

Edge absorption and circular photogalvanic effect in 2D topological insulator edges

*M. V. Entin^{+*1)}, L. I. Magarill^{+*}*

⁺*Rzhanov Institute of Semiconductor Physics SB of the RAS, 630090 Novosibirsk, Russia*

^{*}*Novosibirsk State University, 630090 Novosibirsk, Russia*

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The electron absorption on the edge states and the edge photocurrent of a 2D topological insulator (TI) are studied. We consider the optical transitions within linear edge branches of the energy spectrum. The interaction with impurities is taken into account. The circular polarization is found to produce the edge photocurrent, the direction of which is determined by light polarization and edge orientation.

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Introduction. The 2D topological insulator (TI) is one of the most actively developing areas of the solid state physics. One of the most known 2D TI is the 2D CdTe/HgTe/CdTe quantum well with central layer width $d > 6.3$ nm [1–4]. It attracts attention due to the gapless linear energy spectrum and the absence of backscattering connected with the topological protection. The absence of backscattering leads to non-locality of electron transport [3–5].

Here we are interested in the microwave absorption and photocurrent caused by this absorption in the intrinsic 2D TI. Although the energy-momentum conservation permits the elastic optical transitions at small frequency between edge states of TI, the topological protection forbids these transitions. One way to obtain the microwave optical transitions in TI (and the photocurrent) due to magneto-dipole transitions between spin subbands was offered by [6] (see Fig. 1a). Unfortunately, magneto-dipole transition is weaker than the electric dipole one by a small parameter of the ratio of electron and light velocities v/c .

Meanwhile, the experiment [7] on the 2D TI demonstrates that the edge states possess strong enough electromagnetic field absorption, accompanied by the photoconductivity. It is hardly to expect so strong effects due to the magnetic dipole transitions. More probably to expect that the electric dipole transitions become permitted for some solid-state reasons, say, the impurity scattering.

The photogalvanic effect (PGE) induced by microwave electric dipole transitions in graphene or gapless two-dimensional HgTe strips was theoretically studied

in [8]. It was found that the PGE results from the asymmetry of the generation together with the scattering of electrons on the strip edges. This process produces the contra-propagating photocurrents along the strip edges. In contrast to [6], the edge currents are concentrated in a distance of mean free path at edges instead the edge state width. This consideration is applicable in 2D TI when the energy of quantum exceeds the energy gap. However, in experiment the photocurrent is observed for small frequencies, where transitions between edge states exist only.

But according to [9], the absorption and photogalvanic effect are forbidden for a very wide class of the 1D Hamiltonians of shape: $p^2/2m + U(x) + (\sigma\mathbf{a}(x)p + p\sigma\mathbf{a}(x))$. The potential $U(x)$ and the vector field $\mathbf{a}(x)$ can be determined by the impurities. This class includes the one-dimensional edge Hamiltonian if to set $m \rightarrow \infty$.

Unfortunately, it is difficult to obtain the non-zero edge absorption and photocurrent in electro-dipole approximation. We have looked through different mechanisms involving scattering to choose which can give rise to these processes. The set of considered mechanisms includes different kinds of scattering: electron-electron, non-magnetic and magnetic impurities. We also considered bulk-induced and surface-induced asymmetry (BIA and SIA, accordingly) corrections to the single-electron Hamiltonian, assuming intra- and interband transitions. Unfortunately, all but one of these mechanisms do not give finite absorption and photocurrent on the edge states. The only suitable variant taking into consideration both 1D edge states and 2D states of the TI is the topic of the present paper.

The proposed mechanism of PGE is shown in Fig. 1b. Electrons are excited by light from the edge state σ, p to

¹⁾e-mail: entin@isp.nsc.ru

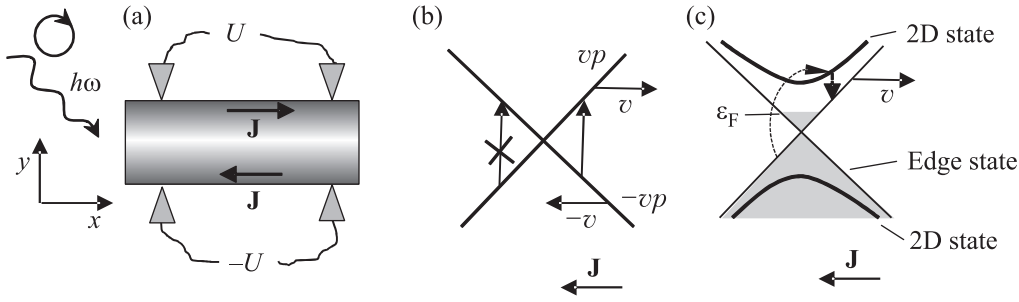


Fig. 1. (a) – PGE according to [6]: selective magnetodipole transitions excite electrons with momentum $p > 0$ from $\sigma = -1$ to $\sigma = 1$. (b) – The effect considered here: indirect impurity-stimulated transitions preferably involving electrons with positive velocity v . The indirect excitation includes virtual impurity-induced transition from the 1D to 2D state with a vertical electric-dipole-induced return to the edge state. (c) – A sketch of the effect. The topological insulator illuminated by circular-polarized microwave. The photocurrent is concentrated near the edges

σ, p' due to their interaction with impurities via intermediate 2D states. This leads to the non-zero probability of phototransitions caused by both in-plane components of electric field. Besides, for circular-polarized light the transition probability has momentum asymmetry, which is determined by the direction of this polarization. This asymmetry yields the edge photocurrent.

First, we will consider the 2D and edge states of TI. The elastic electric dipole transition amplitudes are given by the intersubband matrix elements of electron velocity. We will show that these elements vanish. Then the problem will be considered with the participation of non-magnetic impurities. This process is studied in the first order of impurity potential. In accordance with the abovesaid, the impurities will not lead to transitions if the 1D states are involved in the process only. The appearance of impurity-induced optical transitions requires the mixing of 1D and 2D states. The microwave absorption and PGE will be found using these mixed transition amplitudes. At last, we will discuss the results.

Electron states. For the calculation of absorption and PGE current one should find wave functions and energies of the 2D and edge states of the TI. We start from the Hamiltonian of 2D TI [10–12]:

$$\mathcal{H}(\mathbf{k}) = \begin{pmatrix} H(\mathbf{k}) & 0 \\ 0 & H^*(-\mathbf{k}) \end{pmatrix}, \quad (1)$$

$$H(\mathbf{k}) = \epsilon_k + \mathbf{d}\sigma,$$

where σ_a are the Pauli matrices, $\epsilon_k = -Dk^2$, $d_1 = Ak_x$, $d_2 = Ak_y$, $d_3 = M - Bk^2$. Parameters A, B, D, M are determined by the quantum well thickness and the material parameters.

First, let us consider eigenfunctions and eigenvalues of the Hamiltonian Eq. (1) in an unbounded 2D system.

Keeping in mind the fact that the edge states participating in absorption have small momenta one can suppose that the same is valid for intermediate 2D states. For simplicity, below we will neglect the boundary conditions for 2D states. In this approximation one can write the wave functions as

$$\begin{aligned} \Psi_{\mathbf{k};c,+1} &= R(\mathbf{r})(Ake^{-i\gamma}, 2|M|, 0, 0), \\ \Psi_{\mathbf{k};c,-1} &= R(\mathbf{r})(0, 0, -Ake^{i\gamma}, 2|M|), \\ \Psi_{\mathbf{k};v,+1} &= R(\mathbf{r})(2|M|, -Ake^{i\gamma}, 0, 0), \\ \Psi_{\mathbf{k};v,-1} &= R(\mathbf{r})(0, 0, 2|M|, Ake^{-i\gamma}), \end{aligned} \quad (2)$$

$$R(\mathbf{r}) = \frac{e^{i\mathbf{k}\mathbf{r}}}{\sqrt{L_x L_y (4M^2 + A^2 k^2)}}. \quad (3)$$

Here $\mathbf{r} = (x, y)$, \mathbf{k} is the 2D momentum, $k_x + ik_y = ke^{i\gamma}$. The degenerate in the spin σ conduction (c) and valence (v) band have energies

$$\begin{aligned} \epsilon_{\mathbf{k},c} &= |M| + k^2/2m_c, \quad \epsilon_{\mathbf{k},v} = -|M| - k^2/2m_v, \\ m_{c,v} &= \frac{|M|}{A^2 + 2|M|(B \mp D)}. \end{aligned} \quad (4)$$

Further, consider the edge states of the semi-bounded ($0 \leq y < +\infty$, $-\infty < x < +\infty$) 2D TI. They obey the zero boundary condition at $y = 0$. In the case of small longitudinal momenta p along the axis x one can write for the wave functions with energies $\epsilon_{p,\sigma} = \epsilon_0 + \sigma vp$ ($\sigma = \pm 1$) [12]:

$$\begin{aligned} \psi_{p,\sigma} &= e^{ipx} C g(y) \chi_\sigma / \sqrt{L_x}, \quad g(y) = e^{-\lambda_1 y} - e^{-\lambda_2 y}, \\ \chi_{+1} &= (1, \eta, 0, 0) / \sqrt{1 + \eta^2}, \\ \chi_{-1} &= (0, 0, 1, \eta) / \sqrt{1 + \eta^2}, \end{aligned} \quad (5)$$

$$C^2 \int_0^\infty g^2(y) dy = 1,$$

where

$$v = A\sqrt{(B^2 - D^2)/B^2}, \quad \eta^2 = \frac{B + D}{B - D}, \quad (6)$$

$$\epsilon_0 = -MD/B, \quad C^2 = \frac{2\lambda_1\lambda_2(\lambda_1 + \lambda_2)}{(\lambda_1 - \lambda_2)^2},$$

$$\lambda_{1,2} = \frac{A}{2\sqrt{B^2 - D^2}} \pm \sqrt{\frac{A^2}{4(B^2 - D^2)} - \frac{M}{B}}. \quad (7)$$

For a typical HgTe well with width 7 nm $\lambda_1/\lambda_2 = 41.7$, so this ratio can be treated as a large parameter.

Probability of indirect optical transitions. The probability of direct transitions between edge states is given by matrix elements of the electron velocity operator $\mathbf{V} = \nabla_{\mathbf{k}}\mathcal{H}$. With the use of wave functions Eq. (5) we see that $(V_x)_{p,\sigma;p,-\sigma} = (V_y)_{p,\sigma;p,-\sigma} = 0$. Thus, such optical transitions are forbidden. Due to the forbiddenness of the direct optical transitions, we should consider the indirect transitions with the participation of non-magnetic impurities. The probability of these processes, induced by the classical alternating electric field $\text{Re}(\mathbf{E}e^{-i\omega t})$, where $\mathbf{E} = (E_x, E_y, 0)$ is the complex amplitude of electric field and ω is its frequency, can be written as

$$W_{\gamma\beta} = \frac{\pi}{2}\delta(\epsilon_\gamma - \epsilon_\beta - \omega) \left| \sum_{\alpha} \left(\frac{U_{\gamma\alpha}F_{\alpha\beta}}{\epsilon_\alpha - \epsilon_\gamma} + \frac{F_{\gamma\alpha}U_{\alpha\beta}}{\epsilon_\alpha - \epsilon_\beta} \right) \right|^2 +$$

$$+ \frac{\pi}{2}\delta(\epsilon_\gamma - \epsilon_\beta + \omega) \left| \sum_{\alpha} \left(\frac{U_{\gamma\alpha}F_{\alpha\beta}^+}{\epsilon_\alpha - \epsilon_\gamma} + \frac{F_{\gamma\alpha}^+U_{\alpha\beta}}{\epsilon_\alpha - \epsilon_\beta} \right) \right|^2. \quad (8)$$

Here indices β (γ) label initial (final) edge states with energies $\epsilon_{\beta(\gamma)}$, index α labels the intermediate state with energy ϵ_α , $U = \sum_i u(\mathbf{r} - \mathbf{r}_i)$ is the interaction Hamiltonian with impurities, $u(\mathbf{r})$ is the potential of individual impurity center with coordinate \mathbf{r}_i , $F = -ie\mathbf{E}\mathbf{V}/\omega$. The quantity $\mathcal{F} = (Fe^{-i\omega t} + F^\dagger e^{i\omega t})/2$ gives the dipole interaction with the electric field.

Generally speaking, the transition probability is determined by all intermediate states α . We expected that the edge states should be mainly involved in this process due to small denominators in Eq. (8). However, if *only* edge states are involved in the process, the impurity-induced photo-transitions inside the edge state vanish due to vanishing of parenthesis inside the sums in Eq. (8). This agrees with the result of [9], mentioned in the Introduction. Hence, we should consider the transitions through intermediate 2D states.

The absorption is determined by the components of conductivity tensor $\text{Re}(\sigma_{jj}(\omega))$, $j = (x, y)$. Using Eq. (8) we have

$$\text{Re}(\sigma_{jj}(\omega)) = \frac{\pi e^2}{\omega L_x} \sum_{p,p',\sigma} \delta(v\sigma(p' - p) + \omega) \times$$

$$\times (f^{(0)}(vp'\sigma) - f^{(0)}(vp\sigma)) \times$$

$$\times \left| \sum_{k_y, l} \left(\frac{V_{p,\sigma;\mathbf{k},l,\sigma}^j U_{\mathbf{k},l,\sigma;p',\sigma}}{\epsilon_{\mathbf{k},l} - \epsilon_{p',\sigma}} + \frac{U_{p,\sigma;\mathbf{k},l,\sigma} V_{\mathbf{k},l,\sigma;p',\sigma}^j}{\epsilon_{\mathbf{k},l} - \epsilon_{p,\sigma}} \right) \right|^2. \quad (9)$$

Here $\epsilon_{\mathbf{k},l}$ is 2D states dispersion ($\alpha = (\mathbf{k}, l, \sigma)$, l means “c” or “v”), $f^{(0)}(\epsilon)$ is the equilibrium distribution function with the Fermi energy ϵ_F .

To calculate $\text{Re}(\sigma_{ii}(\omega))$ (and in the following the PGE current), we should find the matrix elements of operators V and U (we will consider the case of short-range impurities $U = u_0 \sum_i \delta(\mathbf{r} - \mathbf{r}_i)$). Using wave functions Eqs. (2) and (5) we get

$$V_{p,\sigma;\mathbf{k},l,\sigma}^x = \pm i\sigma V_{p,\sigma;\mathbf{k},l,\sigma}^y = \sigma AC\tilde{g}(k_y) \sqrt{\frac{B \mp D}{2BL_y}} \delta_{p,k_x}, \quad (10)$$

$$U_{p,\sigma;\mathbf{k}',l,\sigma} = C \sum_i g(y_i) \sqrt{\frac{B \pm D}{2BL_y}} \frac{u_0}{L_x} e^{-i(p-k'_x)x_i + ik'_y y_i}. \quad (11)$$

Here $\tilde{g}(k_y) = \int_0^\infty dy g(y) \exp(ik_y y)$, the upper (lower) sign in Eqs.(10)–(11) corresponds to $l = c$ (v), respectively. After substitution of Eqs. (10)–(11) into Eq. (12) we arrive at

$$\text{Re}(\sigma_{xx}(\omega)) = 2n_s e^2 \lambda_2 \left(\frac{u_0 B D}{M(B^2 - D^2)} \right)^2, \quad (12)$$

$$\text{Re}(\sigma_{yy}(\omega)) = \frac{4n_s e^2 \lambda_2}{\pi} \frac{(u_0 D)^2 B^6 \omega^2}{M^4 (B^2 - D^2)^4}, \quad (13)$$

where n_s is the areal density of impurities. According to Eq. (12), longitudinal conductivity σ_{xx} does not depend on the frequency, while the transversal σ_{yy} is proportional to ω^2 . The first fact is in imaginary contradiction with the infiniteness of the static edge conductivity. In fact, the calculations here were made in the limit when the frequency is *larger* than the scattering rate $1/\tau$, while the static limit needs the opposite relation. The dependence of σ_{yy} on ω corresponds to the expected absence of the static absorption across the edge.

Let us estimate $\text{Re}(\sigma_{xx})$ with the use of parameters for the 7 nm well from [13], $A = 364.5 \text{ meV} \cdot \text{nm}$, $B = -686 \text{ meV} \cdot \text{nm}^2$, $D = -512 \text{ meV} \cdot \text{nm}^2$, $M = -10 \text{ meV}$. The product $n_s u_0^2$ can be estimated from 2D electron mobility, for which the value $10^5 \text{ cm}^2/\text{V} \cdot \text{s}$ is chosen. This gives $\text{Re}(\sigma_{xx}) \approx 5.4 \cdot 10^{-11} \text{ S} \cdot \text{cm}$.

Photogalvanic effect. A sketch of a possible measurement of the edge photocurrent is shown in Fig. 1c. The probes are connected to the wide strip of TI. The

voltage between probes can be attributed to the edge photocurrent.

Unlike the absorption, the photogalvanic effect is determined by a product of E_x and E_y . The phenomenological expression for PGE current $\mathbf{j} = (j_x, 0, 0)$ is

$$\mathbf{j} = \alpha_s(\mathbf{E}(\mathbf{n}_y \mathbf{E}^*) - \mathbf{n}_y |(\mathbf{n}_y \mathbf{E})|^2 + c.c.) + i\alpha_a[\mathbf{n}_y[\mathbf{E}\mathbf{E}^*]],$$

where α_s, α_a are the real values, \mathbf{n}_y is the unit vector along the y -axis.

To calculate the PGE current, we will solve the kinetic equation for distribution function f_γ ($\gamma = (p, \sigma)$). This equation can be written as

$$\sigma v \partial_x f_\gamma^{(1)} = -\frac{f_\gamma^{(1)}}{\tau_\gamma} + G_\gamma, \quad (14)$$

where $f_\gamma^{(1)} = f_\gamma - f^{(0)}(\epsilon_\gamma)$, τ_γ is the relaxation time (see below), the generation $G_\gamma = \sum_\beta W_{\gamma,\beta}(f^{(0)}(\epsilon_\beta) - f^{(0)}(\epsilon_\gamma))$.

At relatively small edge length L_x (ballistic regime) we will consider the spatially inhomogeneous kinetic equation, keeping in mind the fact that the elastic backscattering is absent and the inelastic processes are weak at low temperatures and the excitation energy. If so, the relaxation can occur at the contacts. *Vice versa*, if $L_x \rightarrow \infty$ (kinetic regime) the only way to establish the stationary distribution function is backscattering of edge electrons within the edge states. This process is permitted for phonon scattering owing to the finite excitation energy. We restrict ourselves by the τ approximation.

In the kinetic regime from Eq. (14) we have $f_\gamma^{(1)} = \tau_\gamma G_\gamma$. We assume that the relaxation time depends on γ only via energy, $\tau_\gamma = \tau(\epsilon_\gamma)$. It is convenient to use instead of quantum numbers p, σ another set $\epsilon \equiv \sigma p v$ and σ . Here and below we count energy from ϵ_0 . For the PGE current we find

$$j_x^{kin} = \frac{e}{2\pi} \int_{-\infty}^{\infty} d\epsilon \tau(\epsilon) (G_{\epsilon,1} - G_{\epsilon,-1}). \quad (15)$$

It is seen that, as usual, appearance of PGE requires the asymmetry of the generation: $G_{\epsilon,\sigma}$ should be non-equal to $G_{\epsilon,-\sigma}$.

Using Eq. (8) we get to

$$j_x^{kin} = Z |\mathbf{E}|^2 \zeta \int_{-\infty}^{\infty} d\epsilon \tau(\epsilon) \hat{D}_\omega^{(2)} f^{(0)}(\epsilon), \quad (16)$$

$$Z = \frac{4n_s \lambda_2 e^3}{\pi v |M| \omega} \left(\frac{A u_0 B D}{M(B^2 - D^2)} \right)^2. \quad (17)$$

Here, $\zeta = \text{Im}(E_x E_y^*) / |\mathbf{E}|^2$ is the circular polarization degree, $\hat{D}_\omega^{(2)}$ denotes the operator of finite difference: $\hat{D}_\omega^{(2)} f(\epsilon) = f(\epsilon + \omega) + f(\epsilon - \omega) - 2f(\epsilon)$.

Now let us consider the ballistic case of electron motion along the edge. In this case the relaxation occurs only in the contacts to the edge while the relaxation time tends to infinity. The kinetic equation should be supplemented by the conditions on the contacts. The simple situation when the electrons outgoing the contacts are in equilibrium is insufficient. It is supposed that the edge electrons are partly reflected back by contacts. In other words, at the left (right) contact $x = -L_x/2$ ($x = +L_x/2$) the non-equilibrium distribution function of the electrons moving to the right (left) is proportional (but not equal) to the distribution function of the electrons moving to the left (right): $f^{(1)}(\epsilon, \pm 1; x = \mp L_x/2) = P(\epsilon) f^{(1)}(\epsilon, \mp 1; x = \mp L_x/2)$. We have assumed that both contacts have the same reflectivities P .

Solving the kinetic equation Eq. (14) with $\tau = \infty$ we come to

$$j_x^{bal} = \frac{L_x}{v} Z |\mathbf{E}|^2 \zeta \int_{-\infty}^{\infty} d\epsilon \frac{1}{1 + P(\epsilon)} \hat{D}_\omega^{(2)} f^{(0)}(\epsilon). \quad (18)$$

In Eq. (18) the flight time between contacts L_x/v replaces the relaxation time τ in Eq. (16).

In particular, when ω is less than characteristic sizes of functions $\tau(\epsilon)$ and $P(\epsilon)$, ϵ_τ and ϵ_P , $\hat{D}_\omega^{(2)} f(\epsilon) \rightarrow -\omega^2 \delta'(\epsilon - \epsilon_F)$, and the expressions Eqs. (16) and (18) convert to

$$j_x^{kin} = Z |\mathbf{E}|^2 \zeta \omega^2 \tau'(\epsilon_F), \quad (19)$$

$$j_x^{bal} = -Z |\mathbf{E}|^2 \zeta \frac{\omega^2 L_x}{v} \frac{P'(\epsilon_F)}{(1 + P(\epsilon_F))^2}. \quad (20)$$

If $\omega \gg \epsilon_\tau$ or $\omega \gg \epsilon_P$ the PGE current one should use more general Eqs. (16) and (18).

The expression $\hat{D}_\omega^{(2)} f^{(0)}(\epsilon)$ is an odd function of $\epsilon - \epsilon_F$. Eqs. (16) and (18) show that if $\tau(\epsilon)$ or $P(\epsilon) = \text{const}$ (in particular, $P = 0$), the PGE current disappears. Just for this reason we use a more complicated model with finite $P(\epsilon)$ depending on the electron energy²⁾.

The system spectrum near the conic point has the electron-hole symmetry. If this is the case for other functions entering Eqs. (16), (18)–(20), the current is odd as a function of ϵ_F and, thus, it vanishes at $\epsilon_F = 0$. The interaction with impurities, in principle, depends on the sign of potential and, hence, on the carrier sign. However, in the considered Born approximation this

²⁾In fact, the backscattering in the contact to the 1D system is not an ordinary thing, because the reversion of the electron motion in a 2D TI edge needs a spin-flip. Hence, it has some smallness. A question arises, what reason can reflect electrons back in the contact? The possible origins of this scattering are the exchange interaction of the edge electron near the contact with electrons of a 2D sea or the spin-orbit interaction in the vicinity of the contact. The first variant looks more reasonable.

symmetry exists and the transition probability has the same symmetry. If this symmetry applies to the relaxation time, the current Eq. (16) vanishes in the neutrality point and changes sign as a function of ϵ_F . The same is valid for Eq. (18) when the reflectivity of the contacts is equal for electrons and holes. Note, that the reflectivity not obligatorily has the same value for electrons and holes, because the e-h symmetry can be violated by contacting media.

Let us estimate the photocurrent value. The parameter $n_s u_0^2$ can be found using the 2D mobility. We take the abovementioned parameters of the 2D layer and, in addition, the 2D mobility $10^5 \text{ cm}^2/\text{V}\cdot\text{s}$, $\hbar\omega = 1 \text{ meV}$, the mean backscattering length $\tau v = 10^{-3} \text{ cm}$. Besides, we estimate $\tau'(\epsilon_F)$ as $\tau'(\epsilon_F) \sim \tau/\text{meV}$. In this case, for $E = 1 \text{ V/cm}$, we have from Eq. (16) $j \simeq 34 \text{ pA}$.

Discussion. We have calculated the electro-dipole absorption by the edge states of the 2D topological insulator which takes place for the energy of quantum less than the 2D TI gap. The electro-dipole absorption vanishes in the collisionless limit. The impurity scattering was taken into account. The transitions with a virtual participation of 2D states have their crucial significance for the effect. It was found that the intrabranched transitions occur only. The consideration is carried out in the limit, when frequency ω exceeds the collision rate (but is yet less than the 2D gap). We have stated that the absorption exists for the electric field directed along and across the edges. This absorption is not described by the static conductivity: σ_{yy} vanishes and $\sigma_{xx} \rightarrow \infty$. Unlike [6], the effect does not need the magnetodipole transitions. Our results on the optical absorption correspond to the experimental findings [7]; the detailed comparison will follow. Note, that the experimental discrimination of magnetodipole and electro-dipole mechanisms can be simply done by the control of the standing wave phase, namely, by positioning of the sample into the electric field node or antinode.

The edge photogalvanic effect has been studied in kinetic and ballistic approaches for the intrabranched transitions caused by electro-dipole transitions with the participation of impurity scattering. We have found that only the circular PGE exists while the linear PGE vanishes. In the most cases the effect changes the sign when the Fermi level transits the conic point. The photocurrent appears due to the photoinduced redistribution of carriers together with the dependence of the backscattering time on the energy. This condition is important, but it does not obligatory hold. In particular, the backscattering by the magnetic impurities with a contact spin-spin interaction should give the backscattering time independent on the energy. The complication of the picture by

taking into account the form-factor of magnetic impurities yields the needed dependence.

We have used the assumption that all phototransitions occur within the edge states only. This assumption is valid if the distances between the Fermi level and the valence or conduction bands of the 2D TI exceed the energy of quantum. For a larger frequency, the transitions between the edge states and the 2D states arise, and that can also give the edge photocurrent. This variant will be considered elsewhere.

It is useful to compare the photocurrent on the 1D edge states with the 2D edge photocurrent [8] for gapless 2D HgTe. While both photocurrents are concentrated near the edges, the first has the dimension of the edge state width and the second has a much larger dimension of the mean free path. The 2D electrons can freely be scattered by spinless impurities, while the edge state electrons can not. Note, that the absence of the gap in [8] is not an important factor, because these results remain valid in a system with a finite gap if the electron energy exceeds the gap value (and can be freely recalculated in the case of a finite gap).

The phototransitions considered here are weak due to the participation of impurity scattering in the absorption. In the photocurrent their weakness is partly compensated by the weak backscattering of edge-state electrons.

Another comparison should be done with the PGE induced by the magnetic dipole transitions [6]. The smallness of magnetic dipole transitions v/c should be compared with the ratio of the scattering rate to electron energy $2/\omega\tau$, which determines the impurity-induced PGE in the edge states.

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