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Full replica symmetry breaking in p-spin-glass-like systems[1)]

T. I. Schelkacheva[a], N. M. Chtchelkatchev[a,b,c,d,e 2)]

[a] Institute for High Pressure Physics, Russian Academy of Sciences, 142190, Troitsk, Moscow, Russia

[b] L.D. Landau Institute for Theoretical Physics, Russian Academy of Sciences, 142432 Chernogolovka, Russia

[c] Ural Federal University, 620002 Ekaterinburg, Russia

[d] Department of Theoretical Physics, Moscow Institute of Physics and Technology, 141700 Moscow, Russia

[e] All-Russia Research Institute of Automatics, 127055 Moscow, Russia

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The basis of understanding glasses is the Sherrington–Kirkpatrick (SK) model [1]: the Ising model with random links. A stable solution for SK model was obtained by Parisi [2, 3] with a full replica symmetry breaking (FRSB) scheme. Later it was realised that replica symmetry is not abstract and academic question but it corresponds to formation of the specific hierarchy of basins in the energy landscape of the glass forming system.

A natural generalization of the SK model with pair interaction of spins is a model with p-spin interactions [3, 4]. Unlike of the SK model, p-spin model has a stable first replica symmetry breaking (RSB) solution that arises abruptly. A very low-temperature boundary of the 1RSB stability region is given by so called Gardner transition temperature intensively discussed last time [4, 5] where a valley in configuration space transforms to a multitude of separated basins.

Now there is a reborn of interest to spin models showing glassy behaviour [5–14]. It turned out that these models can qualitatively explain physics of "real" glasses [15]. On the other hand, there is limited number of analytically solvable glassy models and each such model is interesting itself. Here we propose analytical solution of p-spin-glass-like system and discuss physical applications of this model.

For a long time there was a conjecture that there are more or less two classes of models, depending on how the replica symmetry breaking appears [16]. In one class of models FRSB occurs continuously at the transition point from the paramagnetic state to the glass state (like, e.g., in SK-model). The second class of models can be called 1RSB-models (p-spin model, Potts models). In this case there is a finite range of temperatures where stable 1RSB glass solution occurs. What is important that this 1RSB solution mostly appears abruptly.

1RSB-models and especially p-spin glasses in recent years attract much interest in connection with the fact that there is a close relationship between static replica approach and dynamic consideration. For example the Random First Order Transition theory for structural glasses is inspired by the p-spin glass model [5–15]. It should be also noted that the two classes of models mentioned above distinguish essentially by their energy landscape [17], which is an important concept in the dynamics of liquids and glasses.

In the context of SK-like and 1RSB-models it is possible to develop very advanced theoretical tools that can be reused in other contexts. These relatively simple models based on well-designed solutions allow to explore qualitatively an extensive range of issues, ranging from magnetic to structural glass transitions [5–18].

Therefore, a natural generalizations of these basic models leads to a successful description of the various types of glasses, such as orientation glasses and cluster glasses. Replacing the simple Ising spins by more complex operators $\hat{U}$ dramatically expands the range of solvable tasks. The operator $\hat{U}$ can be, for example, the axial quadrupole moment (quadrupole glass in molecular solid hydrogen at different pressures) or the role of $\hat{U}$ can be played by certain combinations of the cubic harmonics (orientational glass of the clusters, for example, in C_{60} in a wide pressure range), see [16–26].

It was shown recently that many statements related to the models based on Ising spins can be applied to p-spin-like models that use instead of spins diagonal operators $\hat{U}$ when there is *"reflection symmetry"*: $\mathrm{Tr}\hat{U}^{2k+1} = 0$, $k = 1, 2....$ For such models we can build a stable 1RSB solution for $p > 2$ (see Fig. 1) in a wide re-

[1)] See Supplemental material for this paper on JETP Letters suit: www.jetpletters.ac.ru

[2)] e-mail: nms@itp.ac.ru

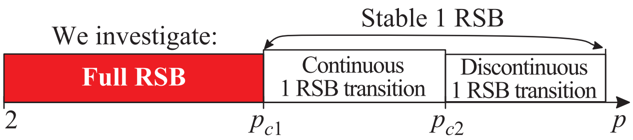

Fig. 1. (Color online) We have got FRSB glass solution of a p-spin-like model when the effective number of interacting particles $2 < p < p_{c1}$. We considered general diagonal operators $\hat{U}$ with $\mathrm{Tr}\hat{U}^{2k+1} \neq 0$, $k = 1, 2, ...$ instead of Ising spins

gion of temperatures. It was shown that the point $p = 2$ is special for such models [26–28].

For p-spin-like model we previously have got significantly different solutions when diagonal operators $\hat{U}$ have *broken reflective symmetry*, then $\mathrm{Tr}\hat{U}^{2k+1} \neq 0$ like (e.g., for quadrupole operators). And for these models 1RSB glass solution behavior have been recently investigated near the glass transition at different (continuous) p [26, 29]. It turned out that there is a finite region of instability of 1RSB solutions for $2 \leq p < p_{c_1}$ where p_{c_1} is determined by the specific form of $\hat{U}$. 1RSB solution is stable for $p > p_{c_1}$. Wherein the transition from paraphase to 1RSB glass is continuous for $p_{c_1} < p < p_{c_2}$. When $p > p_{c_2}$ [26, 29] 1RSB glass occurs abruptly just as in the conventional p-spin model. We should note that p_{c_2} is not universal, but it depends on the particular type of $\hat{U}$. We have built [30, 31] FBSB solution for these models with a pair interaction $p = 2$.

In this letter we investigate in detail the generalised p-spin glass forming models in the region $2 \leq p < p_{c_1}$ where instability of 1RSB glass solution is expected. We build a solution with full replica symmetry breaking. The very existence of the domain with full replica symmetry breaking is a surprising result especially compared with the traditional p-spin model of Ising spins. Continuously changing the effective number of interacting particles in p-spin model allows us to describe the crossover from the full replica symmetry breaking glass solution to stable first replica symmetry breaking glass solution (in our case of non-reflective symmetry diagonal operators used instead of Ising spins). For illustrating example we take operators of axial quadrupole moments in place of Ising spins. For this model we find the boundary value $p_{c_1} \cong 2.5$.

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