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Dirac and Weyl fermions: from Gor'kov equations to Standard Model (in memory of Lev Petrovich Gor'kov)

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Application of quantum field theory to condensed matter physics began in Soviet Union around 1956–1957 [1]. In this approach the Fermi sea serves as an analog of the relativistic quantum vacuum – the Dirac sea. The Gor'kov theory of superconductivity [2] has been the fundamental step in this direction, which in turn triggered the development of the relativisic theories. The composite models developed by Nambu and Jona-Lasinio [3] and by Vaks and Larkin [4], where the Higgs bosons appear as a composite states of the fermion pairs, are the direct consequences of the Gor'kov theory. In such models the original Weyl fermions of Standard Model (such as top quarks) play the role of the electrons in metals, while the composite Higgs bosons are analogs of the collective modes of the order parameter in superconductors [5, 6].

Here we consider another consequence of the Gor'kov theory of superconductivity, where the Weyl fermions emerge in superconductors as Bogoliubov quasiparticles. This in particular takes place for superconductors of the symmetry class $O(D_2)$ [7–9], where the 4 left-handed and 4 right-handed topologically protected chiral fermions emerge, see Fig. 1. Expansion of the Gor'kov Green's function in the vicinity of each topologically protected Weyl point leads to the effective relativistic quantum field theory with effective gauge fields and the effective gravity. This provides the hint for possible emergent origin of the "fundamental" Weyl fermions, gauge fields, and general relativity [10–12].

The majority of the superconductivity classes discussed in Refs. [7–9] contain the topologically protected Weyl points and/or Dirac lines. The systems with Weyl fermions should experience the effects related to chiral anomaly [20, 21] – production of the chiral charge in the external electric and magnetic fields, or in the effective gauge fields produced by deformations. The latter has been observed in chiral superfluid ^{3}He-A as an extra

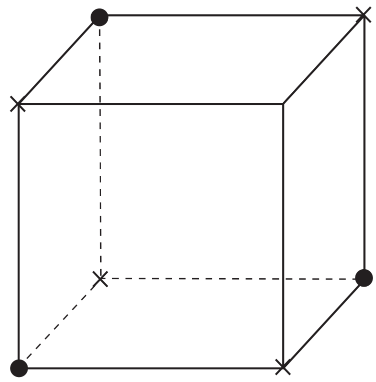

Fig. 1.(from Ref. [8]). Arrangement of the nodes in the energy spectrum in superconductors of class $O(D_2)$. The points denote four Weyl nodes with topological charge $N = +1$, and crosses denote four Weyl nodes with $N = -1$. In the modern language the topological charge represent the charge of the Berry phase monopole in momentum space [13], which can be analytically expressed in terms of the Gor'kov–Green's function [14]. In the vicinity of each Weyl node with $N = +1$ the chiral right-handed Weyl fermions emerge, while $N = -1$ is the topological charge of the left-handed quasiparticles. In a different form the topological stability of the Weyl nodes has been discussed for the neutrino sector in Refs. [15, 16]. In case of Fermi pockets discussed in Ref. [17] the number of Weyl fermions may increase. If the pockets are on the three-fold axes, one would have 8 right-handed and 8 left-handed fermions. This can be compared with 8 right-handed and 8 left-handed particles (quarks and leptons) in each generation of Standard Model fermions, and in the four-dimensional graphene [18, 19]

force acting on vortex-skyrmions [22]. This is the spectral flow force or the Kopnin force, see e.g. Ref. [23]. It can be expressed in terms of the effective "magnetic" **B** and "electric" **E** fields acting on Weyl fermions. These fields are produced by deformation in superconductors, or by the moving skyrmion texture in ^{3}He-A [12]. The Kopnin force is proportional to $\mathbf{B}\cdot\mathbf{E}$, and the measured

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coefficient in front of $\mathbf{B}\cdot\mathbf{E}$ was found to be in agreement with the Adler–Bell–Jackiw equation [20, 21], describing production of the fermionic charge generated by the chiral anomaly.

The effective quantum field theory emerging in the vicinity of the Dirac nodal lines is still waiting for its development, as well as in the vicinity of nodes of arbitrary co-dimension. Probably this may have relation to branes in string theory [11].

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