Pis'ma v ZhETF, vol. 105, iss. 2, pp. 110 – 111

Role of qubit-cavity entanglement for switching dynamics of quantum interfaces in superconductor metamaterials

S. V. Remizov[+*], *D. S. Shapiro*[+*×1)], *A. N. Rubtsov*[+#]

[+] *N.L. Dukhov Research Institute of Automatics (VNIIA), 127055 Moscow, Russia*

[*] *V.A. Kotel'nikov Institute of Radio Engineering and Electronics, Russian Academy of Sciences, 125009 Moscow, Russia*

[×] *National University of Science and Technology MISIS, 119049 Moscow, Russia*

[#] *Russian Quantum Center, 143025 Skolkovo, Russia*

Submitted 25 October 2016
Resubmitted 24 November 2016

DOI: 10.7868/S0370274X17020126

Quantum metamaterials are hybrid systems consisting of arrays of qubits coupled to the photon modes of a cavity [1–4]. In solid state structures the qubits are realized using nitrogen-vacancy (NV) centers in diamonds [5], and spins of ^{31}P dopants in ^{28}Si crystals [6] or Cr^{3+} in Al$_2$O$_3$ samples [7], and superconducting Josephson qubits [8]. Among of others, the Josephson qubits are particularly perspective for an implementation of quantum gates [9] due to their high degree of tunability. Frequency of excitation, given by an energy difference between ground and excited states, can be controllably tuned in a wide range using the external magnetic flux threading a loop of the qubit. Modern technology allows for a production of metematerial structures obeying sophisticated geometry and low decoherence effects.

High nonlinearity of the qubit excitation spectrum, combined with low decoherence, gives rise to unusual properties of quantum metamaterials, distinguishing them from the linear-optical metastructures. These unusual features are associated with intrinsic quantum dynamics of qubits and photon degrees of freedom. They are revealed in the optical response of a metamaterial to the external strong pump field, driving the system away from its ground state. A textbook example is the rotation on a Bloch sphere of the state of a single qubit subjected to an external field pulse. The well-understood solution for dynamics of a single qubit is commonly used as a key building block in the mean-field description of complex metamaterials containing a number of qubits and cavity modes.

Assuming no correlations between the qubits and photons, one comes to the set of Maxwell–Bloch equations [10] virtually describing qubits coupled to a classical field of the cavity and (or) external pump. This article is devoted to the role of quantum entanglement between qubit and cavity modes of the superconducting metamaterial. Whereas it is generally clear that these correlation effects beyond Maxwell–Bloch scheme are revealed in strong-driving regimes, their quantitative role in an experimentally/technologically relevant situation is not yet studied. At the same time, such study is highly motivated by the quantum technology development, because a realization of qubit gates and operation of quantum simulators assume applying of driving fields of strengths comparable with qubit-cavity coupling energy g. An important distinction between the problem considered and that typical for laser physics is that the pumping in our case is applied to qubit while photon mode is not subjected to the drive. For a weak driving, the system studied behaves classically, since the qubit virtually acts as a linear (Gaussian) degree of freedom; this regime cannot reveal a difference between the quantum and linear-optical metamaterials. At strong driving, we demonstrated that the two-level nature of the qubit plays an essential role, giving rise a non-coherent photon state which is highly entangled with the qubit. Therefore the cavity field cannot be described (semi)classically although the photon occupation number is large.

This is shown from a comparison of steady state solution of the standard Maxwell–Bloch equations and numerical solution of Lindblad equation on a many-body density matrix. Speaking more concretely, we have shown that mean-field approach, where the density matrix of the system can be represented via direct product of isolated qubit and photon ones $\rho_{\mathrm{mf}} = \rho_{\mathrm{ph}} \otimes \rho_{\mathrm{q}}$, provides a good steady state solution up to certain threshold f^* but at $f > f^*$ the strong discrepancy from the many-body result is observed. The model of relaxation, based on Lindblad equation, is commonly used in simulations of experiments and in models of strongly driven

---

1) e-mail: shapiro.dima@gmail.com

superconducting qubits [11], as well as in laser physics [12]. The underlying physical picture is an exponential decay of the coherence [13] with the rates being the same in entire phase space of the qubit and cavity. In Fig. 1 we plot the relation between the photon number n_{ph} and driving f in stationary regime. We observe a remarkable artifact which follows from the Maxwell–Bloch approach, but is not present in the many-body description: a hysteresis in photon number as a function of f. This behavior shows up in a certain range around of the threshold f^* if a coupling energy between photons and the qubit is large enough. This bistability regime is similar to the one in [14] where a driving was applied to photon mode. We insist, however, that the solution of the Lindblad equation for the many-body density matrix does not contain such a bistable regime and we therefore interpret it as an artifact of the mean-field approximation.

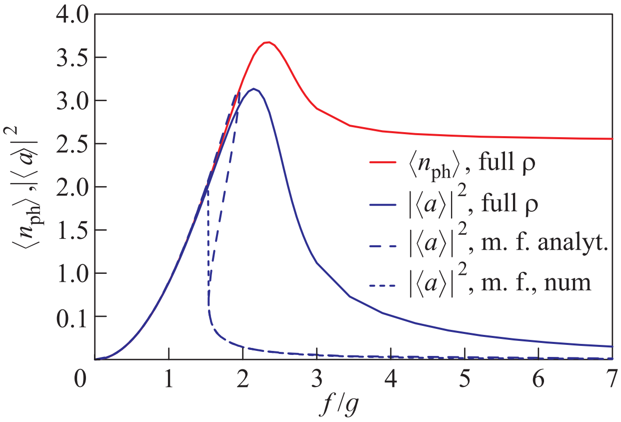

Fig. 1. (Color online) Photon number vs driving amplitude f in the steady state regime

For a non-adiabatic switching, the many-body effects are important even for weaker driving. Following our line of argumentation, one can expect that the mean-field description of this regime cannot be cured by an account of e.g. higher order correlations, just because the entanglement is essential and cannot be seen as a perturbation of a semiclassical state of the system. We indeed performed a calculation of this kind. Namely, we took into account $\langle\sigma^+ a\rangle - \langle\sigma^+\rangle\langle a\rangle$ and similar terms in the "extended" mean-field equations (this part of the study is not presented in the article) and observed no significant improvement of the Maxwell–Bloch result. The study was funded by the Russian Science Foundation (Grant # 16-12-00095).

Full text of the paper is published in JETP Letters journal. DOI: 10.1134/S0021364017020060

1. O. Astafiev, A. M. Zagoskin, A. A. Abdumalikov, Yu. Pashkin, T. Yamamoto, K. Inomata, Y. Nakamura, and J. S. Tsai, Science **327**, 840 (2010).
2. P. Macha, G. Oelsner, J.-M. Reiner, M. Marthaler, S. André, G. Schön, U. Hübner, H.-G. Meyer, E. Il'ichev, and A. V. Ustinov, Nature Commun. **5**, 5146 (2014).
3. A. L. Rakhmanov, A. M. Zagoskin, S. Savel'ev, and F. Nori, Phys. Rev. B **77**, 144507 (2008).
4. D. S. Shapiro, P. Macha, A. N. Rubtsov, and A. V. Ustinov, Photonics **2** (2), 449 (2015).
5. S. Putz, D. O. Krimer, R. Amsuss, A. Valookaran, T. Nobauer, J. Schmiedmayer, S. Rotter, and J. Majer, Nat. Phys. **10**, 720 (2014).
6. J. J. L. Morton, A. M. Tyryshkin, R. M. Brown, Sh. Shankar, B. W. Lovett Brendon, A. Ardavan, T. Schenkel, E. E. Haller, J. W. Ager, and S. A. Lyon, Nature **455**, 1085 (2008).
7. D. I. Schuster, A. P. Sears, E. Ginossar, L. DiCarlo, L. Frunzio, J. J. L. Morton, H. Wu, G. A. D. Briggs, B. B. Buckley, D. D. Awschalom, and R. J. Schoelkopf, Phys. Rev. Lett. **105**, 140501 (2010).
8. Y. Makhlin, G. Schön, and A. Shnirman, Rev. Mod. Phys. **73**, 357 (2001).
9. P. D. Nation, J. R. Johansson, M. P. Blencowe, and F. Nori, Rev. Mod. Phys. **84**, 1 (2012).
10. H. J. Carmichael, *Statistical Methods in Quantum Optics 1*, Springer-Verlag, Berlin–Heidelberg (1999).
11. J. Braumüller, J. Cramer, S. Schlör, H. Rotzinger, L. Radtke, A. Lukashenko, P. Yang, S. T. Skacel, S. Probst, M. Marthaler, L. Guo, A. V. Ustinov, and M. Weides, Phys. Rev. B **91**, 054523 (2015).
12. Y. Mu and C. M. Savage, Phys. Rev. A **46**, 5944 (1992).
13. A. Shnirman, Yu. Makhlin, and G. Schön, Physica Scripta **102**, 147 (2002).
14. C. M. Savage and H. J. Carmichael, IEEE J. Q. Electr. **24**(8), 1495 (1988).