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Non-Abelian vortex in four dimensions as a critical Superstring

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In this paper we review the recent discovery of non-Abelian solitonic vortices [1–4] supported in certain four-dimensional $\mathcal{N}=2$ supersymmetric Yang–Milles theory with matter which behave as critical ten-dimensional superstrings.

In QCD, the Regge trajectories show almost perfect linear J behavior (J stands for spin). However, in all controllable examples at weak coupling a solitonic confining string exhibits linear behavior for the Regge trajectories only at asymptotically large spins [5, 6]. The reason for this is that at $J \sim 1$ the physical "string" becomes short and thick and cannot yield linear Regge behavior. Linear Regge trajectories at $J \sim 1$ have a chance to emerge only if the string at hand satisfies the thin-string condition [7],

$$T \ll m^2, \tag{1}$$

where T is the string tension and m is a typical mass scale of the bulk fields forming the string. The former parameter determines the string length, while the latter determines the string thickness. At weak coupling $g^2 \ll 1$, where g^2 is the bulk coupling constant, we have $m \sim g\sqrt{T}$. The thin-string condition (1) is therefore badly broken.

For the majority of solitonic strings in four dimensions, such as the Abrikosov–Nielsen–Olesen (ANO) vortices [8], the low-energy two-dimensional effective theory on the string world sheet – the Nambu–Goto theory – is not ultraviolet (UV) complete. To make the world sheet theory fully defined one has to take into account higher derivative corrections [9]. Higher derivative terms run in inverse powers of m and blow up in UV making the string worldsheet "crumpled" [10]. The blow up of higher derivative terms in the worldsheet theory corresponds to the occurrence of a thick and short "string".

The question weather one can find an example of a solitonic string which might produce linear Regge trajectories at $J \sim 1$ was addressed and answered in [7]. Such a string should satisfy the thin-string condition (1). This condition means that higher derivative correction are parametrically small and can be ignored. If so, the low-energy world-sheet theory must be UV complete. This implies the following necessary conditions:

(i) The low-energy world-sheet theory must be conformally invariant;

(ii) The theory must have the critical value of the Virasoro central charge.

As well-known, these conditions are satisfied by fundamental (super)string in 10D.

In [7] it was shown that (i) and (ii) above are met by the non-Abelian vortex string [1–4] supported in four-dimensional $\mathcal{N}=2$ supersymmetric QCD with the U(N) gauge group, $N_f = 2N$ matter hypermultiplets and the Fayet-Iliopoulos (FI) parameter ξ [11]. The non-Abelian part of the gauge group has the vanishing β function. (We will need to consider $N = 2$.)

The non-Abelian vortex string is 1/2 BPS saturated and, therefore, has $\mathcal{N} = (2, 2)$ supersymmetry on its worldsheet. In addition to translational moduli characteristic of the ANO strings, the non-Abelian string carries orientational moduli, as well as size moduli provided that $N_f > N$ [1–4], see [12–15] for reviews. Their dynamics is described by two-dimensional sigma model with the target space

$$\mathcal{O}(-1)^{\oplus(N_f-N)}_{\mathbb{CP}^1}, \tag{2}$$

to which we will refer as WCP$(N, N_f - N)$ model. For $N_f = 2N$ the model becomes conformal and condition (i) above is satisfied. Moreover for $N = 2$ the dimension of orientational/size moduli space is six and they can be combined with four translational moduli to form a ten-dimensional space required for critical superstrings. Thus the second condition is also satisfied [7]. For $N = 2$ the sigma model target space is a six-dimensional non-compact Calabi–Yau manifold Y_6, namely, the resolved conifold.

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Given that the necessary conditions are met, a hypothesis was put forward [7] that this non-Abelian vortex string does satisfy thin-string condition (1) at strong coupling regime in the vicinity of a critical value of $g_c^2 \sim 1$. This implies that $m(g^2) \to \infty$ at $g^2 \to g_c^2$.

The vortices in the $\mathrm{U}(N)$ theories under consideration are topologically stable, therefore the finite length strings are closed. Thus, we focus on the closed strings emerging from four dimensions. The goal is to identify closed string states with hadrons of the four-dimensional bulk theory. The first step of this program, namely the identification of massless string states was performed in [16, 17].

In particular, we identified a single matter hypermultiplet associated with the deformation of the complex structure of the conifold as the only 4D massless mode of the string. Other states arising from the ten-dimensional graviton are not dynamical in four dimensions. In particular, 4D graviton and unwanted vector multiplets are absent. This is due to non-compactness of the Calabi–Yau manifold we deal with and non-normalizability of the corresponding modes.

It was also discussed how the states seen in the bulk theory at weak coupling are related to what we obtain from the string theory at strong coupling. In particular the hypermultiplet associated with the deformation of the complex structure of the conifold is interpreted as a monopole-monopole baryon [16, 17].

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