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Theoretical treatment of pulsed Overhauser DNP: consideration of a general periodic pulse sequence

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A general theoretical approach to pulsed Overhauser-type Dynamic Nuclear Polarization (DNP) is presented. DNP is a powerful method to create non-thermal polarization of nuclear spins, thereby enhancing their nuclear magnetic resonance signals. The theory presented can treat pulsed microwave irradiation of electron paramagnetic resonance transitions for periodic pulse sequences of general composition. DNP enhancement is analyzed in detail as a function of the microwave pulse length for rectangular pulses and pulses with finite rise time. Characteristic oscillations of the DNP enhancement are found when the pulse-length is stepwise increased, originating from coherent motion of the electron spins driven by the pulses. Experimental low-field DNP data are in very good agreement with this theoretical approach.

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Dynamic Nuclear Polarization (DNP) [1–4] is becoming a widely used tool to enhance weak Nuclear Magnetic Resonance (NMR) signals by transferring thermal polarization from electronic to nuclear spins in the presence of microwave (MW) pumping. Since the electronic gyromagnetic ratio, γ_e, is much larger than the nuclear one, γ_n, the thermal electron polarization is also much higher; consequently, efficient electron-nuclear polarization transfer leads to strong NMR signal enhancement. In liquids, DNP is operative due to the Overhauser mechanism [5], i.e., due to electron-nuclear cross-relaxation in the presence of MW pumping of electronic spin transitions.The expression for the resulting NMR enhancement, ε, is usually written as follows:

$$\varepsilon = \frac{I_z}{I_0} = 1 - \xi \cdot f \cdot s \left|\frac{\gamma_e}{\gamma_n}\right|. \qquad (1)$$

Here I_z and I_0 stand for the nuclear spin polarization in the presence and in the absence of MW pumping; the enhancement is due to the fact that $\left|\frac{\gamma_e}{\gamma_n}\right| \gg 1$. The polarization transfer efficiency is characterized by the following three factors: ξ, termed the coupling factor, describes the efficiency of cross-relaxation as compared to the nuclear T_1-relaxation; $f = \left(1 - \frac{T_{1p}}{T_1}\right)$ (here T_{1p}, T_1 are the longitudinal nuclear relaxation times with and without paramagnetic additives, respectively), termed the leakage factor, describes the nuclear spin relaxation coming from interaction with the electron spins compared to other mechanisms; s, termed the saturation factor, stands for the efficiency of MW-pumping. The value of s is given by the following expression

$$s = \frac{S_{eq} - S_z}{S_{eq}}, \qquad (2)$$

where S_z is the electronic z-magnetization in the presence of MW-pumping and S_{eq} is its value at thermal equilibrium. For a radical with a complex multi-line EPR (Electron Paramagnetic Resonance) spectrum it is necessary to average $(S_z - S_{eq})$ over all spectral lines in order to calculate s.

In DNP experiments, particularly in aqueous solutions, the maximum achievable saturation factor is usually limited by MW heating of the sample; hence, it is important to find ways for optimizing the s factor at a given MW-power. A promising way to do this [6] is offered by MW-pumping done in the pulsed mode. In this situation, the electron spins are flipped by a short MW-pulse; after that, the magnetization relaxes back to thermal equilibrium during the long waiting period between subsequent pulses. It has been shown [7, 8] that such a method can indeed lead to strong DNP enhancements. Interestingly, the dependence of ε on the MW-pulse length, τ_p, exhibits an oscillatory component. This observation is a clear indication that coherent motion of the electron spin magnetization driven by the MW-field comes into play.

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In previous works [7, 8] we have proposed a theoretical description of pulsed Overhauser-type DNP and obtained the following expression for ε:

$$\varepsilon = 1 - \xi \cdot f \cdot s_{av} \cdot \left|\frac{\gamma_e}{\gamma_n}\right|. \qquad (3)$$

Eq. (3) is very similar to Eq. (1); only the s_{av} factor is different from the saturation factor in Eq. (1): s_{av} is the relative deviation of S_z from S_{eq} averaged over one period of the pulse sequence of a duration τ:

$$s_{av} = \frac{1}{\tau}\int_0^\tau \frac{S_{eq} - S_z(t)}{S_{eq}} dt. \qquad (4)$$

During each period S_z varies significantly with time, in contrast to the stationary value at cw-irradiation. Now, the electron spins are always far away from saturation. For this reason, we need to replace the "saturation factor" by the average "deviation factor", i.e., the DNP efficiency depends on how far (on average) the electron spin magnetization deviates from its equilibrium value.

Although the previous work [7, 8] has provided initial insight into pulsed DNP, calculation of s_{av} still remains a problem in the general case. Previously, we calculated $S_z(t)$ by using the Bloch equations; however, this method meets considerable difficulties in the cases of multi-pulse sequences or MW-pulses having non-rectangular shape or variable phase. For this reason, a general theoretical approach to pulsed DNP is required, which can treat an arbitrary pulse sequence. In this contribution, we bridge the existing gaps in DNP theory by developing such an approach and demonstrate the effects coming from variation of the pulse shape and phase.

To describe the electron spin evolution under the action of MW-pulses we introduce the density matrix, ρ, of a stable radical, which obeys the following equation:

$$\dot\rho = -i\left[\hat H(t), \rho\right] + \hat{\hat R}\rho = \left(i\hat{\hat H}(t) + \hat{\hat R}\right)\rho. \qquad (5)$$

Here $\hat H(t)$ is the Hamiltonian of the radical in the MW-rotating frame (written in the angular frequency units), $\hat{\hat H}_i$ is the corresponding super-operator in the Liouville space and $\hat{\hat R}$ is the relaxation super-operator. We consider a periodic pulse sequence; each cycle of the sequence has a duration τ and can be split into N periods of a duration δt_i, such that within each of these short periods the Hamiltonian can be considered time independent and equal to $\hat H_i$ (the form of $\hat H_i$ is given below). $\hat{\hat R}$ is taken time-independent; importantly, in Eq. (5) there is no source term, i.e., the relaxation term is written as $\hat{\hat R}\rho$ but not as $\hat{\hat R}(\rho - \rho_{eq})$ (as it has to be done when the Bloch equations are used). Consequently, to describe relaxation of the electronic spin system to the thermal equilibrium we need to bear in mind that the elements of $\hat{\hat R}$, which describe T_1-relaxation, differ by the electronic Boltzmann factor. For instance, the elements $W_\downarrow$ and $W_\uparrow$, which stand for the EPR transition lowering the energy of the electronic spin and the EPR transition increasing this energy, respectively, are given by expressions:

$$\frac{W_\downarrow}{W_\uparrow} = \exp\left(\frac{\hbar\gamma_e B_0}{k_{\rm B}T}\right) \approx 1 + 2\alpha,$$

$$W_\downarrow + W_\uparrow = \frac{1}{T_{1e}} \Rightarrow W_\downarrow \approx \frac{1+\alpha}{2T_{1e}}, \quad W_\uparrow = \frac{1-\alpha}{2T_{1e}}. \qquad (6)$$

Here B_0 is the external magnetic field strength, $k_{\rm B}$ is the Boltzmann constant, T is the absolute temperature, $\alpha = \frac{\hbar\gamma_e B_0}{2k_{\rm B}T} \ll 1$ and T_{1e} is the electronic T_1-relaxation time. Transverse relaxation (i.e., T_2-relaxation) is introduced in the usual way as decay of electronic spin coherences with time T_{2e}. The Hamiltonian $\hat H_i$ in the rotating frame is (in units of $\hbar$)

$$\hat H_i = \delta\omega\hat S_z + \omega_{1i}(\cos\varphi_i\hat S_x + \sin\varphi_i\hat S_y). \qquad (7)$$

Here $\delta\omega$ is the frequency offset for the relevant component of the EPR spectrum (for several components the average over them should be taken); ω_{1i} is the B_1-field strength for the i-th time interval and φ_i is its phase. For numerical solution of Eq. (5), the Hamiltonian $\hat H_i$ should be transformed into the Liouville space in the usual way to obtain the corresponding super-operator $\hat{\hat H}_i$. Then the relation between the density matrix, ρ_m, before the m-th cycle and immediately after it, ρ_{m+1}, is:

$$\rho_{m+1} = \hat{\hat W}_1\hat{\hat W}_2 \cdot \ldots \cdot \hat{\hat W}_n\rho_m = \hat{\hat A}\rho_m, \qquad (8)$$

where $\hat{\hat W}_i = \exp(\{i\hat{\hat H}_i + \hat{\hat R}\}\delta t_i)$. In the quasi-stationary regime (which is rapidly established when $t > T_{1e}$, T_{2e} [7, 8]) the density matrix is exactly the same before and after the cycle and is equal to ρ_{st}:

$$\rho_{st} = \hat{\hat A}\rho_{st}. \qquad (9)$$

This is a homogeneous linear set of equations for the elements of ρ_{st}. When $\hat{\hat A}$ is an $n \times n$ matrix its rank is equal to $(n-1)$; therefore to solve the system (9) we need to exclude one equation (linearly dependent on the other) and add one more equation coming from the fact that $\mathrm{Tr}\{\rho\} = \sum_i^n \rho_{ii} = 1$. As a result, we arrive to a linear inhomogeneous system of equations, which is easy to solve:

$$\hat{\hat X}\rho_{st} = y \Rightarrow \rho_{st} = \{\hat{\hat X}\}^{-1}y. \qquad (10)$$

In this way we can compute the density matrix in the beginning of each cycle of the pulse sequence, consequently, the density matrix, $\rho(t)$, can be calculated at each instant of time by solving Eq. (5) with the initial condition $\rho(t=0)=\rho_{st}$, as well as $S_z(t)$, which is equal to $\mathrm{Tr}\{\hat{S}_z(t)\}$. Thus, this method allows one to compute s_{av} and, consequently, ε as functions of the parameters of the pulse sequence and the electron spin relaxation times T_{1e} and T_{2e}. When the EPR line has several components, s_{av} should be computed separately for each component and summed over all components taking into account their statistical weights. Using the scheme for calculating ε we computed the enhancement as a function of the pulse length. First, we considered a sequence with a cycle comprising only a single ideal rectangular pulse of a duration τ_p; the delay between subsequent pulses is $\tau_w=\tau-\tau_p$, i.e., the duty cycle for such a sequence is $D=\frac{\tau_p}{\tau_p+\tau_w}\times 100\,\%=\frac{\tau_p}{\tau}\times 100\,\%$. The resulting DNP enhancement as a function of τ_p (for a fixed D) is shown in Fig. 1 for a radical with two EPR

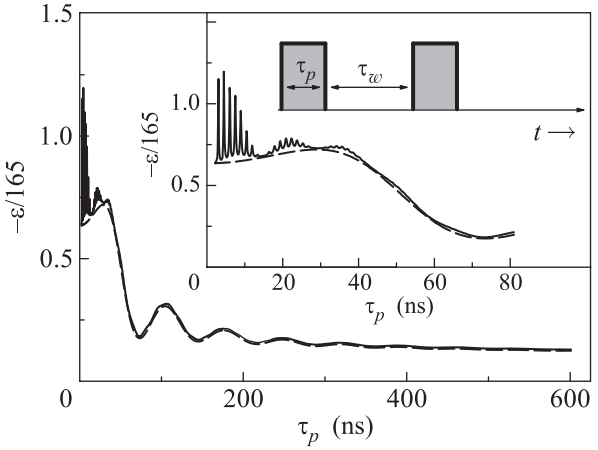

Fig. 1. DNP enhancement as a function of the EPR pulse length, τ_p. The total enhancement is shown by solid line; the contribution from the resonant component is shown by dashed line. Pulses are ideal; two cycles of the pulse sequence are schematically presented. Parameters of the calculation: $\xi\cdot f=\frac{1}{2}$; nuclei are protons, duty cycle is $D=10\,\%$; EPR spectrum has two components with splitting $\Delta=2.4\,\mathrm{mT}$ (i.e., the maximal contribution to ε from each component is -165); EPR pumping is performed in resonance with one of the components; $\omega_1/\gamma_e=0.5\,\mathrm{mT}$, $T_{1e}=300\,\mathrm{ns}$, $T_{2e}=150\,\mathrm{ns}$

lines. When the MW-field is on resonance with one of the lines, ε is predominantly due to excitation of this component: $\varepsilon\approx\varepsilon_{\mathrm{res}}$. The dependence of $\varepsilon_{\mathrm{res}}$ on τ_p contains pronounced oscillations, which correspond to nutation of the electron spin about the MW-field. Maxima in this dependence correspond to rotation of the electron spin by π, 3π, 5π, ...; damping of oscillations and decay of the enhancement is caused by (i) T_1- and T_2-relaxation of the electron spin during the pulse and (ii) by the fact that long τ_p also corresponds to long τ_w (as $D=\mathrm{const}$), such that electrons are staying at thermal equilibrium most of the time. Interestingly, at short τ_p there is an additional contribution to ε with fast oscillations, which come from the non-resonant spectral component, see Fig. 1. When the frequency off-set, Δ, between the EPR lines is much larger than ω_1 a single MW-pulse can flip the electron spin magnetization for an off-resonant component by a small angle only. However, a periodic train of pulses can excite the off-resonant component almost as well as the resonant one, consequently, at certain short τ_p times the enhancement is almost doubled as compared to $\varepsilon_{\mathrm{res}}$ (Fig. 1). The frequency of fast oscillations corresponds to the precession of the electron spin magnetization for the off-resonant component in the MW-rotating frame, i.e., it is given by Δ.

In experiments, a behavior similar to that for $\varepsilon_{\mathrm{res}}$ has already been reported [7, 8]; however, no fast oscillations were visible. We attribute the lack of the fast component to MW-pulse imperfections, namely, to the finite rise- and fall-times of the MW-pulses. With the approach developed here we can verify this idea by using in the calculation a time-dependent MW-field strength $\omega_1(t)$. We consider here pulses with exponentially rising and decreasing ω_1 strength:

$$\omega_1(t)=\omega_{1b}+\begin{cases}\omega_{1p}\left(1-e^{-\frac{t}{\tau_r}}\right), & \text{at } 0<t<\tau_p,\\ \omega_1(t=\tau_p)e^{-\frac{t-\tau_p}{\tau_r}}, & \text{at } \tau_p<t<\tau_p+\tau_w.\end{cases}\qquad(11)$$

Here $\omega_{1b}=\omega_{1p}e^{-\frac{\tau_w}{\tau_r}}(1-e^{-\frac{\tau_p}{\tau_r}})/(1-e^{-\frac{\tau_p+\tau_w}{\tau_r}})$. The $\omega_1(t)$ dependences are shown in Fig. 2 for the ideal pulses and pulses with finite τ_r. Here ω_{1p} is the peak MW-field strength; τ_r stands for the pulse rise-time. It can be proven that for the chosen $\omega_1(t)$ function the integral $\int_0^\tau\omega_1(t)dt$ is the same for ideal and non-ideal pulses having the same peak MW-field strength, ω_{1p} (and when all other parameters are the same).

The calculation results for such non-ideal pulses are shown in Fig. 3. The contribution from the resonant component, giving the slower oscillations at the frequency ω_{1p}, almost does not depend on τ_r, but the results for the off-resonant component are very different, namely, the enhancement coming from this component (fast oscillation) almost vanishes at longer τ_r. The reason is that pulses with slow rise-times cannot flip this component; thus, they do not excite the electron spins at $\Delta\gg\omega_1$. Therefore, for achieving significant enhance-

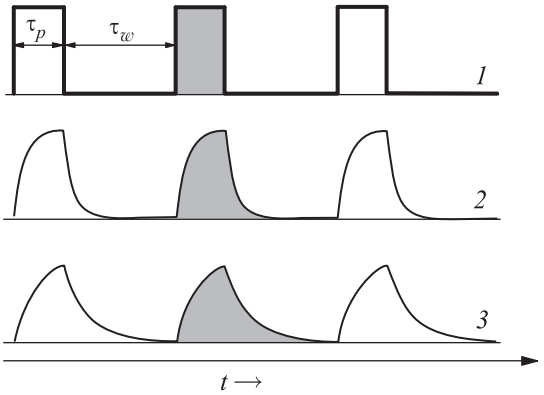

Fig. 2. Time dependence of ω_1 (shown for three periods of the pulse sequence) for ideal pulses, i.e., $\tau_r = 0$ (*1*), $\tau_r = \tau_p/5$ (*2*), and $\tau_r = \tau_p/2$ (*3*); here $D = 30\,\%$ for the ideal pulses. The shaded area, given by the integral $\int_0^\tau \omega_1(t)dt$, is exactly the same in all cases when ω_{1p} is the same

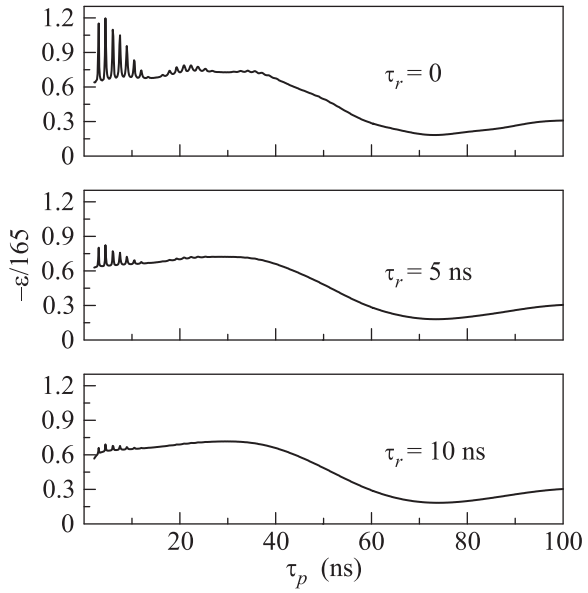

Fig. 3. DNP enhancement as a function of the EPR pulse length, τ_p, shown for different τ_r equal to 0 (ideal pulses), 5 ns and 10 ns. Other parameters of the calculation are the same as those used in Fig. 1

ment from off-resonant components it is necessary to use MW-pulses with short rise-times. The threshold τ_r value, at which the fast oscillations from the off-resonant component become pronounced, depends on the splitting between the EPR lines and on the ω_{1p} value. For the parameters chosen in Figs. 1 and 3, this threshold τ_r is about 5 ns as follows from our numerical calculations (compare the three traces in Fig. 3). The threshold τ_r value can be estimated by using the relation $\frac{\omega_{1p}}{\tau_r} > \Delta^2$, as follows from the criteria for non-adiabatic variation of the spin Hamiltonian [9].

Our approach can also be used to analyze the effect of the phase of subsequent MW pulses. To do so, we treated two different periodic pulse sequences in addition to the simplest sequence, see Fig. 1. In the first sequence (PS1), the period consists of two ideal rectangular pulses (i.e., $\tau_r \to 0$) having the same duration, τ_p, but different phase, which is 0 and $\pi/2$, separated by the same delay τ_w. In the second sequence (PS2), the period contains eight rectangular equally spaced pulses of the same duration with their phases, φ_i, taking random values distributed between 0 and 2π.

The effect of variable phase of the MW pulses is demonstrated in Fig. 4. This effect is pronounced only

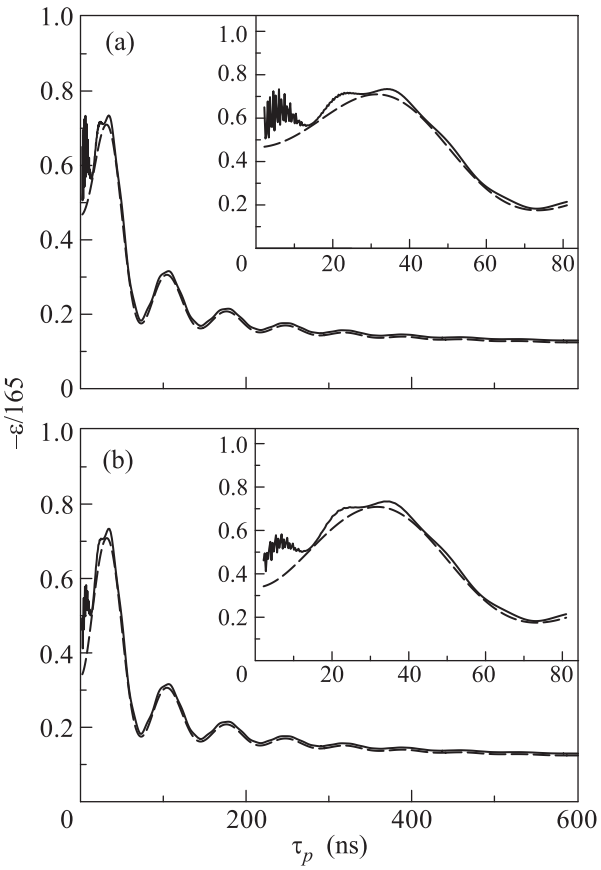

Fig. 4. DNP enhancement as a function of the EPR pulse length, τ_p, for the pulse sequence PS1 (a) and PS2 (b). The total enhancement is shown by solid line; the contribution from the resonant component is shown by dashed line. Other parameters of the calculation are the same as those used in Fig. 1

when the delay between the pulses is shorter than (or, at least, comparable to) T_{1e}, therefore, at long τ_p and τ_w the phases of subsequent pulses have no effect on

the enhancement and can be set arbitrarily. At short τ_r and τ_w the results are considerably different for the three pulse sequences considered here (compare Figs. 1 and 4). Specifically, the fast oscillations coming from the off-resonant component are strongly suppressed by phase alterations, see Fig. 4. The behavior of the resonant component is also affected by the MW pulse phases; however, this effect is smaller and vanishes for τ_p corresponding to a π-pulse or longer. Suppression of the enhancement at short τ_r is the highest for arbitrarily set phases. Thus, for achieving the sought enhancement from the off-resonant EPR component it is necessary to control the phases of the MW-pulses.

Finally, let us demonstrate an application of our theoretical approach to experimental results for pulsed DNP. Experiments were performed in the same way as before [7] at EPR pumping frequency of 300 MHz (i.e., by RF-pulses). To match EPR conditions the sample was brought to low magnetic fields (around 10 mT). Experiments were done for D_2O solutions of protonated TEMPOL radical, which has three EPR components (due to hyperfine interaction of the electron spin with the spin-1 ^{14}N nucleus) with a splitting of 1.7 mT. The enhancement measured for a set of different D values for the the residual water protons exhibits a pronounced dependence on the DNP pulse length, τ_p. We are able to fit the entire dataset using a common set of fitting parameters, which are the electron relaxation times, ω_{1p} and the product $\xi \cdot f$. The agreement between theory and experiment is good; this fact allows us to determine the following parameters: $\omega_1/\gamma_e = 0.51$ mT, $T_{1e} = 730$ ns, $T_{2e} = 70$ ns, $\xi \cdot f = 0.475$, i.e., $f = 0.95$, since at low fields $\xi \approx \frac{1}{2}$ (valid for dipolar-induced cross-relaxation). We attribute the difference between T_{1e} and T_{2e} to inhomogeneous broadening of the EPR lines by hyperfine couplings to protons, which shortens T_{2e} but does not have a strong effect on T_{1e}. The T_{2e} value corresponds to hyperfine interactions of about 0.12 mT.

It is worth noting that in the example presented in Fig. 5 the off-resonant components significantly contribute to the signal. Indeed, when only the resonant component is responsible for DNP the maximal enhancement is $\varepsilon = -\frac{1}{3}\xi \cdot f \cdot \left|\frac{\gamma_e}{\gamma_n}\right| \approx -110$ for $\xi \cdot f = \frac{1}{2}$ and $\varepsilon \approx 105$ when $\xi \cdot f = 0.475$. The fact that the maximal enhancement is over 120 is an indication that off-resonant components contribute to the signal; their contribution has been taken into account in the modeling.

Thus, we have developed a general approach to pulsed Overhauser-type DNP. With this approach we are able to treat an arbitrary pulse sequence, to take

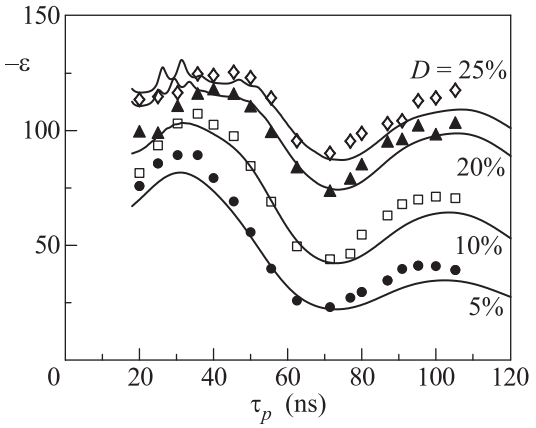

Fig. 5. DNP enhancement as a function of the EPR pulse length, τ_p at different duty cycle D. Experimental results are shown by circles ($D = 5\,\%$), squares ($D = 10\,\%$), triangles ($D = 20\,\%$) and diamonds ($D = 25\,\%$); simulation results are shown by lines. Experiments were done using TEMPOL radicals having three EPR lines split by 1.7 mT; pumping was performed at the central component, $\tau_r = 8.7$ ns. Parameters determined from fitting: $\xi \cdot f = 0.475$; $\omega_1/\gamma_e = 0.51$ mT, $T_{1e} = 730$ ns, $T_{2e} = 70$ ns

quantitatively into account EPR pulses of realistic shape, or to consider EPR pulses having variable phase. We have predicted strong DNP effects coming from excitation of non-resonant EPR components; however, such components are manifest only when the rise-time of EPR pulses is sufficiently short. Last, but not least, we have applied our approach to describe our experimental results for pulsed DNP at low magnetic fields, being able to model the experimental τ_p-dependences and to determine relevant experimental parameters, such as T_{1e} and T_{2e}. These results demonstrate that pulsed DNP can be a useful method providing strong NMR signal enhancements when a broad EPR spectrum is excited by short pulses.

We anticipate that our theory can be used to predict DNP enhancements in more complex situations, for instance, when "composite" MW-pulses consisting of several interleaved pulses of different frequencies, or chirped pulses are used in order to excite a broad EPR spectrum and thus to give guide-lines for efficient experiments. Additionally, EPR spectral exchange (coming from the Heisenberg exchange [10, 11] or nuclear quadrupolar relaxation in the case of ^{14}N nitroxides [12, 13]) can be treated, resulting in an increased "saturation" factor. Another potential application is modeling of low-field DNP experiments in the hyperfine-dominated range with low-frequency EPR pumping [14]. Hence, our approach gives a solid theoretical ground for modeling pulsed DNP experiments, for predicting the resulting NMR enhancement and for determining EPR

parameters of paramagnetic polarizing agents from the dependence of ε on the timing of MW pulse sequences.

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