Topological superconductivity and fractional Josephson effect in quasi-one dimensional wires on a plane

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A time-reversal invariant topological superconductivity is suggested to be realized in a quasi-one dimensional structure on a plane, which is fabricated by filling the superconducting materials into the periodic channel of dielectric matrices like zeolite and asbestos under high pressure. The topological superconducting phase sets up in the presence of large spin-orbit interactions when intra-wire s-wave and inter-wire d-wave pairings take place. Kramers pairs of Majorana bound states emerge at the edges of each wire. We analyze effects of Zeeman magnetic field on Majorana zero-energy states. In-plane magnetic field was shown to make asymmetric the energy dispersion, nevertheless Majorana fermions survive due to protection of a particle-hole symmetry. Tunneling of Majorana quasi-particle from the end of one wire to the nearest-neighboring one yields edge fractional Josephson current with 4π-periodicity.

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Introduction. The recent theoretical prediction and experimental observation of topological phase of matter with time-reversal symmetry in a number of materials [1] have attracted great interest in this subject. The time-reversal invariant (TRI) topological superconductors (SCs) was predicted by theoretical classification of Bogolyubov–de Gennes (BdG) Hamiltonian [2–4] which constitute a completely distinct symmetry class $DIII$. Due to the presence of intrinsic particle-hole symmetry (PHS) the gappless zero-modes in the topological superconductors constitute Majorana fermions (MFs), obeying non-Abelian braiding statistics which is useful in implementing fault-tolerant topological quantum computer [5]. A variety of condensed-matter systems hosting localized Majorana quasi-particles have been proposed, notably quantum Hall states [6] and topological superconductors [7–11], as well as systems with the charge-density wave instability [12].

Recent works [13–18] have proposed TRI topological superconductivity (class $DIII$) with a $\mathbb{Z}_2$ invariant, which takes a value $\nu = 1$, in a number of systems with intrinsic or proximity induced superconductivity of p-wave, spin-triplet or d-wave and $s_{\pm}$-wave spin-singlet pairings. TRI topological SC is assumed to be realized in the variety class of natural quasi-one dimensional (quasi- 1D) materials such as Lithium molybdenum purple bronze ($Li_{0.9}Mo_6O_{17}$) and some organic superconductors [19–21].

A possible application of Majorana fermions as topological qubits requires, on the one hand, finding unique fingerprints that unambiguously confirm their existence and, on the other hand, developing techniques that allow their detection. The majority of previous proposals for their detection rely on the zero-energy excitation features in the tunneling experiments. Alternative experimental routes to observe Majorana bound states may be based on unconventional 4π-periodic oscillation of the Josephson current. It has first been shown by Kitaev [22] for two single-channel topological superconducting wires, brought into contact to form SNS Josephson junction, that the two Majorana edge states across the junction couple to each other and generate a fractional Josephson effect: instead of the usual 2π, the Josephson current exhibits a periodicity of 4π with the phase difference between the superconductors. This doubling of the periodicity can be interpreted as the tunneling of "half" of a Cooper pair. This prediction has later been extended to many different systems [23, 24].

Nowadays, the challenge is to find a real material which supports the topological SC properties. In this work we show that the time-reversal invariant topological superconducting phase can be realized in quasi-1D wires on a plane in the presence of s-wave intra-wire and d-wave inter-wire pairings. Similar structures have been fabricated by Bogomolov's group [25, 26] by

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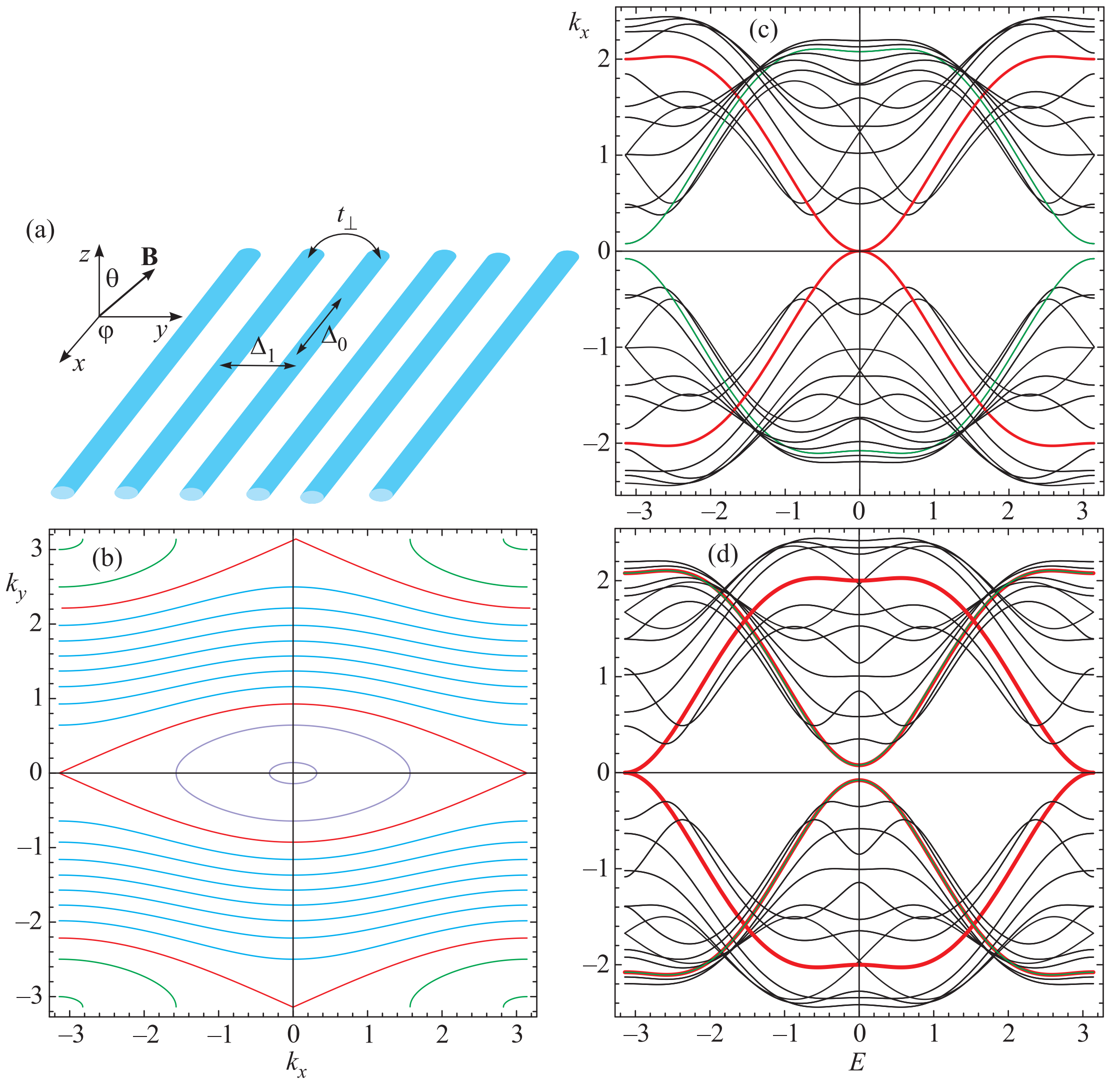

Fig. 1. (Color online) (a) – Structure of quasi-1D superconducting wires on a plane with arbitrary directed magnetic field $\mathbf{B}$. (b) – Fermi surface of the equidistant wires on plane, $-2t_\parallel \cos k_x - 2t_\perp \cos k_y = \mu$. Strong anisotropy is provided by the parameter $t \equiv t_\perp/t_\parallel = 0.2$. The Fermi energy μ varies in the interval $|\mu| \leq 2t_\parallel + 2t_\perp$. The Fermi surfaces are opened for $|\mu| \leq 2t_\parallel - 2t_\perp$, which are shown by blue curves. For $2t_\parallel - 2t_\perp < |\mu| \leq 2t_\parallel + 2t_\perp$ the Fermi surfaces are closed (green curves). The red curves, corresponding to $|\mu| = 2t_\parallel - 2t_\perp$, separate the opened Fermi surfaces from closed ones. The energy spectrum of strongly anisotropic 2D superconductor as a function of k_x for different values of $k_y = 0.0,\ \pm 0.5,\ \pm\pi/3,\ \pm 1.5,\ \pm 2\pi/3,\ \pm 2.5$, and ± 3.0 for $\Delta_0 > 0$ (c) with a zero-energy state (red curves) at the center of the Brillouin zone and $\Delta_0 < 0$ (d) with a zero energy state (red curves) at the boundaries of the Brillouin zone. The parameters are chosen in the unit of the band width $2t_\parallel = 1$. In the both cases we choose $|\Delta_0| = \Delta_1 = 0.5 \cdot 2t_\parallel$, $t_\perp = 0.2 \cdot 2t_\parallel$, $\alpha = 0.6 \cdot 2t_\parallel$, and $\beta = 0$, also $\mu = -0.061 \cdot 2t_\parallel$ and $0.061 \cdot 2t_\parallel$ for panels c and d correspondingly

filling a superconducting material into the cavities or channels of dielectric matrices like zeolite and asbestos crystals under high pressure up to 30 kbar. The regular set of channels or cavities of 5–10 Å diameter in zeolite and from 20–30 Å up to 100–150 Å diameter in asbestos [27] form a periodic lattice of different geometry in one-, two-, and three-dimension, e.g. like several equidistant filaments with 5–20 Å separation in zeolite and with 150–500 Å separation in asbestos on a plane or quasi-1D space lattice. The critical temperatures T_c of such structures become higher than the T_c of the bulk superconductors by factors of 2–5 [25]. High stress field around the filaments may guarantee higher value of spin-orbit interactions in the structures. An increase in T_c may be caused by an excitonic mechanism of inter-wire pairings due to polarization of the dielectric matrix between the wires. We investigate topological phases of such kind quasi-1D superconductor with combined s- and d-wave pairings in the presence and absence of the time-reversal invariance. In difference from chiral superconductors, belonging to $DIII$ symmetry-class too, the zero mode in the TRI topological superconductors come in pairs due to Kramers's theorem. Multiple Majorana–Kramers pairs with strongly spatial overlapping wave functions are protected by time-reversal symmetry, and they persist at zero energy. At each end of a superconducting wire are localized two Majorana fermions that form a Kramers doublet and are protected by time-reversal symmetry. An external Zeeman magnetic field breaks the Kramers degeneracy. Calculations of the energy dispersion of the topological SC reveal two kind asymmetries: first, an interplay of Rashba and Dresselhaus SOIs makes the energy dispersion strongly asymmetric even in the absence of the magnetic field, and second, in-plane Zeeman field introduces an additional anisotropy into the dispersion in the presence of Rashba or/and Dresselhaus SOIs even for a single wire when $t_\perp = 0$. In the non-trivial topological phase, Majorana particles, resided at the ends of each wire, tunnel from one wire to the nearest-neighboring wire yielding fractional Josephson current over the wires' end with 4π periodicity.

Time reversal invariant topological superconductor. The equidistant superconducting wires, aligned along x-axes in $\{x, y\}$ plane, with s-wave intra- and d-wave inter-wire pairings (Fig. 1a), in the presence of spin-orbit interactions (SOIs) and arbitrary directed homogeneous magnetic field $\mathbf{B}$ are described by Hamiltonian

$$\hat{H} = \sum_j \left\{ \hat{H}_{j,j} + \hat{H}_{j,j+1} + \hat{H}_{j+1,j} \right\}, \quad (1)$$

where $\hat{H}_{j+1,j} = \hat{H}^\dagger_{j,j+1}$, and

$$\hat{H}_{j,j} = \sum_{\sigma,\sigma'} \int \frac{dk_x}{2\pi} \Big(\psi^\dagger_{j,\sigma}(k_x) \xi_{k_x} \psi_{j,\sigma}(k_x) + \\ + \psi^\dagger_{j,\sigma} \Big\{ 2 \sin k_x \left[\alpha(\sigma_y)_{\sigma,\sigma'} + \beta(\sigma_x)_{\sigma,\sigma'} \right] + \epsilon_{\mathrm{Z}} \big[(\sigma_z)_{\sigma,\sigma'} \cos\theta + \\ + (\sigma_x)_{\sigma,\sigma'} \sin\theta \ \cos\varphi + (\sigma_y)_{\sigma,\sigma'} \sin\theta \sin\varphi \big] \Big\} \psi_{j,\sigma'} + \\ + \Delta_0 \psi^\dagger_{j,\uparrow}(k_x) \psi^\dagger_{j,\downarrow}(-k_x) + \Delta_0^* \psi_{j,\downarrow}(-k_x) \psi_{j,\uparrow}(k_x) \Big), \quad (2)$$

$$\hat{H}_{j,j+1} = \sum_{\sigma,\sigma'} \int \frac{dk_x}{2\pi} \Big\{ t_\perp \psi^\dagger_{j,\sigma}(k_x) \psi_{j+1,\sigma}(k_x) + \\ + i \psi^\dagger_{j,\sigma} \left[\bar{\alpha}(\sigma_x)_{\sigma,\sigma'} + \bar{\beta}(\sigma_y)_{\sigma,\sigma'} \right] \psi_{j+1,\sigma'} + \\ + \Delta_1 \psi^\dagger_{j,\uparrow}(k_x) \psi^\dagger_{j+1,\downarrow}(-k_x) + \Delta_1^* \psi_{j,\downarrow}(-k_x) \psi_{j+1,\uparrow}(k_x) \Big\}. \quad (3)$$

Here, $\psi^\dagger_{j,\sigma}(k_x)$ ($\psi_{j,\sigma}(k_x)$) is a creation (annihilation) operator of an electron with a longitudinal momentum k_x and spin σ in a jth wire, $t_\parallel$ ($t_\perp$) and μ are the longitudinal (transverse) overlap integral and the Fermi energy, $\xi_{k_x} = (-2t_\parallel \cos k_x - \mu)$ is the energy dispersion in a single wire, $\alpha(\beta)$ and $\bar{\alpha}(\bar{\beta})$ are the longitudinal and transverse components of Rashba (Dresselhaus) spin-orbit constant; $\epsilon_{\mathrm{Z}} = g\hbar\mu_{\mathrm{B}} B/2$ is the Zeeman energy, θ and φ are correspondingly the polar and the azimuthal angles of the magnetic field $\mathbf{B} = B\{\sin\varphi\cos\theta, \sin\varphi\sin\theta, \cos\theta\}$. Note that the orbital effects of the magnetic field, which is neglected here, will be considered elsewhere. The existence of intra- and inter-wire order parameters Δ_0 and Δ_1 allows one to introduce the effective order parameter $\Delta(k_y)$ in momentum space, $\Delta(k_y) = \Delta_0 + 2\Delta_1 \cos k_y = |\Delta(k_y)| e^{i\phi}$, [28]. Further, we will take $\alpha = \bar{\alpha}$ and $\beta = \bar{\beta}$. The structure is strongly anisotropic under $t_\perp \ll t_\parallel$, and the Fermi surface is opened (see Fig. 1b), consisting of two goffered lines. Note that a conventional Josephson coupling between the wires is realized under $t_\perp < k_{\mathrm{B}} T_{c0} \ll 2t_\parallel$ [29], where T_{c0} is the critical temperature evaluated by means of the mean field theory.

We write Hamiltonian, given by Eqs. (1)–(3), as $N \times N$ tridiagonal matrix $\hat{H} = \int \frac{dk_x}{2\pi} \Psi^\dagger_N \mathcal{H}_N \Psi_N$ in the basis of the generalized Nambu wave function of N superconducting wires, $\Psi^\dagger_N = (\Psi^\dagger_{1,k_x}, \Psi^\dagger_{2,k_x}, \ldots, \Psi^\dagger_{j,k_x}, \ldots, \Psi^\dagger_{N,k_x})$, which is $4N$ dimensional vector with $\Psi^\dagger_{j,k_x} = \left(\psi^\dagger_{j,\uparrow}(k_x), \psi^\dagger_{j,\downarrow}(k_x), \psi_{j,\downarrow}(-k_x), -\psi_{j,\uparrow}(-k_x) \right)$ for a single wire. This expression can be exactly written in the two-wire Nambu basis, $\Psi_j^{(2)\,\dagger} = (\Psi^\dagger_{j,k_x}, \Psi^\dagger_{j+1,k_x})$,

$$\hat{H} = \int \frac{dk_x}{2\pi} \sum_{j=1}^{N} \Psi_j^{(2)\,\dagger} \mathcal{H}_j^{(2)} \Psi_j^{(2)}, \quad (4)$$

where

$$\mathcal{H}_j^{(2)} = \begin{pmatrix} \mathcal{H}_{j,j} & \mathcal{H}_{j,j+1} \\ \mathcal{H}_{j+1,j} & \mathcal{H}_{j+1,j+1} \end{pmatrix}, \quad (5)$$

and each $\mathcal{H}_{j,j'}$ entry is 4×4 matrix,

$$\mathcal{H}_{j,j} = \frac{1}{2} \Big[\xi_{k_x} \tau_z + 2\alpha \sin k_x \tau_z \sigma_y + \\ + 2\beta \sin k_x \tau_z \sigma_x + \epsilon_Z (\cos\theta \sigma_z + \sin\theta \cos\varphi \sigma_x + \\ + \sin\theta \sin\varphi \sigma_y) + |\Delta_0| \cos\phi_{0j} \tau_x - |\Delta_0| \sin\phi_{0j} \tau_y \Big], \quad (6)$$

and $\mathcal{H}_{j+1,j} = \mathcal{H}^\dagger_{j,j+1}$,

$$\mathcal{H}_{j,j+1} = \frac{1}{2} \Big[t_\perp \tau_z + i\bar{\alpha} \tau_z \sigma_x + i\bar{\beta} \tau_z \sigma_y + \\ + |\Delta_1| \cos\phi_{1j} \tau_x - |\Delta_1| \sin\phi_{1j} \tau_y \Big]. \quad (7)$$

Note that $\mathcal{H}_{N,N+1} = \mathcal{H}_{N+1,N} = 0$ and $\mathcal{H}_{N+1,N+1} = \mathcal{H}_{1,1}$ for $\mathcal{H}_j^{(2)}$ and $\Psi_N^{(2)\,\dagger} = (\Psi_{N,k_x}^\dagger, \Psi_{1,k_x}^\dagger)$ in Eq. (5). Hamiltonian, given by Eqs. (1)–(3), is transformed from discrete wire-number representation to the momentum space, and is written in the extended Nambu spinor basis $\Psi_{\mathbf{k}}^\dagger = (\psi_{\mathbf{k},\uparrow}^\dagger, \psi_{\mathbf{k},\downarrow}^\dagger, \psi_{-\mathbf{k},\downarrow}, -\psi_{-\mathbf{k},\uparrow})$, as

$$\hat{H} = \int_{-\pi}^{\pi} \frac{dk_x}{2\pi} \int_{-\pi}^{\pi} \frac{dk_y}{2\pi} \Psi_{\mathbf{k}}^\dagger \mathcal{H} \Psi_{\mathbf{k}}, \tag{8}$$

where

$$\mathcal{H} = \xi_{\mathbf{k}}\tau_z + 2(\alpha \sin k_x - \bar{\beta}\sin k_y)\tau_z\sigma_y + 2(\beta \sin k_x - \bar{\alpha}\sin k_y)\tau_z\sigma_x + \epsilon_Z(\cos\theta\sigma_z + \sin\theta\cos\varphi\sigma_x + \sin\theta\sin\varphi\sigma_y) + |\Delta|\cos\phi\ \tau_x - |\Delta|\sin\phi\ \tau_y, \tag{9}$$

where $\xi_{\mathbf{k}} = -2t_\parallel \cos k_x - 2t_\perp \cos k_y - \mu$, and ϕ is the total phase of the effective order parameter $\Delta(k_y)$.

In the absence of the magnetic field and for real order parameter ($\phi = 0$), the time-reversal symmetry and the particle-hole symmetry are protected. Hamiltonian $\mathcal{H}(B = 0) \equiv \mathcal{H}_0$ satisfies the relations $\Theta\mathcal{H}_0(\mathbf{k}) = \mathcal{H}_0(-\mathbf{k})\Theta$ with TRI operator $\Theta = i\sigma_y\mathcal{K}$, where $\mathcal{K}$ is the anti-unitary complex conjugation operator. The PHS emerges from the intrinsic structure of the BdG Hamiltonian, which relates quasi-particle excitations at $\pm E$ through the second quantized operator relation $\gamma_E = \gamma_{-E}^\dagger$, providing a ground for formation of zero-mode Majorana state. This symmetry satisfies the anti-commutation relation $\Xi\mathcal{H}_0(\mathbf{k}) = -\mathcal{H}_0(-\mathbf{k})\Xi$ with the particle-hole operator $\Xi = \tau_y\sigma_y\mathcal{K}$, obeying $\Xi^2 = -1$. The presence of TRS and PHS leads to a chiral symmetry $\Pi\mathcal{H}_0(\mathbf{k}) = -\mathcal{H}_0(\mathbf{k})\Pi$, where the unitary chiral operator Π is the product of Θ and Ξ, $\Pi = -i\Theta\Xi = \tau_y\sigma_0$.

The energy spectrum, obtained from $\det|E_0 - \mathcal{H}_0| = 0$, reads

$$E_0 = s\sqrt{(\xi_{\mathbf{k}} \pm \epsilon_S)^2 + \Delta^2(k_y)}, \tag{10}$$

where $s = \pm$ and $\epsilon_S = 2\Big[\left(\sin^2 k_x + \sin^2 k_y\right)(\alpha^2+\beta^2) - 4\alpha\beta\sin k_x \sin k_y\Big]^{1/2}$, is the spin-orbit interactions energy. The condition $\Delta(k_y) = 0$ determines the nodal points of the order parameter. $\Delta(k_y)$ changes sign at $\cos k_y = -\frac{\Delta_0}{2\Delta_1}$ if $|\Delta_0| < 2\Delta_1$ as moving along k_y from $k_y = 0$ to $k_y = \pi$. A nontrivial TRI superconductor with $\nu = 1$ is realized if there are an odd number of Fermi surfaces with a negative pairing order parameter [3]. At the nodal points

$$E_{0N} = s\Big[2t_\parallel \cos k_x + \mu - t_\perp \frac{\Delta_0}{2\Delta_1} \pm \epsilon_{SN}(k_x, k_{yN})\Big], \tag{11}$$

where $\delta = \sqrt{1 - \frac{\Delta_0^2}{4\Delta_1^2}}$; and $\epsilon_{SN}(k_x, k_{yN}) = 2\sqrt{\left(\sin^2 k_x + \delta^2\right)(\alpha^2+\beta^2) - 4\alpha\beta\delta\sin k_x}$ is the value of ϵ_S at the nodal point $k_{yN} = \arccos\left(-\frac{\Delta_0}{2\Delta_1}\right)$. ϵ_{SN} varies between the maximal $\epsilon_{SN}^{\max}(\pm\pi/2, k_{yN}, \alpha, \beta) = 2\sqrt{\alpha^2+\beta^2}\sqrt{1+\delta^2 \mp \frac{4\alpha\beta}{\alpha^2+\beta^2}\delta}$ and minimal $\epsilon_{SN}^{\min}(k_{x0}, k_{yN}, \alpha, \beta) = 2\delta\frac{(\alpha+\beta)|\alpha-\beta|}{\sqrt{\alpha^2+\beta^2}}$ values.

The order parameter switches sign as the nodal point is crossed. On the other hand, the SOIs split the Fermi surfaces. The splitted Fermi surfaces around the nodal points lie in the energetic interval of $\epsilon_{SN}(k_x, k_{yN})$ from each other. Non-trivial TRI topological phase with $\nu = 1$ is realized when the maximal value of the kinetic energy term (the first three terms) in Eq. (11) is smaller than the minimal value of the SOIs mediated splitting energy $\epsilon_{SN}^{\min}(k_x, k_{yN}, \alpha, \beta)$, $\left|2t_\parallel + \mu - t_\perp\frac{\Delta_0}{\Delta_1}\right| < \epsilon_{SN}^{\min}(k_{x0}, \alpha, \beta)$. The SC is fully gapped when $\left|2t_\parallel + \mu - t_\perp\frac{\Delta_0}{\Delta_1}\right| > \epsilon_{SN}^{\max}(k_{x0}, \alpha, \beta)$. The calculation of the BdG quasi-particles' energy spectrum is simplified for $\beta = 0$, for which $\epsilon_{SN}^{\min}(k_{x0}, \alpha, \beta) = 2\alpha\delta$ and $\epsilon_{SN}^{\max}(k_{x0}, \alpha, \beta) = 2\alpha\sqrt{1+\delta^2}$. The band structure of the topological SC with zero energy surface states for this case is drawn in Figs. 1c and d. For $\Delta_0 < 0$, Majorana edge states appear at $k_x = 0$, which are shown in Fig. 1c by red curves. The zero energy states move to the Brillouin zone boundaries, $k_x = \pm\pi$, for $\Delta_0 > 0$ (see Fig. 1d). The topologically non-trivial superconductor belongs to $DIII$ symmetry class with $\mathbb{Z}_2$ invariant, which takes a value $\nu = 1$. In this case unpaired MFs at each end of a single wire form topologically protected Majorana–Kramers pairs, yielding four zero-energy modes.

Even though a pair of zero energy states are localized at the same end, they are protected by time-reversal symmetry against hybridization preventing to split them to finite energies. Braiding the end-states in these TS results in an exchange of the Kramers pairs rather than isolated Majorana mode, which complicates an application of the TRI topological superconductors to quantum computation. Although braiding of two Majoranas in chiral topological superconductors yields Abelian operators, braiding of Majorana end states in $DIII$-class topological superconductors was shown [30] to represent by non-Abelian operators due to protection of the TRS.

Effects of in-plane magnetic field. An external magnetic field destroys the time-reversal symmetry in the system. The superconductor turns into topological state for an appropriate choice of the magnetic field and

SOI strengths as well as the value of the chemical potential. The energy dispersion for the BdG quasi-particles in the presence of an external magnetic field is expressed according to Eq. (9) as

$$(E^2-\xi_{\mathbf{k}}^2-\epsilon_S^2-\epsilon_{\mathrm{Z}}^2-|\Delta|^2)^2-8\xi_{\mathbf{k}}\epsilon_{\mathrm{Z}}\sin\theta\,\Phi_{\mathbf{k}}(\varphi)E-$$
$$-4\xi_{\mathbf{k}}^2(\epsilon_S^2+\epsilon_{\mathrm{Z}}^2)-4(\epsilon_z\sin\theta\Phi_{\mathbf{k}})^2-4|\Delta|^2\epsilon_{\mathrm{Z}}^2=0, \quad (12)$$

where $\Phi_{\mathbf{k}}(\alpha,\beta,\varphi) = \sin k_x(\alpha\sin\varphi+\beta\cos\varphi) - \sin k_y(\alpha\cos\varphi+\beta\sin\varphi)$ determines the azimuthal angle φ dependence of the energy. $\Phi_{\mathbf{k}}(\alpha,\beta,\varphi)$ emerges due to interference between the co-planar vector fields, such as the magnetic field and the spin-orbit interactions, and introduces a linear in E term in Eq. (12). This linear term destroys a symmetry of the energy dispersion, and it vanishes for a magnetic field, perpendicular ($\theta=0$) to the superconducting plane, yielding from Eq. (12) for the energy spectrum,

$$E^2=\xi_{\mathbf{k}}^2+\epsilon_S^2+\epsilon_{\mathrm{Z}}^2+|\Delta(k_y)|^2\pm2\sqrt{\xi_{\mathbf{k}}^2(\epsilon_S^2+\epsilon_{\mathrm{Z}}^2)+|\Delta|^2\epsilon_{\mathrm{Z}}^2}, \quad (13)$$

which hosts zero energy state $E(0) = s\left|\epsilon_{\mathrm{Z}}\pm\pm\sqrt{\tilde{\mu}^2+|\Delta(0)|^2}\right|$ at the center of the Brillouin zone. Here, $\tilde{\mu}=\mu+2t_{\parallel}+2t_{\perp}$ and $s=\pm$. This expression shows that for $\epsilon_{\mathrm{Z}}>\sqrt{\tilde{\mu}^2+|\Delta|^2}$ the topological non-trivial phase is realized, where one Majorana bound-state resides at $k=0$; while for $\epsilon_{\mathrm{Z}}<\sqrt{\tilde{\mu}^2+|\Delta|^2}$ a topologically trivial gapped state takes place with one Majorana bound-state at the edges. The energy dispersion depends on orientation of the in-plane magnetic field ($\theta=\pi/2$), and the dependence of E on k_x and k_y becomes asymmetric. It is well known [31, 32], that interplay of Rashba- and Dresselhaus-SOIs in the absence of Zeeman magnetic field makes anisotropic the Fermi surface and the kinetic properties of $2D$ electron gas. In-plane magnetic field introduces an additional anisotropy in the energy dispersion. Note that the asymmetries introduced by these two factors differ each other. Indeed, the anisotropy e.g. in the conductivity was shown [31] to increase with the strength of SOIs (the case $\alpha=\pm\beta$ is particular [33]). Nevertheless, the dispersion asymmetry oscillates with the in-plane magnetic field orientation. E vs. k_x dependence is drawn in Fig. 2 for two different cases: the energy dispersion in Fig. 2a is calculated in the absence of the Dresselhaus SOI ($\beta=0$) and for different orientations of the in-plane magnetic field; whereas Fig. 2b shows the dispersion for fixed value of the SOI constants ($\alpha=0.5\cdot2t_{\parallel}$, $\beta=0.4\cdot2t_{\parallel}$) but different values of the azimuthal angle φ. Symmetry of E vs. $\{k_x,k_y\}$ in Eq. (12) strongly differs for $B=0$ but $\beta\neq0$ from $B\neq0$ but $\beta=0$. Indeed,

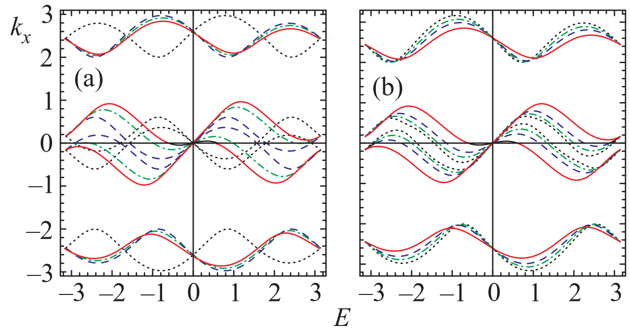

Fig. 2. (Color online) Energy dispersion vs k_x ($k_y=0$) for $\beta=0$ and different values of the in-plane magnetic orientation: $\varphi=3\pi/2$, $2\pi/3$, $\pi/2$, and $\pi/4$ for dotted (black), dot-dashed (green), dashed (blue), and solid (red) curves, respectively (a); $\varphi=\pi/4$ and different values of Dresselhaus SOI constant: $\beta=0.4$, 0.3, 0.2, and 0.0 for dotted (black), dot-dashed (green), dashed (blue), and solid (red) curves, respectively (b). The dimensionless parameters for both figures are chosen (in the unit of $2t_{\parallel}$) to be $2t_{\perp}=0.1$, $\mu=0$, $\alpha=0.5$, $\epsilon_Z=\sqrt{1.7}$, and $|\Delta|=0.7$

$E(-k_x,-k_y)=\pm E(k_x,k_y)$ in the former case declares the existence of both time-reversal and particle-hole symmetries. Nevertheless, $E(-k_x,-k_y)=-E(k_x,k_y)$ in the latter case demonstrates that the particle-hole symmetry only is preserved in the system. Note that the energy dispersion becomes asymmetric even for a single wire in the absence of the transverse tunneling $t_{\perp}=0$ (also $\bar{\alpha}=\bar{\beta}=0$), when a linear in E term in Eq. (12) survives and becomes proportional to $\Phi_{\mathbf{k}}(\alpha,\beta,\varphi) = \sin k_x(\alpha\sin\varphi+\beta\cos\varphi)$. The chosen parameters allow a realization of the zero-energy state at the origin with asymmetric dispersion. While the Hamiltonian (9) respects the particle-hole symmetry even in the presence of the magnetic field, a zero energy states obtained constitute Majorana fermions.

The presence of the midgap mode alters the transport properties of the Josephson junction as the Majorana edge states mediate the transfer of single electrons, as opposed to Cooper pairs, across the junction. Kitaev first predicted [22] that a pair of Majorana fermions fused across a junction of two topological superconducting wires generates a Josephson current $I\propto\sin(\phi_l-\phi_r)/2$, exhibiting thus a remarkable 4π periodicity in the phase difference $\phi_l-\phi_r$ between the left and right wires, in contrast to the 2π periodic current in conventional Josephson contacts. Such a "fractional" Josephson effect was later established in other structures supporting Majorana edge states.

In order to calculate the Majorana coupling between two nearest-neighboring wires, one needs to find the wave function of individual Majorana at the end of jth wire. A unitary transformation $\mathbb{H}_j = \mathbb{U}^\dagger \mathcal{H}_j^{(2)} \mathbb{U}_j$ with the operator

$$\mathbb{U}_j = \begin{pmatrix} e^{-i\frac{\phi_{0j}}{2}\tau_z} & 0 \\ 0 & e^{-i\frac{\phi_{0j+1}}{2}\tau_z} \end{pmatrix}, \quad (14)$$

transfers the phase ϕ_{0j} dependence from the diagonal term $\mathcal{H}_{jj}$ to the off-diagonal term $\mathcal{H}_{j,j+1}$ and to the wave function $|\Psi_j\rangle = \mathbb{U}_j|\psi_{0j}\rangle$. The tunneling amplitude between jth and $(j+1)$th wires, which is proportional to $t_\perp$ in the off-diagonal Hamiltonian, acquires an oscillating multiplier after transformation $\mathbb{H}_{j,j+1} = e^{i\frac{\phi_{0j}}{2}\tau_z}\mathcal{H}_{j,j+1}e^{-i\frac{\phi_{0j+1}}{2}\tau_z} \propto \frac{t_\perp}{2}\cos\frac{\phi_{j+1}-\phi_j}{2}\ \tau_z$. Then, the coupling energy $\Delta E_{j,j+1}$ due to the tunneling between two nearest-neighboring wires is $\Delta E_{j,j+1} \propto$ $\propto t_\perp \cos\frac{\phi_{j+1}-\phi_j}{2}$, yielding

$$j_{j,j+1} = j_0^S(t_\perp)\sin\frac{\phi_{j+1}-\phi_j}{2} \quad (15)$$

for the Josephson current. Note that each wire is chosen rather long, which ensures a negligible overlapping of Majorana wave functions at two ends of a single wire. Nevertheless, Majoranas with significant spatially overlapping wave functions in the nearest-neighboring wires survive [34] due to a spatial reflection symmetry. Tunneling of Majorana quasi-particles between the end of jth and $(j+1)$th wires results in Josephson current, flowing along the wires' end points.

Conclusions. In this paper we argue that a TRI topological superconducting phase may realizes in the novel class of materials, consisting of regular, weakly-coupled superconducting wires in dielectric matrices [25–27]. The structures are fabricated under high pressures, which guarantee higher value of spin-orbit interactions. Experimentally observed enhancement of the critical temperature of these structures allows us to suggest that apart from the intra-wire s-wave pairing, the inter-wire d-wave pairing sets up too, yielding an effective nodal order parameter. The order parameter changes sign by crossing the nodal point between two Fermi surfaces, splitted due to spin-orbit interactions. Note that the only requirement for realization of a nontrivial topological superconductor is that the superconducting pair potential switches sign between the two Fermi surfaces. Time-reversal symmetric topological superconductor belongs to a $DIII$ symmetry-class and is classified by the $\mathbb{Z}_2$ topological invariant.

The distance between the superconducting wires in the cavities or channels of the dielectric matrices is enough large which allows us to detect the position of each wire under scanning tunneling microscope (STM). Topological phase in a single wire can be manipulated by supplying an electrical potential by means of the cantilever of the STM, which derives the wire from the superconducting phase to a normal metallic phase destroying Majorana quasi-particle in a particularly chosen wire.

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