	1.049	−0.167	−0.329	0.830	−1.456	−1.1845(11)
^{208}Pb-nn	D_2^-	0.778	−0.275	0.174	0.205	−0.113	0.665	0.63009(11)
	D_2^-	−1.156	0.443	−0.691	−0.021	0.165	−0.991	−1.2478(17)

Table 2. Difference δD_2 between theoretical and experimental values of DMD for different versions of the theory

	$\gamma=0$	$\gamma=0.06$	$(\gamma=0.06)^{\rm PC}$	$(\gamma=0.03)^{\rm PC}$	$(\gamma{=}0)^{\rm PC}$	$D_2^{\rm exp}$
^{132}Sn-pp	−1.529	−0.641	−0.210	−0.117	−0.035	−1.234(6)
^{132}Sn-nn	0.169	−0.390	−0.440	−0.231	0.008	2.132(9)
	0.062	0.327	0.348	0.260	0.156	−1.227(6)
^{208}Pb-pp	1.053	0.373	0.091	0.188	0.308	0.627(22)
	−1.101	−0.282	0.065	−0.091	−0.271	−1.1845(11)
^{208}Pb-nn	0.148	−0.100	−0.136	−0.060	0.035	0.63009(11)
	0.092	0.427	0.428	0.349	0.257	−1.2478(17)
$\langle\delta D_2\rangle_{\rm rms}$	0.82138	0.39298	0.28605	0.20827	0.19323	

surface. In practice, we solve the PC corrected equation (15) limiting ourselves with two shells nearby the Fermi level. In Table 1, the effect of each PC correction to a DMD value is given separately. In this set of calculations we put $\gamma = 0$ in Eq. (3) which determines the EPI of the semi-microscopic model, hence $D_2^{(0)}$ means the direct prediction for the DMD of the Brueckner theory. The next columns present separate PC corrections to this quantity. So, the 2-nd column shows the result of application of Eq. (24) with $\mathcal{V}_{\rm ind} = 0$, whereas the 3-rd one presents the effect of $\mathcal{V}_{\rm ind}$ itself with $Z_1^{\rm PC} = ... = Z_{2'}^{\rm PC} = 1$. The column 4 shows the effect of PC corrections to the single-particle energies in Eq. (15) only. At last, column 5 presents the total PC effect $\delta D_2^{\rm PC} = D_2^{\rm PC} - D_2^{(0)}$, where $D_2^{\rm PC}$ (column 6) is the solution of Eq. (15) with all PC corrections included. As it should be, the value of $\delta D_2^{\rm PC}$ does not equal the sum of the values in previous three columns because of an interference between different PC effects. Experimental DMD values are found from the mass table [34].

The Z-factor effect (column 2) always has the sign opposite to that of $D_2^{(0)}$ value thus suppressing the absolute value of $D_2^{(0)}$. This is a trivial consequence of the $Z^{\rm PC} < 1$ condition. The scale of the suppression varies from $\simeq 20\,\%$ (the neutron D_2^+ mode in ^{132}Sn) to $\simeq 50\,\%$ (both proton modes in ^{132}Sn and ^{208}Pb). It agrees with average values of the $Z^{\rm PC}$-factors, $Z_\lambda^{\rm PC} \simeq 0.7{-}0.9$, of these nuclei found in [27] or [35, 36]. In all cases where the PC effect due to the induced interaction (column 3) is big, its sign coincides with that of $D_2^{(0)}$, i.e. it corresponds to an additional attraction. Two exceptions, the proton D_2^- mode in both nuclei, occur in the cases of very small value of this effect, much less than that due to the Z-factor. At last, go to the single-particle energy effect (column 4). Here there are five cases where this effect is rather big and three, where it is negligible. In all the cases of the first part, this effect helps to make agreement with the data better. The total PC correction (column 5) has always the correct sign with one exception, the neutron D_2^- mode in ^{208}Pb. Fortunately, in this "bad" case the PC correction is not big and spoils agreement not much. On the contrary, in many "good" cases this correction is large and helps to improve the initial $D_2^{(0)}$ value significantly. In all the cases, the PC effect results in a suppression of the initial DMD value, i.e. it acts qualitatively as the phenomenological term in Eq. (3) for the EPI of the semi-phenomenological model. This makes it reasonable to try to search a new optimal value of the parameter γ with account for the PC effects.

Results of such attempt are given in Table 2. To make the comparison with experiment more transparent, we present differences between each theoretical prediction and the corresponding experimental value. We exclude from the analysis one case, the proton D_2^- mode in ^{132}Sn, where the experimental datum does not possess sufficiently high accuracy. In the last line, we put the rms deviation of each version of the theory from the

data. Of course such average is not so much indicative for so small number of averaged quantities, nevertheless it helps to feel a tendency. The column 2 corresponding to $\gamma = 0.06$ without PC corrections has, of course, better accuracy than the column 1 corresponding to the pure Brueckner theory. However, it gives way to all three next columns corresponding different values of γ with PC corrections. It is difficult to choose between two columns, 4 and 5 with $\gamma = 0.03$ and 0 correspondingly, but it looks highly believable that the initial value $\gamma = 0.06$ of the semi-microscopic model should be taken smaller after explicit inclusion of the PC corrections.

To conclude, we made a self-consistent calculation of the PC corrections to the DMD values for double-magic ^{132}Sn and ^{208}Pb nuclei. It makes the agreement with the experimental data better provided the phenomenological parameter γ of the semi-microscopic model is taken between zero and 0.03. A wider amount of nuclei should be analyzed with PC corrections included for more definite conclusions on the optimal value of the parameter γ. It can include other magic nuclei and non-superfluid subsystems of semi-magic nuclei as well. However, a careful choice should be made of nuclei where the perturbation theory in the PC coupling vertex is valid.

The work was partly supported by the Grant NSh-932.2014.2 of the Russian Ministry for Science and Education, and by the RFBR Grants 13-02-00085-a, 13-02-12106-ofi_m, 14-02-00107-a, 14-22-03040-ofi_m. Calculations were partially carried out on the Computer Center of Kurchatov Institute. E.S. thanks the INFN, Seczione di Catania, for hospitality.

1. S. S. Pankratov, M. Baldo, M. V. Zverev, U. Lombardo, E. E. Saperstein, and S. V. Tolokonnikov, JETP Lett. **90**, 612 (2009).
2. S. S. Pankratov, M. V. Zverev, M. Baldo, U. Lombardo, and E. E. Saperstein, Phys. Rev. C **84**, 014321 (2011).
3. E. E. Saperstein, M. Baldo, U. Lombardo, S. S. Pankratov, and M. V. Zverev, Phys. At. Nucl. **74**, 1644 (2011).
4. N. V. Gnezdilov and E. E. Saperstein, JETP Lett. **95**, 603 (2012).
5. E. E. Saperstein, M. Baldo, N. V. Gnezdilov, U. Lombardo, and S. S. Pankratov, EPJ Web. Conf. **38**, 05002 (2012).
6. N. V. Gnezdilov and E. E. Saperstein, Phys. At. Nucl. **77**, 185 (2014).
7. F. Barranco, R. A. Broglia, H. Esbensen, and E. Vigezzi, Phys. Lett. B **390**, 13 (1997).
8. F. Barranco, R. A. Broglia, G. Colo, G. Gori, E. Vigezzi, and P. F. Bortignon, Eur. Phys. J. A **21**, 57 (2004).
9. A. Pastore, F. Barranco, R. A. Broglia, and E. Vigezzi, Phys. Rev. C **78**, 024315 (2008).
10. A. V. Smirnov, S. V. Tolokonnikov, and S. A. Fayans, Sov. J. Nucl. Phys. **48**, 995 (1988).
11. I. N. Borzov, S. A. Fayans, E. Krömer, and D. Zawischa, Z. Phys. A **335**, 117 (1996).
12. S. A. Fayans, JETP Lett. **68**, 169 (1998).
13. S. A. Fayans, S. V. Tolokonnikov, E. L. Trykov, and D. Zawischa, Nucl. Phys. A **676**, 49 (2000).
14. S. V. Tolokonnikov and E. E. Saperstein, Phys. At. Nucl. **73**, 1684 (2010).
15. E. Chabanat, P. Bonche, P. Haensel, J. Meyer, and R. Schaeffer, Nucl. Phys. A **627**, 710 (1997).
16. M. Baldo, U. Lombardo, S. S. Pankratov, and E. E. Saperstein, J. Phys. G: Nucl. Phys. **37**, 064016 (2010).
17. E. E. Saperstein and M. Baldo, in *50 Years of Nuclear BCS*, World Scientific Review Volume, ed. by R. Broglia and V. Zelevinsky, Singapure (2012), ch. 19, p. 263.
18. T. Duguet and T. Lesinski, Eur. Phys. J. Special Topics **156**, 207 (2008).
19. K. Hebeler, T. Duguet, T. Lesinski, and A. Schwenk, Phys. Rev. C **80**, 044321 (2009).
20. S. K. Bogner, T. T. S. Kuo, and A. Schwenk, Phys. Rep. **386**, 1 (2003).
21. L.-W. Siu, J. W. Holt, T. T. S. Kuo, and G. E. Brown, Phys. Rev. C **79**, 054004 (2009).
22. T. Duguet, in *50 Years of Nuclear BCS*, World Scientific Review Volume, ed. by R. Broglia and V. Zelevinsky, Singapure (2012), ch. 17, p. 229.
23. E. Epelbaum, H.-W. Hammer, and U.-G. Meissner, Rev. Mod. Phys. **81**, 1773 (2009).
24. A. B. Migdal, *Theory of Finite Fermi Systems and Applications to Atomic Nuclei*, Wiley, N.Y. (1967).
25. D. J. Thouless, Ann. Phys. **10**, 553 (1960).
26. E. E. Saperstein and M. A. Troitsky, Yad. Fiz. **1**, 400 (1965).
27. N. V. Gnezdilov, I. N. Borzov, E. E. Saperstein, and S. V. Tolokonnikov, Phys. Rev. C **89**, 034304 (2014).
28. E. Litvinova and P. Ring, Phys. Rev. C **73**, 044328 (2006).
29. L.-G. Cao, G. Colò, H. Sagawa, and P. F. Bortignon, Phys. Rev. C **89**, 044314 (2014).
30. D. Tarpanov, J. Dobaczewski, J. Toivanen, and B. G. Carlsson, Phys. Rev. Lett. **113**, 252501 (2014).
31. M. Baldo, P. F. Bortignon, G. Coló, D. Rizzo, and L. Sciacchitano, J. Phys. G: Nucl. Phys. **42**, 085109 (2015).
32. A. P. Platonov, Sov. J. Nucl. Phys. **34**, 612 (1981).
33. A. Bohr and B. R. Mottelson, *Nuclear Structure*, Benjamin–N.Y.–Amsterdam (1969), v. 1.
34. M. Wang, G. Audi, A. H. Wapstra, F. G. Kondev, M. MacCormick, X. Xu, and B. Pfeiffer, Chin. Phys. C **36**, 1603 (2012).
35. N. V. Gnezdilov, E. E. Saperstein, and S. V. Tolokonnikov, EPL **107**, 62001 (2014).
36. N. V. Gnezdilov, E. E. Saperstein, and S. V. Tolokonnikov, Phys. At. Nucl. **78**, 24 (2015).