

Strong coupling constant from QCD analysis of the fixed-target DIS data

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Deep inelastic scattering data on F_2 structure function obtained in fixed-target experiments were analyzed in the valence quark approximation with a next-to-next-to-leading-order accuracy. The strong coupling constant is found to be $\alpha_s(M_Z^2) = 0.1157 \pm 0.0022$ (total exp.error), which is seen to be well compatible with the average world value. This study is meant to at least partially explain differences in the predictions for observables at the LHC found recently, caused by usage of various sets of parton distribution functions obtained by different groups.

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1. Introduction. The cross-section values in LHC experiments, along with the extracted parameters, such as, for example, the mass of t quark and the strong coupling constant $\alpha_s(M_Z^2)$, depend strongly on the type of parton distribution functions (PDFs) used in the analyses. Recently, large differences are found in both the cross-section values and extracted parameters, which were obtained by using Alekhin–Blumlein–Moch (ABM) [1] and Jimenez–Delgado–Reya (JR) PDF sets [2]. The latter were in turn derived mostly by fitting deep inelastic scattering (DIS) data. Other groups doing such an analysis, namely, CTEQ [3], NN21 Collaborations [4], and MSTW group [5], included in their fits additional experimental data (see the recent review [6] and references therein).

The differences are sometimes seen to be much larger than the individual PDF uncertainties [6, 7] and give rise to mostly different shapes of gluon densities and strong coupling constant $\alpha_s(M_Z^2)$, which are in turn strongly correlated. The values of $\alpha_s(M_Z^2)$ obtained using the ABM sets [1, 8] are considerably lower than those derived in other cases and can partially be explained [9] by the usage of the fixed flavor number scheme in the the ABM sets.

In the present brief report we will focus on the strong coupling constant value. Let us note another way of decreasing the value of $\alpha_s(M_Z^2)$ observed in [1, 8], which is associated with a so-called *BCDMS effect*. The effect comes about upon analyzing stiffly accurate BCDMS

data [10–12], which are very important in fitting the value of $\alpha_s(M_Z^2)$, especially in the analyses based on mostly DIS data, which is the case for ABM sets. However, as it was shown in [13], those precise data were collected with large systematic errors within certain ranges, which can presumably be responsible for an effective decrease in the value of $\alpha_s(M_Z^2)$ (see [13–15]).

One of the most accurate processes to extract $\alpha_s(M_Z^2)$ values is the valence part of DIS structure function (SF) F_2 , which is free from any correlations with gluon density. Here we will only consider the valence part²⁾. The study closely follows those devoted to similar analyses [14, 15] performed at the next (NLO) and next-to-next-to-leading-order (NNLO) levels, respectively, which means that we consider systematic errors in BCDMS data in a different manner than it was done in [15] in order the study their influence on our results obtained in [14, 15], where $\alpha_s(M_Z^2)$ value was shown to increase when we cut out BCDMS data with the largest systematic errors. Those results have recently been criticized in [8], where it was found that this effect is negligible. The authors of [8] supposed that the $\alpha_s(M_Z^2)$ value increased due to systematic errors being neglected in BCDMS data in the analyses done in [14, 15].

Here, we will show that including the systematic errors in BCDMS data in a different way does not signif-

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²⁾In the present paper we restrict analysis to the large x region. Consequently, the analysis is dubbed a “valence quark” one (simply signaling the absence of gluons) but actually the data on the total structure function $F_2(x, Q^2)$ will be considered.

icantly alter our results derived in [14, 15]. Upon omitting BCDMS data with the largest systematic errors we obtain larger values of the coupling constant normalization $\alpha_s(M_Z^2)$ fitted to the experimental data. Moreover, the effect does not strongly depend on specific cut values, as it was observed earlier in [14, 15].

DIS structure function (SF) $F_2(x, Q^2)$ is dealt with by analyzing SLAC, NMC, and BCDMS experimental data [10–12, 16–18] at NNLO of massless perturbative QCD. As in our previous papers the function $F_2(x, Q^2)$ is represented as a sum of the leading twist $F_2^{p\text{QCD}}(x, Q^2)$ and twist four terms

$$F_2(x, Q^2) = F_2^{p\text{QCD}}(x, Q^2) \left[1 + \frac{\tilde{h}_4(x)}{Q^2} \right], \quad (1)$$

where $F_2^{p\text{QCD}}(x, Q^2)$ denotes the twist-2 part together with target mass corrections. The part $\sim \tilde{h}_4(x)$ denotes the nonzero twist term corrections. For more details concerning an approach to analyzing the experimental data we adopt refer to [14, 19].

2. Results. As is known a valence quark analysis features no gluons taking part in the analysis; therefore, the cut imposed on the Bjorken variable ($x \geq 0.25$) effectively excludes the region where gluon density is believed to be non-negligible.

Since a twist expansion starts to be applicable only above $Q^2 \sim 1 \text{ GeV}^2$ the cut $Q^2 \geq 1 \text{ GeV}^2$ on data is imposed throughout.

A starting point of the evolution is $Q_0^2 = 90 \text{ GeV}^2$ for BCDMS and all datasets, and $Q_0^2 = 20 \text{ GeV}^2$ – for combined SLAC and NMC datasets. These Q_0^2 values are close to the average values of Q^2 spanning the respective data. The heavy quark thresholds are taken at $Q_f^2 = m_f^2$.

2.1. BCDMS data. Analysis starts with the most precise experimental data [10–12] obtained by the BCDMS muon scattering experiment for large Q^2 values. A complete set of data includes 607 points when the cut $x \geq 0.25$ is imposed.

As in [14, 15] the data with largest systematic errors are cut out by imposing certain limits on the kinematic variable $Y = (E_0 - E)/E_0$ (where E_0 and E are lepton’s initial and final energies, respectively [13]). The following Y cuts depending on the limits put on x are imposed:

$$\begin{aligned} Y \geq 0.14 & \quad \text{for} \quad 0.3 < x \leq 0.4, \\ Y \geq 0.16 & \quad \text{for} \quad 0.4 < x \leq 0.5, \\ Y \geq Y_{\text{cut}3} & \quad \text{for} \quad 0.5 < x \leq 0.6, \\ Y \geq Y_{\text{cut}4} & \quad \text{for} \quad 0.6 < x \leq 0.7, \\ Y \geq Y_{\text{cut}5} & \quad \text{for} \quad 0.7 < x \leq 0.8. \end{aligned}$$

An impact of experimental systematic errors on the results of QCD analysis is studied for a few sets of $Y_{\text{cut}3}$, $Y_{\text{cut}4}$, and $Y_{\text{cut}5}$ cuts given in Table 1.

Table 1

A set of $Y_{\text{cut}3}$, $Y_{\text{cut}4}$, and $Y_{\text{cut}5}$ values used in the analysis ³⁾

$N_{Y_{\text{cut}}}$	1	2	3	4	5
$Y_{\text{cut}3}$	0.16	0.16	0.18	0.22	0.23
$Y_{\text{cut}4}$	0.18	0.20	0.20	0.23	0.24
$Y_{\text{cut}5}$	0.20	0.22	0.22	0.24	0.25

Following the analyses performed in [14, 15], we arrive at similar results: α_s values for both original and modified (by cuts) datasets are shown in Table 2, where a total systematic error is estimated in quadrature by using the method somewhat different from that utilized in our earlier analyses ($N_{Y_{\text{cut}}} = 0$ corresponds to the case without Y cuts). Namely, instead of accounting for those errors by the multiplication procedure (an old approach outlined in [15]), here they are taken altogether in quadrature from the very beginning.

Upon the cuts imposed (in what follows we work with a set $N_{Y_{\text{cut}}} = 5$), only 452 points left available for analysis. Fitting them according to the procedure outlined above the following results are obtained:

$$\begin{aligned} \alpha_s(M_Z^2) &= 0.1155 \pm 0.0016 \text{ (stat)} \pm \\ &\pm 0.0030 \text{ (syst)} \pm 0.0007 \text{ (norm)}, \end{aligned} \quad (2)$$

where an abbreviation “norm” denotes the experimental data normalization error stemming from the difference of the fits with free and fixed normalizations of BCDMS data subsets [10–12] having different values of the beam energy.

Performing the fits of SLAC and NMC experimental data [16–18] we obtain results, which are very similar to those derived in [15] while fitting SLAC, NMC, and BFP data altogether. Therefore, we do not present here results of the analyses with only SLAC and NMC data included, although note that the results are compatible within errors with those given above in (2) based on the analysis of BCDMS data alone. Thus, we can put all the data together and fit them simultaneously.

2.2. SLAC, BCDMS, and NMC datasets. As in the case of BCDMS data analysis the cuts imposed are $x \geq 0.25$ and $N_{Y_{\text{cut}}} = 5$ (see Table 1). Then, an overall set of data consists of 756 points.

In order to determine the region where perturbative QCD is applicable we start by analyzing the data without a contribution of twist-four terms (that is

Table 2

NNLO $\alpha_s(M_Z^2)$ values for various sets of Y cuts

$N_{Y_{\text{cut}}}$	Number of points	χ^2 quad. syst. err. (mult. syst. err.)	$\alpha_s(M_Z^2) \pm$ \pm stat. error quad. syst. err.	$\alpha_s(M_Z^2) \pm$ \pm stat. error mult. syst. err.	Total syst. error
0	607	446 (642)	0.1064 ± 0.0012	0.1056 ± 0.0012	0.0054
1	502	361 (481)	0.1132 ± 0.0015	0.1127 ± 0.0015	0.0039
2	495	357 (477)	0.1135 ± 0.0015	0.1130 ± 0.0015	0.0038
3	489	352 (463)	0.1140 ± 0.0015	0.1136 ± 0.0015	0.0036
4	458	350 (427)	0.1150 ± 0.0016	0.1144 ± 0.0016	0.0031
5	452	325 (421)	0.1155 ± 0.0016	0.1149 ± 0.0016	0.0030

$F_2 = F_2^{p\text{QCD}}$) and perform several fits with the cut $Q^2 \geq Q_{\text{min}}^2$ gradually increased. Table 3 demonstrates that the quality of fits appears to be acceptable already at $Q^2 = 2 \text{ GeV}^2$.

Now, the twist-four corrections are added and the data with a global cut $Q^2 \geq 1 \text{ GeV}^2$ is fitted. As in the previous studies [14, 15] it is clearly seen that higher twists improve the fit quality, with an insignificant discrepancy in the values of the coupling constant to be quoted below.

Finally,⁴⁾ using the valence quark evolution analyses of SLAC, NMC, and BCDMS experimental data for SF F_2 with no account for twist-four corrections and the cut $Q^2 \geq 2 \text{ GeV}^2$, we obtain (with $\chi^2/\text{DOF} = 1.03$)

$$\alpha_s(M_Z^2) = 0.1161 \pm 0.0003 \text{ (stat)} \pm 0.0018 \text{ (syst)} \pm 0.0007 \text{ (norm)}. \quad (3)$$

Upon including the twist-four corrections and imposing the cut $Q^2 \geq 1 \text{ GeV}^2$, the following result is found (with $\chi^2/\text{DOF} = 0.88$):

$$\alpha_s(M_Z^2) = 0.1157 \pm 0.0008 \text{ (stat)} \pm 0.0020 \text{ (syst)} \pm 0.0005 \text{ (norm)}. \quad (4)$$

3. Conclusions. A reanalysis of the BCDMS data performed by cutting off the points with largest systematic errors and accounting for remaining systematic errors in a different manner shows that as in the previous study the values of $\alpha_s(M_Z^2)$ rise sharply with the cuts on systematics imposed. On the other hand, the latter do not depend on the choice of a certain cut within statistical errors. The present results are compatible with those obtained in our earlier paper [15], where systematic errors in BCDMS data were taken into account in a different way. To be more precise, in [15], and in even earlier studies [21, 22, 14], systematics was dealt with as

follows: all fits were done with experimental data multiplied by respective systematic errors for F_2 separately for each source of uncertainties. Then, the differences between fits with different sources taken into account give the total systematic error derived in quadrature. In the present paper, systematics is dealt with in quadrature rather than in a multiplicative manner right from the start.

Taking into account systematic errors in BCDMS data does not change results of the fits obtained in [15] except for just a single detail: now perturbative QCD (without higher twist corrections) is well compatible with the experimental data already at $Q^2 \geq 2 \text{ GeV}^2$ (see Table 3).

Here, we would like to offer some explanations of the absence of the rise in $\alpha_s(M_Z^2)$ value upon cutting out the regions in BCDMS data with largest systematic errors stated in [8]. Note that the values of systematic errors are rather large in the cut out regions but not infinitely large. In the latter case there of course is no any effect of absence/existence of the cut out regions. One of possible explanations relates with the fact that Ref. [8] includes complete analyses⁵⁾, where there is some correlation between $\alpha_s(M_Z^2)$ values and the shape of gluon density. So, cutting out the ranges of BCDMS data with the largest systematic errors could lead in [8] to the shape of gluon density somewhat altered.

It turns out that for $Q^2 \geq 2 \text{ GeV}^2$ the formulæ of pure perturbative QCD (i.e. twist-two approximation along with the target mass corrections) are enough to achieve good agreement with all the data analyzed. The reference result is then found to be

$$\alpha_s(M_Z^2) = 0.1161 \pm 0.0020 \text{ (total exp. error)}, \quad (5)$$

⁴⁾More details of the analysis and results can be found in [20].

⁵⁾The complete analysis deals with sea, valence quark, and gluon densities.

$\alpha_s(M_Z^2)$ and χ^2 in the combined SLAC, BCDMS, NMC analysis

Q_{\min}^2	N of points	HTC	$\chi^2(F_2)/\text{DOF}$	$\alpha_s(90 \text{ GeV}^2) \pm \text{stat}$	$\alpha_s(M_Z^2)$
1.0	756	No	1.41	0.1757 ± 0.0007	0.1160
2.0	731	No	1.03	0.1758 ± 0.0007	0.1161
3.0	704	No	0.84	0.1787 ± 0.0009	0.1173
4.0	682	No	0.79	0.1790 ± 0.0009	0.1174
5.0	662	No	0.79	0.1795 ± 0.0011	0.1177
6.0	637	No	0.79	0.1798 ± 0.0013	0.1178
7.0	610	No	0.78	0.1792 ± 0.0016	0.1175
8.0	594	No	0.79	0.1787 ± 0.0019	0.1173
9.0	575	No	0.78	0.1785 ± 0.0023	0.1172
10.0	564	No	0.77	0.1765 ± 0.0026	0.1164
1.0	756	Yes	0.88	0.1750 ± 0.0019	0.1157

where the total experimental error is obtained from errors in Eq. (3) taken in quadrature.

Upon adding twist-four corrections, fairly good agreement between QCD and the data starting already at $Q^2 = 1 \text{ GeV}^2$, where the Wilson expansion starts to be applicable, is observed. This way we obtain for the coupling constant at Z mass peak:

$$\alpha_s(M_Z^2) = 0.1157 \pm 0.0022 \text{ (total exp. error)}. \quad (6)$$

Note that, in a sense, our results lie between those obtained in [8, 1] and [2] (a dynamical approach) and, respectively, results derived by MSTW [5] and NN21 [4] groups. They are consistent with those obtained in [3], [2] (a standard fit) and also with studies of the recent data of CMS and ATLAS collaborations done in [23–26], respectively (see the recent review [27]). A complete agreement with recent results obtained in lattice QCD [28] is also observed. Our result is slightly below the central world average value

$$\alpha_s(M_Z^2)|_{\text{world average}} = 0.1185 \pm 0.0006, \quad (7)$$

presented in [29], but still compatible within errors.

We hope that our results shed some additional light on the differences in the predictions for observables at the LHC found recently [6, 7], which are resulted from the utilization of various sets of parton distribution functions obtained by different groups. Indeed, excluding the ranges with largest systematic errors in BCDMS data increases the value of $\alpha_s(M_Z^2)$ in the fits based on mostly DIS experimental data and could therefore potentially lead to decrease in those differences.

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