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# A theory of slightly fluctuating ratchets

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In Brownian ratchet theory, as in any theoretical field, the choice of an appropriate approximation or simplifying assumption is an effective way of obtaining analytical results [1]. In a number of natural or artificial systems exhibiting ratchet effect, the ratchet potential can be presented as a sum of a time-independent main contribution and a small time-dependent correction. Thus developing an analytical theory of slightly fluctuating ratchets is an important problem, especially for the design of molecular nanomachines. We present the solution of this problem, which gives integrated consideration of ratchets of two basic types, rocking and flashing, with an arbitrary time dependence of potential energy fluctuations. Our approach allows rejecting the restrictions of instantaneous switching of the ratchet potential. This idealization is used in a variety of theoretical studies [1–3], but cannot, for instance, interpret correctly the behavior of photomotors [4], in which there always exist relaxation processes of finite duration, so that one can expect new effects as a result of what place relaxation times occupy in the hierarchy of the photomotor characteristic times.

We consider overdamped transport of a Brownian particle with the time-dependent potential energy $U(x,t)$ of an additive-multiplicative form, $U(x,t) = u(x) + \sigma(t)w(x)$, which includes the stationary main contribution $u(x)$ and the small correction $\sigma(t)w(x)$. This form embraces the majority of both practically and theoretically significant variants of potential energy changes [1, 3, 5]. With an assumption that the functions $u(x)$ and $w'(x) \equiv dw(x)/dx$ are $L$-periodic, we can analyse in a similar way two ratchet models famous in the theory of Brownian motors: flashing $[w(x+L) = w(x)]$ and rocking $[w(x) = Fx,\ F = \text{const}]$ ratchets. The function $\sigma(t)$ describes the character of time fluctuations of the particle potential energy; it must have a zero average value, $\langle\sigma(t)\rangle = 0$ [the symbol $\langle\ldots\rangle$ means the operation of averaging over the fluctuations, the definition of which depends on the nature of the quantity $\sigma(t)$].

The main quantity in ratchet theory, which the ratchet effect exhibits itself through, is the average particle velocity [1], $\langle v\rangle$. In order to obtain a general analytical expression for it, we use the retarded Green's function $g(x,x',t)$ $[g(x,x',t) = 0$ at $t<0]$ of diffusion in the time-independent part $u(x)$ of the potential $U(x,t)$ and the perturbation theory of the Smoluchovski equation with respect to the small quantity $w'(x)$ of order $O(w)$. This has allowed us to write $\langle v\rangle$ in an elegant analytical form:

$$\langle v\rangle = L\beta^2 D^2 \int_0^L dx \rho_+(x) w'(x) \times$$
$$\times \int_0^L dx' S(x,x') \frac{\partial}{\partial x'} w'(x')\rho_-(x') + O(w^3),$$
$$S(x,x') = \int_0^\infty dt g(x,x',t) K(t), \tag{1}$$
$$\rho_\pm(x) = e^{\pm\beta u(x)} \Big/ \int_0^L dx e^{\pm\beta u(x)},$$

$D$ is the diffusion coefficient, $\beta = (k_B T)^{-1}$ is the inverse thermal energy ($k_B$ is the Boltzmann constant, $T$ is the absolute temperature), and $K(t) \equiv \langle\sigma(t_0+t)\sigma(t_0)\rangle$ is the correlation function. Eq. (1) is valid for an arbitrary, both stochastic and deterministic, time dependence $\sigma(t)$ and applicable to both flashing and rocking ratchets. Moreover, from this equation, all known basic analytical expressions for $\langle v\rangle$, obtained under the various approximations and additionally expanded over small $w(t)$, fol-

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low. The main features of the dependence of the average velocity on the parameters of the model are determined by the peculiarities of the function $S(x, x')$ describing the particle propagation from the point $x'$ to the point $x$ in the potential $u(x)$ under the conditions considered.

As an important example of using the high-temperature approximation of Eq. (1), we have analysed periodic relaxation processes, with the large period $\tau$ ($\tau \gg \tau_L$, $\tau_L = L^2/D$ is the characteristic diffusion time on the period $L$) and the finite relaxation time $\tau_{\rm rel}$, described by the supersymmetric function $\sigma(t) = -\sigma(t + \tau/2)$ which is defined in the interval $0 < t < \tau/2$ as $\sigma(t) = 1 - 2\exp(-t/\tau_{\rm rel})$ at $\tau \gg \tau_{\rm rel}$ [inset (b) of Fig. 1]. One of the bright application-

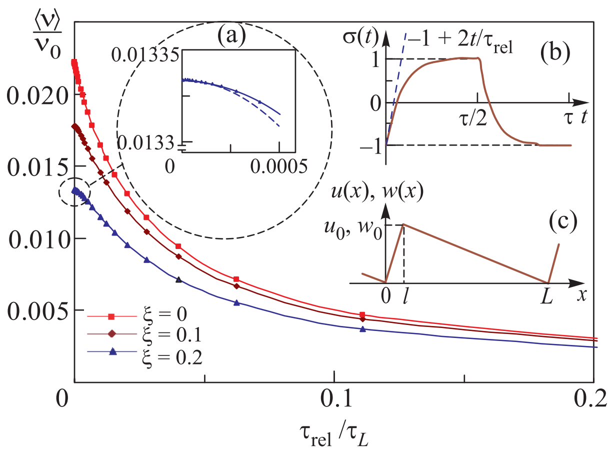


Fig. 1. (Color online) The average velocity $\langle v\rangle$, in units of $v_0 \equiv \beta^3 u_0 w_0^2 (L/\tau)$, of the Brownian ratchet with the sawtooth profiles of the main and fluctuation parts of the potential energy [the inset (c)] vs the normalized relaxation time $\tau_{\rm rel}/\tau_L$ for several normalized sawtooth lengths $\xi \equiv l/L$. The dashed curve in the inset (b) is a rough linear approximation, used in Ref. [7], of the exponential relaxation process $\sigma(t) = 1 - 2\exp(-t/\tau_{\rm rel})$ (the solid curve). The inset (a) details the quadratic $\tau_{\rm rel}$ – dependence of $\langle v\rangle$ at small $\tau_{\rm rel}$, obtained in Ref. [7] provided that the transient process is exponential (the dashed curve)

oriented illustrations of a ratchet model with relaxation of this type can be, for instance, the so-called Brownian photomotor, i.e., a system in which the ratchet effect originates as a result of photo-excitation [4]. The sought-for velocity $\langle v\rangle$ is compactly expressed in terms of the effective fluctuating potential introduced in Ref. [6]. The comparison of the obtained relaxation contribution to $\langle v\rangle$ with the velocity in Ref. [6] of the stochastic ratchets with dichotomous changes, having the exponential correlation function, of the potential profile shows the deep similarity of two mechanisms responsible for the ratchet effect.

The curves of Fig. 1 (main plot) illustrate the behavior of the ratchet average velocity as an effect of interplay of two basic parameters of the model: the dimensionless relaxation time $\tau_{\rm rel}/\tau_L$ (it can be any at $\tau \gg \tau_L$ and $\tau \gg \tau_{\rm rel}$) and the normalized length $\xi \equiv l/L$ (it determines the asymmetry of the potential profile and begets a new system characteristic time, the diffusion time $\tau_l \equiv l^2/D = \xi^2\tau_L$ over the small length $l$). The character of $\tau_{\rm rel}/\tau_L$-dependence of $\langle v\rangle$ essentially changes when the extremely asymmetric potential ($\xi = 0$) loses this quality ($\xi \neq 0$): at $\tau_{\rm rel} \ll \tau_L$, the linear decreasing law taking place at $\xi = 0$ turns into the quadratic dependence at $\xi \neq 0$ (compare squared curves with rhombus and triangle curves). The quadratic dependence of the velocity $\langle v\rangle$ on the duration of the transient process at $\xi \neq 0$ completely reproduces the result from Ref. [7] at some assumptions [see the dashed curve in the inset (a) of Fig. 1, and the details in the caption].

Summarizing, the Green's function technique provides a high-level description and great opportunities in calculating ratchet characteristics. An arbitrary time dependence of a slightly fluctuating potential allows consideration of complex ratchet systems with the interplay of characteristic time scales, that usually leads to the subtleties in ratchet behavior [1, 3], especially in asymptotic one [6]. As an example we have considered a Brownian photomotor for which two potentially competing characteristic times exist.

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