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# The occurrence of a Mott-like gap in single-particle spectra of electron systems possessing flat bands

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An unconventional type of the Mott's insulators where the gap in the spectrum of single-particle excitations is associated with repulsive effective interactions between quasiparticles is shown to exist in strongly correlated electron systems of solids that possess flat bands. The occurrence of this gap is demonstrated to be the consequence of violation of particle-hole symmetry, inherent in such systems. The results obtained are applied to elucidate the Fermi arc structure observed at temperatures up to 100 K in angle-resolved photoemission spectra of the compound $Sr_2IrO_4$, not showing superconductivity down to low $T$.

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The present article is devoted to the analysis of spectra $\epsilon(\mathbf{p})$ of single-particle excitations of strongly correlated electron systems of solids within a flat-band scenario developed in Refs. [1–5]. In this scenario, non-Fermi-liquid (NFL) behavior of such systems, experimentally studied for more than 20 years, is attributed to the occurrence of flat bands (zero-energy fermions), a $T = 0$ dispersionless portion $\epsilon = 0$ of the single-particle spectrum, frequently called the fermion condensate (FC). Originally, basic aspects of theory of fermion condensation that properly elucidates NFL behavior of strongly correlated Fermi systems (see, e.g., [6–14]) were developed on the base of the Landau approach to FL theory where the ground state energy $E$ is treated as a functional of the quasiparticle momentum distribution $n(\mathbf{p})$. A crucial point is that the ground-state momentum distribution $n_*(\mathbf{p})$ of a system with the FC is found with the aid of variational condition [1]

$$\frac{\delta E}{\delta n(\mathbf{p})} - \mu = 0, \quad \mathbf{p} \in \Omega. \tag{1}$$

Since in normal states, examined in this article, the l.h.s. of this condition is nothing but the quasiparticle energy $\epsilon(\mathbf{p})$ measured from the chemical potential $\mu$, this equation implies the formation of the FC in the momentum region $\mathbf{p} \in \Omega$, where $n_*(\mathbf{p})$ changes continuously between 0 and 1. In the complementary domain $\mathbf{p} \notin \Omega$, associated with quasiparticles, not belonging to the FC, the distribution $n_*(\mathbf{p})$ coincides with the FL one, being 1 for the occupied states and 0, otherwise.

Within theory of fermion condensation [1–5], the dispersion of the spectrum of such quasiparticles, called further normal, is evaluated in terms of a phenomenological interaction function $f(\mathbf{p}, \mathbf{p}_1)$ with the aid of Landau equation

$$\frac{\partial \epsilon(\mathbf{p})}{\partial \mathbf{p}} = \frac{\mathbf{p}}{M} + \int f(\mathbf{p}, \mathbf{p}_1) \frac{\partial \epsilon(\mathbf{p}_1)}{\partial \mathbf{p}_1} d\mathbf{p}_1, \quad \mathbf{p} \notin \Omega, \tag{2}$$

where $d\mathbf{p}$ is the volume element in momentum space, including the factor $(2\pi)^i$ in the denominator, with $i$, being dimensionality of the problem.

Results of numerous calculations (see, e.g., Ref. [14]) demonstrate that there is no gap, separating this normal part of the spectrum $\epsilon(\mathbf{p})$ from the dispersionless FC one. The purpose of the present article is to check whether this feature holds, going beyond the scope of the existing version of theory of fermion condensation. As we will see, it does not: if interactions between the FC and normal quasiparticles are taken into account properly, a gap emerges that separates the FC spectrum from that of normal quasiparticles.

Here we address the case $T = 0$. Since at finite $T$, the FC dispersion changes linearly with $T$ [3], the $T = 0$ results obtained below hold at low $T$, as long as the gap value $D(0)$ exceeds the FC width $\propto \rho_{\rm FC} T$. Otherwise, corrections to results obtained within the existing version of theory are of no interest, being small.

To gain insight into the problem it is advantageous to employ the Belyaev's diagram technique developed in his work on theory of Bose liquid [15]. In doing so we treat results of numerical solving the set of Eqs. (1) and

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(2) for the momentum distribution $n_*(\mathbf{p})$ and energy spectrum, denoted further $\epsilon_0(\mathbf{p})$, as an initial iterate.

Since every integration over the FC domain introduces an additional small dimensionless factor $\eta = \rho_{\rm FC}/\rho$, the full set of the diagrams under consideration can be divided into subsets vs. the amount of the FC lines displayed. This situation is opposite to that in low-density Bose gas where just occupation numbers of normal quasiparticles, proportional to the difference between the total density and condensate one, are small [16].

The simplest way to generalize the existing version of theory of fermion condensation is to straightforwardly evaluate the imaginary part $\Sigma''$ of the mass operator of the normal quasiparticle and then calculate the real part $\Sigma'$ with the aid of the Kramers–Kronig dispersion relation. Therefore it is instructive to begin the analysis with remembering basic points of evaluation of $\Sigma$ in FL theory where formula for $\Sigma''$ reads [17]:

$$\Sigma''(\mathbf{p},\varepsilon) \propto -\iint |\Gamma^2(\mathbf{p},\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3)|\delta(\varepsilon+\epsilon_3-\epsilon_1-\epsilon_2)$$
$$\Big(n_3(1-n_1)(1-n_2)-(1-n_3)n_1n_2\Big)d\mathbf{p}_1 d\mathbf{p}_2. \quad (3)$$

Here $\Gamma$ stands for the scattering amplitude, and $n_k = \theta(-\epsilon_k)$ are $T=0$ quasiparticle occupation numbers, where $\epsilon_k = \epsilon(\mathbf{p}_k)$, with $k=1,2,3$ and $\mathbf{p}_3 = \mathbf{p}_1+\mathbf{p}_2-\mathbf{p}$, are single-particle energies.

Strictly speaking, the exact $T=0$ formula for $\Sigma''$ does contain the product of three spectral functions

$$A(\mathbf{p},\varepsilon) \propto \frac{|\Sigma''(\mathbf{p},\varepsilon)|}{[\varepsilon-\epsilon_0(\mathbf{p})-\Sigma'(\mathbf{p},\varepsilon)]^2+[\Sigma''(\mathbf{p},\varepsilon)]^2}, \quad (4)$$

associated with imaginary parts of respective quasiparticle Green functions $G(\mathbf{p},\varepsilon) = (\varepsilon-\epsilon_0(\mathbf{p})-\Sigma(\mathbf{p},\varepsilon))^{-1}$. However, in conventional Fermi liquids, the damping $\gamma(\varepsilon)$ of single-particle excitations is quadratic in energy:

$$\gamma(\varepsilon) \propto -\Sigma''(\varepsilon>0) \propto \varepsilon^2, \quad (5)$$

implying that it is small compared with energy. Indeed, in Eq. (3) integration virtually occurs over 3 positive energies $\epsilon_1, \epsilon_2$, and $-\epsilon_3$, confined to the interval $[0,\varepsilon]$, the number of integrations reducing to 2 by virtue of the presence of $\delta(\varepsilon+\epsilon_3-\epsilon_1-\epsilon_2)$ in the integrand. As a result, each of two remaining integrations introduces the factor $\varepsilon$ to yield Eq. (5) and justify the replacement $A(\mathbf{p},\varepsilon)\to\delta(\varepsilon-\epsilon(\mathbf{p}))$.

Once the imaginary part $\Sigma''$ of the mass operator changes continuously through the Fermi surface, so does its real part $\Sigma'(\mathbf{p},\varepsilon)$ as well. In this case, the single-particle spectrum $\epsilon(\mathbf{p})$, evaluated from standard equation

$$\epsilon(\mathbf{p}) = \epsilon_0(\mathbf{p}) + \Sigma'(\mathbf{p},\epsilon(\mathbf{p})), \quad (6)$$

with the bare spectrum $\epsilon_0(\mathbf{p})$, turns out to be gapless.

However, in Fermi systems with flat bands, Eq. (5) *fails*, since in calculations of Eq. (3) two energies associated with FC quasiparticles *identically vanish*, so that the number of energy integrations reduces from 3 to 1, and consequently, the factor $\varepsilon^2$, identifying conventional Fermi liquids, disappears. As a result, the damping $\gamma(\varepsilon)$ turns out to be *energy independent*, and therefore

$$\Sigma''(\varepsilon\to 0) \propto -\frac{\varepsilon}{|\varepsilon|} \quad (7)$$

that rules out the conjecture $A(\mathbf{p},\varepsilon)\to\delta(\varepsilon-\epsilon(\mathbf{p}))$.

Nevertheless, the result (7) itself remains unchanged [8]. Indeed, by virtue of the dispersionless character of the FC spectrum, this subsystem behaves as a set of impurities. Therefore in the amplitude of scattering of normal quasiparticles by the FC, there is a pure elastic term. This circumstance straightforwardly leads to Eq. (7).

In Fermi gas with impurities, the role of the real part $\Sigma'$ of the mass operator $\Sigma$ reduces to a slight renormalization of the chemical potential $\mu$. Contrariwise, in Fermi systems with flat bands, $\Sigma'(\varepsilon)$ acquires a logarithmically divergent term

$$\Sigma'(\varepsilon\to 0) \propto -\frac{\varepsilon}{|\varepsilon|}\ln|\varepsilon|, \quad (8)$$

due to *violation of particle-hole symmetry*, inherent in these systems (see below). This implies the occurrence of the gap in the single-particle spectrum, verified by inserting Eq. (8) into Eq. (6), a basic result of the analysis performed in the present article.

Let us now turn to a more detailed analysis of second-order FC contributions to the imaginary part $\Sigma''$ of the mass operator $\Sigma$, coming from diagrams that contain two FC lines. (The total contribution of diagrams with the single FC line was shown long ago not to provide the gap in the spectrum $\epsilon(\mathbf{p})$ [6, 18]). The second-order part of $\Sigma''$ is found with the aid of a modified formula (3) where two functions $n(\mathbf{p})$ are replaced by $n_*(\mathbf{p})$. There are several options to do that. However, violation of particle-hole symmetry, discussed above, occurs only in a diagram displayed in Fig. 1 where normal quasiparticles, depicted by solid lines, convert to the FC quasiparticles, drawn by dashed ones. This diagram is reminiscent of that, relevant to inhomogeneous Larkin–Ovchinnikov–Fulde–Ferrell (LOFF) pairing with certain total momentum $\mathbf{P}\neq 0$ [19, 20]. However, in the case under consideration, where LOFF pairing is supposed to be forbidden by virtue of the repulsive character of the interaction between quasiparticles in the Cooper channel, integration over all accessible momenta $\mathbf{P}$ is carried

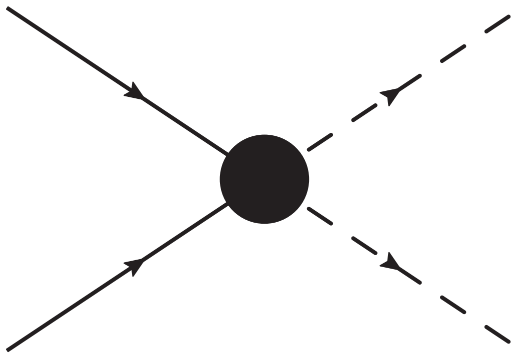

Fig. 1. The graphical representation for the transfer of two normal quasiparticles (solid lines) to the FC (dotted lines)

out that provides the restoration of homogeneity of the ground state. The explicit expression for $\Sigma''(\mathbf{p} \notin \Omega, \varepsilon)$ is obtained from Eq. (3) with replacements: i) $\epsilon_1 = \epsilon_2 \to 0$, $n(\mathbf{p}_1) \to n_*(\mathbf{p}_1), n(\mathbf{p}_2) \to n_*(\mathbf{p}_2)$, ii) $\delta(\varepsilon + \epsilon(\mathbf{p}_3)) \to \to A(\mathbf{p}_3, -\varepsilon)$, and iii) $A(\mathbf{p}_1 \in \Omega, \varepsilon_1) \to \delta(\varepsilon_1)$, $A(\mathbf{p}_2 \in \in \Omega, \varepsilon_2) \to \delta(\varepsilon_2)$ (the latter replacement holds provided fourth-order FC contributions are neglected).

As a result, we find

$$|\Sigma''(\mathbf{p}, \varepsilon > 0)| = \int_C |\Gamma^2(\mathbf{p}, \mathbf{p}_2, \mathbf{P} - \mathbf{p}, \mathbf{P} - \mathbf{p}_2)|$$

$$(1 - n_*(\mathbf{p}_2))(1 - n_*(\mathbf{P} - \mathbf{p}_2))A(\mathbf{P} - \mathbf{p}, -\varepsilon)d\mathbf{P}d\mathbf{p}_2, \quad (9)$$

while

$$|\Sigma''(\mathbf{p}, \varepsilon < 0)| = \int_C |\Gamma^2(\mathbf{p}, \mathbf{p}_2, \mathbf{P} - \mathbf{p}, \mathbf{P} - \mathbf{p}_2)|$$

$$n_*(\mathbf{p}_2)n_*(\mathbf{P} - \mathbf{p}_2)A(\mathbf{P} - \mathbf{p}, -\varepsilon)d\mathbf{P}d\mathbf{p}_2. \quad (10)$$

From aforesaid we infer that the scattering amplitude $\Gamma$, entering this expression, is evaluated at $\eta = 0$. The domain $C$ of integration over momenta $\mathbf{p}_2$ and $\mathbf{P}$ is determined by conditions

$$\mathbf{p}_2 \in \Omega, \quad \mathbf{P} - \mathbf{p}_2 \in \Omega. \quad (11)$$

As seen, the integration over $\mathbf{p}_2$ is separated from that over $\mathbf{P}$, so that upon inserting the explicit form of the spectral function $A$ we arrive at

$$|\Sigma''(\mathbf{p}, \varepsilon)| = \int \frac{K(\mathbf{p}, \mathbf{s}, \varepsilon)|\Sigma''(\mathbf{s}, -\varepsilon)|d\mathbf{s}}{(e(\mathbf{s}, \varepsilon) + \epsilon_0(\mathbf{s}))^2 + (\Sigma''(\mathbf{s}, -\varepsilon))^2}, \quad (12)$$

where $\mathbf{s} = \mathbf{P} - \mathbf{p}$ and

$$e(\mathbf{s}, \varepsilon) = \varepsilon + \Sigma'(\mathbf{s}, -\varepsilon), \quad (13)$$

while the normal component $\epsilon_0(\mathbf{p})$ of the single-particle spectrum is found from Eq. (2). The function $K(\varepsilon)$ is defined as $K(\varepsilon > 0) = K^+$, and $K(\varepsilon < 0) = K^-$, with

$$K^+ = \int_C |\Gamma^2(\mathbf{p}, \mathbf{p}_2, \mathbf{P})|(1 - n_*(\mathbf{p}_2))(1 - n_*(\mathbf{P} - \mathbf{p}_2))d\mathbf{p}_2,$$

$$K^- = \int_C |\Gamma^2(\mathbf{p}, \mathbf{p}_2, \mathbf{P})|n_*(\mathbf{p}_2)n_*(\mathbf{P} - \mathbf{p}_2)d\mathbf{p}_2. \quad (14)$$

Evidently, both the functions $K^\pm$ change linearly with the FC density $\eta$.

The presence of two different expressions, containing the FC momentum distribution $n_*(\mathbf{p})$ in these formulas does ensure *violation of particle-hole symmetry*, since the FC distribution is not invariant with respect to the replacement $n_*(\mathbf{p}) \to 1 - n_*(\mathbf{p})$, and then $|\Sigma''(\mathbf{p}, \varepsilon \to 0^+)| \neq |\Sigma''(\mathbf{p}, \varepsilon \to 0^-)|$. We will employ this fact in derivation of Eq. (8).

In what follows we focus on 2D electron liquid, placed in an external field of the quadratic lattice; such a situation is relevant to cuprates, the most extensively studied family of high-$T_c$ superconductors. In this case, the FC domain consists of four small spots [21], adjacent to saddle points $(0, \pm\pi), (\pm\pi, 0)$, associated with van Hove points (VHPs) where the density of states diverges (see Fig. 2). The association between the FC

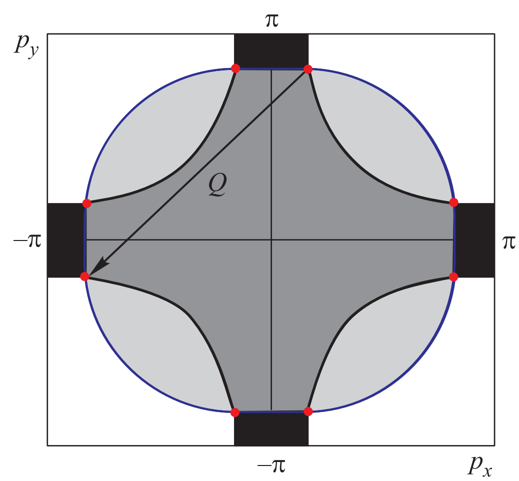


Fig. 2. (Color online) Fermi line (black) and its counterpart (blue) for the bare tight-binding spectrum $\epsilon^0_{\mathbf{p}} = -2\,t_0\,(\cos p_x + \cos p_y) + 4\,t_1\,\cos p_x \cos p_y$, with $t_1/t_0 = 0.45$. The FC regions [21] are colored in black

spots and VHPs stems from observation that the onset of fermion condensation is triggered by violation of the *necessary stability condition* for the Landau state (for detail, see, e.g., Ref. [22]), occurring just beyond critical points where the density of states diverges.

First, let momenta $\mathbf{p}_2$ and $\mathbf{P} - \mathbf{p}_2$ be related to opposite FC spots. Total momentum $\mathbf{P}$ is then close to 0 that resembles the case of Cooper pairing. Boundaries of the integration region $C$ are found with the aid of Eqs. (11). In the case where momentum $\mathbf{p}_2$ is related, e.g., to the FC spot, situated near the saddle point $(0, \pi)$, while momentum $\mathbf{P} - \mathbf{p}_2$, to the FC spot, located close to the opposite saddle point $(0, -\pi)$, one obtains

$$
\begin{gathered}
-\frac{L_x}{2} \le (P_x - p_{2x}) \le \frac{L_x}{2}, \\
-\pi \le (P_y - p_{2y}) \le -\pi + \frac{L_y}{2}, \\
-\frac{L_x}{2} \le p_{2x} \le \frac{L_x}{2}, \quad \pi - \frac{L_y}{2} \le p_{2y} \le \pi,
\end{gathered} \tag{15}
$$

so that

$$
-L_x \le P_x \le L_x, \quad -L_y/2 \le P_y \le L_y/2, \tag{16}
$$

where quantities $L_x, L_y \simeq \eta^{1/2}$ determine the FC range.

The implementation of the mean value theorem (MVT) for integrals, applicable due to positivity of all functions, standing in the integrand, allows one to recast Eq. (12) in the form

$$
|\Sigma''(\mathbf{p},\varepsilon)| = K_{av}(\mathbf{p},\varepsilon) \int \frac{|\Sigma''(\mathbf{s},-\varepsilon)| d\mathbf{s}}{(e(\mathbf{s},\varepsilon)+\epsilon_0(\mathbf{s}))^2 + (\Sigma''(\mathbf{s},-\varepsilon))^2} \tag{17}
$$

where $K_{av}(\mathbf{p},\varepsilon) \propto \eta$ is an averaged value of the function $K(\mathbf{p},\mathbf{P},\varepsilon)$ in the integration region $C$, controlled by Eq. (11).

Inequalities (16) imply that the distance between vectors $\mathbf{p}$ and $\mathbf{s}$ is small that allows one to replace the function $\Sigma(\mathbf{s})$, standing in the denominator of the integrand (17), by $\Sigma(\mathbf{p})$. Furthermore, in the region near the Fermi surface where $|\varepsilon|, |\epsilon_0(\mathbf{p})| \le v_{\rm F} P \simeq \epsilon^0_{\rm F}\eta$, both the term $\Sigma'$ and $\varepsilon$ can be neglected (see below). With this simplifications, Eq. (12) becomes

$$
|\Sigma''(\mathbf{p},\varepsilon)| = K_{av}(\mathbf{p},\varepsilon) \int \frac{|\Sigma''(\mathbf{p},-\varepsilon)| d\mathbf{s}}{\epsilon_0^2(\mathbf{s}) + (\Sigma''(\mathbf{p},-\varepsilon))^2}. \tag{18}
$$

The two-dimensional integration in this equation, whose limits are specified by Eq. (16), is performed with the aid of formula $d\mathbf{s} = dndt = d\epsilon_0 dt/|\mathbf{v}_0|$, where $n$ and $t$ are normal and transversal components of the vector $\mathbf{s}$, while $\mathbf{v}_0(\mathbf{s}) = \nabla\epsilon_0(\mathbf{s})$ where $\mathbf{s} \simeq \mathbf{p} \notin \Omega$. The repeated application of the MVT then yields

$$
\begin{aligned}
\Sigma''(\mathbf{p},\varepsilon>0) &= -\pi l \zeta(\mathbf{p}) K^+_{av}(\mathbf{p}), \\
\Sigma''(\mathbf{p},\varepsilon<0) &= \pi l \zeta(\mathbf{p}) K^-_{av}(\mathbf{p}),
\end{aligned} \tag{19}
$$

where $l \simeq L \propto \eta^{1/2}$ is the length of the interval of integration over $t$, and $\zeta(\mathbf{p})$, an averaged value of the function $1/v_0(\mathbf{s})$.

Evidently, the result obtained is in agreement with Eq. (7), discussed above, justifying that in systems with flat bands, the absolute value of the imaginary part of the mass operator of a normal quasiparticle experiences a *discontinuity* at the Fermi surface. Its magnitude, being of order $\eta^{3/2}$, consists of the factor $\eta$, coming from integration over $\mathbf{p}_2$, and an additional factor $\eta^{1/2}$ associated with the limits of integration over $t$. Similar results are obtained in case the FC momenta belong to the same FC spot, since then total momenta $\mathbf{P}$ turn out to be close to $2\pi/a$, and integration is performed over a small region of momenta $\mathbf{P}' = 2\pi/a - \mathbf{P}$. At the same time, contributions from neighbor FC spots are verified to be suppressed.

Having at hand these results, the real part $\Sigma'$ of the mass operator is then found on the base of the Kramers–Kronig dispersion relation

$$
\Sigma'(\varepsilon) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{\Sigma''(\varepsilon') sign(\varepsilon')}{\varepsilon' - \varepsilon} d\varepsilon'. \tag{20}
$$

In conventional Fermi liquids the particle-hole symmetry holds, implying that $\Sigma''(\varepsilon \to 0^+) = -\Sigma''(\varepsilon \to 0^-)$, and therefore the integral (20) identically vanishes. True, at large distances from the Fermi surface, this symmetry is somehow violated. However, this violation leads only to a renormalization of the chemical potential $\mu$. Contrariwise, according to Eqs. (14) and (19), in systems with flat bands, violation of the particle-hole symmetry occurs just at the Fermi surface. This makes the difference. Indeed, upon inserting Eq. (19) into Eq. (20) and simple manipulations we are led to

$$
\begin{aligned}
\Sigma'(\mathbf{p},\varepsilon\to 0^+) &= -\lambda\eta^{3/2}\zeta(\mathbf{p}) \ln|\varepsilon|, \\
\Sigma'(\mathbf{p},\varepsilon\to 0^-) &= \lambda\eta^{3/2}\zeta(\mathbf{p}) \ln|\varepsilon|.
\end{aligned} \tag{21}
$$

In writing Eq. (21) all numerical factors, independent of $\eta$, are absorbed into the effective coupling constant $\lambda \propto K^-_{av}(\mathbf{p}) - K^+_{av}(\mathbf{p})$, whose sign is supposed to be positive to avoid contradictions with the requirement $\partial\Sigma'(\varepsilon)/\partial\varepsilon < 0$.

With the results obtained, approximations made above are easily verified. Indeed, according to Eq. (21), one has $\Sigma'(\mathbf{s}) \propto \eta^{3/2}$. At the same time, $|\epsilon_0(\mathbf{s})| \simeq Pv_{\rm F} \propto \eta$, so that the contribution from $\Sigma'$ to Eq. (18) can be freely neglected. The same is valid for the term $\varepsilon$, as long as $|\varepsilon| < Pv_F$.

Upon inserting Eq. (21) into Eq. (6) where $\varepsilon$ is replaced by the true single-particle energy, denoted further by $E$, we are led to

$$
\begin{aligned}
E(\mathbf{p}) + \lambda\eta^{3/2}\zeta(\mathbf{p}) \ln E(\mathbf{p}) &= \epsilon_0(\mathbf{p}), \quad E>0, \\
E(\mathbf{p}) - \lambda\eta^{3/2}\zeta(\mathbf{p}) \ln|E(\mathbf{p})| &= \epsilon_0(\mathbf{p}), \quad E<0.
\end{aligned} \tag{22}
$$

Upon setting $\epsilon_0(\mathbf{p}) = 0$, two nontrivial solutions of Eq. (22) are found:

$$
E(\mathbf{p}) \simeq \pm\lambda\eta^{3/2}\zeta(\mathbf{p}) \ln(1/\eta). \tag{23}
$$

We emphasize that the occurrence of the gap (23) in the single-particle spectrum is entailed by the divergence

(21) of the real part of the mass operator, (cf. situation in BCS theory where gap solutions $E(\mathbf{p}) = \pm\Delta(\mathbf{p})$ owe their existence to the pole singularity of the Cooper mass operator $\Sigma'(p,\varepsilon) = \Delta^2/(\varepsilon + \epsilon_0(p))$). Since the gap (23) emerges in the normal state, it can be viewed as a unconventional Mott's gap in the spectrum of single-particle excitations of systems possessing flat bands.

Noteworthy, in deriving these results we applied the same perturbation-theory strategy as J. Kondo in his seminal work on the problem of electron scattering by magnetic impurities in metals. Curiously, his result also contains the logarithmic term $\ln|\varepsilon|$, however, in contrast to Eq. (21), the Kondo correction enters the imaginary part of the electron mass operator, rather than the real one. Summation of higher orders of the Belyaev-like expansion employed here is beyond the scope of the present article. Nevertheless, we hope that similarly to the situation with the Kondo effect, such a summation reduces only to the renormalization of input parameters.

The gap (23) has the specific angular dependence associated with the factor $\zeta(\mathbf{n})$, $(\mathbf{n} = \mathbf{p}/p)$. Outside the FC regions, this quantity differs little from $1/v_0(\mathbf{n})$. Therefore by virtue of vanishing of $v_0(\mathbf{p} \in \Omega)$ due to the dispersionless character of the FC spectrum, the gap magnitude rapidly grows toward the saddle points $(0, \pm\pi)$, $(\pm\pi, 0)$ that results in a specific Fermi arc structure (FAS) of the angle-resolved photoemission spectrum, breaking up of the Fermi surface into disconnected segments. Usually this structure exhibits itself in the ARPES data on high-$T_c$ superconductors where it is conventionally attributed to the occurrence of preformed pairs [23]. However, recently the FAS was uncovered in measurements of photoemission spectra of a 2D metal $Sr_2IrO_4$ that shows no superconductivity down to low $T$; nevertheless, the FAS persists up to 100 K [24–26] that rules out the conventional scenario [23].

It is instructive to address the situation where the Mott's gap is sufficiently large to provide profound suppression of the conductivity $\sigma(T) \propto e^{-D/T}$. In this case, the electron system with the flat band behaves as a strongly correlated system of *neutral fermions*, with the magnetic moment $\mu_B \propto e/m_e$. Its thermodynamic properties, associated with the flat band, remain the same as in the situation without the Mott's gap.

Calculations, carried out above, can also be performed in the case of homogeneous matter to yield

$$E(p) \simeq \pm\lambda\eta \frac{\ln(1/\eta)}{v_{\mathrm{F}}}. \qquad (24)$$

With respect to Eq. (23), the corresponding value of the Mott's gap is somewhat enhanced by virtue of different kinematic restrictions in comparison to Eq. (16). The presence of the significant Mott's gap may affect properties of dense quark matter where the FC presumably resides [14]. In the scenario, discussed in the present article, opening the Mott's gap results in the profound suppression of neutrino cooling of hybrid compact stars with a sharp hadron-quark interface. In connection with this idea, it is not improbable that this suppression is relevant to recent observations of a central compact object in the supernova remnant HESSJ1731-347, being the hottest isolated neutron star, in spite of its venerable age [27, 28].

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