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Analysis of the $e^+e^- \to \pi^0\gamma$ Process Using Anomaly Sum Rules Approach

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1. Extended abstract. The transition form factor (TFF) of the pion is attracting the great interest last years.

In the range of the space-like photon virtualities $Q^2 = -q^2$ from 1 to 40 GeV^2 the transition form factor of the π^0 is quite good investigated by CELLO [1], CLEO [2], BaBar [3] and Belle [4] experiments. At the same time, in the region of low Q^2 and in the time-like region the number of direct measurements of the pion TFF is quite small. The more precise data are expected in the future from BES-III [5] and KLOE-2 [6] collaborations.

Our paper is dedicated to the pion transition form factor in the time-like region. In this region pion TFF can be studied in the process $e^+e^- \to \gamma^* \to \pi^0\gamma$. The SND [7] and CMD2 [8] experiments had collected the data that cover the range 0.6–1.38 GeV of $\sqrt{s}$. In the future, CLAS [9] will provide more precise data for the time-like pion TFF.

In our paper the process $e^+e^- \to \gamma^* \to \pi^0\gamma$ will be considered using the pion TFF, obtained by the anomaly sum rules. This approach is based on the dispersive representation of the axial anomaly ([10, 11], see also [12]), and it is model-independent and not relying on the QCD factorization. Also it is valid for the whole region of the $Q^2 > 0$. In [13] this approach was analytically continued to the time-like region $Q^2 < 0$.

The ASR approach leads to the pion TFF:

$$F(q^2) = \frac{1}{2\sqrt{2}\pi^2 f_\pi}\frac{s_3}{s_3 - q^2}. \qquad (1)$$

As were discussed in [11, 13], this result is valid in time-like and space-like regions (expect the pole $q^2 = s_3$, due to $ImF(q^2) \sim \delta(s_3 - q^2)$). The parameter s_3, as was shown in [14], is ≈ 0.61 GeV^2 and asymptotically ($q^2 \to \to \infty$) it goes to 0.67 GeV^2.

Using (1), one obtains a total cross section for the $e^+e^- \to \gamma^* \to \pi^0\gamma$ process:

$$\sigma = \frac{2}{3}\pi^2\alpha^3_{QED}|F^2(q^2)|. \qquad (2)$$

As it can be seen from the Fig. 1 (curve, solid line) corresponding to (2) with $s_3 = 0.61$ GeV^2, can be used to describe the data in the regions far from the pole, and the place of the pole coincides with the experimental peak.

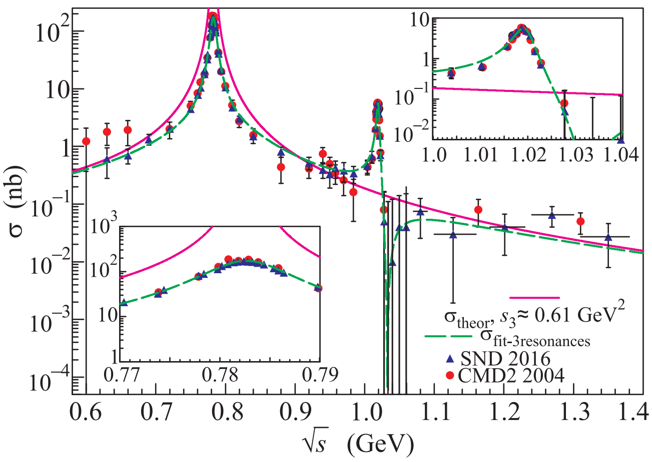

Fig. 1. (Color online) Fit of the whole data range. The inserts are zoom in $\rho - \omega$ and ϕ peaks

Let us emphasize, that the Eq. (2) with $s_3 = 0.61$ indicates the $\rho - \omega$ meson resonance position (in the zero width approximation), but does not indicate the existence of the second resonance, corresponding to the ϕ meson. The reason of this is that the Eq. (1) was obtained for the isovector channel of the axial current and it does not take into account the possible effects of $\pi^0 - \eta - \eta'$ mixing. So the hadrons including s-quarks can't be accounted in such approximation.

In that paper we try to modify the pion TFF (1) in order to make it available to use in the region of resonances. And also to include in it the effect of $\pi^0 - \eta - \eta'$ mixing.

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To achieve approximation with finite width, one should add to the denominator of the (1) term $im_v\Gamma_v$, where m_v and Γ_v are mass and width of the resonance, respectivly. So the modified pion TFF equation will be similar to the relativistic Breit–Wigner amplitude.

The including of mixing leads to appearance of the three terms in the modified pion TFF equation, each one of them will have the coefficient correponds to the mixing angles. Thus the modified pion TFF expression has the form:

$$F(q^2) = \frac{1}{2\sqrt{2}\pi^2 f_{\pi^0}}\Big(\frac{\alpha s_3}{s_3 - q^2 + im_\rho\Gamma_\rho} + \\ + \frac{\beta s_3}{s_3 - q^2 + im_\omega\Gamma_\omega} + \frac{\gamma s_{3\phi}}{s_{3\phi} - q^2 + im_\phi\Gamma_\phi}\Big). \qquad (3)$$

Note, that in the frame of NJL model [15] the mixing effects are attributed to vector chanel, which can be another manifestation of duality between vector and axial channels [13].

We take the assumption, that $s_{3v} \approx m_v^2$. Due to the fact that the accuracy of the ASR approach isn't allowing to distguinish between ρ and ω masses, we suppose that $m_\rho = m_\omega \approx 0.782\,\mathrm{GeV}$ and $s_{3\rho} = s_{3\omega} = m_\rho^2 = m_\omega^2 = s_3 \approx 0.61\,\mathrm{GeV}^2$, but with PDG [16] values for widths: $\Gamma_\rho = 0.149\,\mathrm{GeV}$, $\Gamma_\omega = 0.00849\,\mathrm{GeV}$. For the ϕ meson we also suppose, that $s_{3\phi}$ will be close to the m_ϕ^2. The value of $m_\phi = 1.0194\,\mathrm{GeV}$ and the value of $\Gamma_\phi = 0.00426\,\mathrm{GeV}$ were taken from PDG [16]. Thus in the (3) α, β, γ are free parameters, related to the $\pi^0 - \eta - \eta'$ mixing angles, values of each we find from the fitting of the experimental data.

Substistuting (3) to (2), one obtains the expression for the total cross section. The result is shown on the Fig. 1 (dashed line) and the values of the χ^2 are listed in the Table 1.

Table 1. Values of χ^2 for fit with 3 resonances

	CMD2	**SND2016**	**CMD2+SND2016**
$\chi^2/d.o.f.$	2.53	1.52	1.87

For the fit parameters the following values are obtained: $\alpha = 0.556$, $\beta = 0.49$, $\gamma = -0.036$. Note, that the value of γ has the same order of the magnitude as the $\theta_{\pi-\eta}$ mixing angle ([17, 18], see also [19]), so it confirms our assumptions. Also note that, by use of (3) we can obtain in the limit $q^2 \to 0$ the value

$$\Gamma(\pi^0 \to 2\gamma) \approx 7.9\,\mathrm{eV}$$

in good agreement with experiment [20].

Thus pion TFF (1), calculated by the ASR approach, gives reasonable descprition of the data in region far from the pole. In order to decribe data in the resonance region, one should include to the ASR approach the effect of the $\pi - \eta - \eta'$ mixing and also take finite widths.

The achieved values of the fits coefficients lead to strong restrictions to the values of the $\pi - \eta - \eta'$ mixing angles and can be used in matching with the theoretical values.

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