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Four-form field versus fundamental scalar field

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The discrepancy between the observed almost zero value of the vacuum energy and its estimation in terms of the zero point energy of fermionic and bosonic quantum fields provides the cosmological constant problem. Most plausibly the huge discrepancy of about 120 orders is the result of the naive estimations, which have been based on low-energy effective field theory. In a full equilibrium the huge contribution of the zero point motion is completely cancelled by the microscopic (correspondingly trans-Planckian) degrees of freedom.

The quantum vacuum can be described in terms of their own effective macroscopic variables, which do not depend much on the detailed microscopic structure of the system, such macroscopic approach is represented by the so-called q-theory [1–3].

The particular useful choice for the vacuum variable is the 4-form field strength $F = *(dA_3)$ [4–9], where A_3 is the 3-form gauge field. Such choice satisfies all the requirements needed for the description of the quantum vacuum, especially if instead of the quadratic form in the F-field, one uses the general function of the scalar F. The main difference of this form of the q-theory from the theory of the conventional scalar field is that A_3 is treated as the independent variable instead of the scalar F. This introduces an integration constant to the equations of motion, which serves as the analog of the chemical potential μ in condensed matter with conserved charge. As in the condensed matter, the parameter μ is self-tuned to nullify the proper thermodynamic potential, which enters the Einstein equations as the cosmological constant.

Later it became clear that the q-theory must be extended to include the derivatives of the q-field. The extension of the q-theory with the derivatives of the q-field was used to demonstrate that the oscillations of the q-field during cosmological evolution produce the kind of dark matter [10, 11], and thus the q-field is in the origin of both the dark energy and dark matter.

This extension is also important for the consideration of the q-ball [12] and the interfaces between the vacua with the same energy. Such interfaces take place if the Universe obeys the so-called multiple point principle [13–17], according to which the Universe is at the coexistence point, where different vacua have the same energy density.

In this current paper we present the complete form of the Einstein and Maxwell equations following from the extended q-theory, which includes both the dependence of the gravitational Newton constant on the field strength F and the gradient terms. The latter gives in particular the platform to study the structure of the black hole singularity, which is regularized by the q-field.

The action for the 4-form field interacting with the gravitational field has the following form ($\hbar = c = 1$):

$$S = -\int_{\mathbb{R}^4} d^4x\,\sqrt{|g|}\,\left(\frac{R}{16\pi G(F)} + \epsilon(F) + \right. \tag{1a}$$

$$\left. + \frac{1}{8}\,K(F)\,\nabla^\alpha F^2\,\nabla_\alpha F^2 + \mathcal{L}^{\text{SM}}\right), \tag{1b}$$

$$F_{\kappa\lambda\mu\nu} \equiv \nabla_{[\kappa} A_{\lambda\mu\nu]}, \quad F^2 \equiv -\frac{1}{4!}\,F_{\kappa\lambda\mu\nu}F^{\kappa\lambda\mu\nu} \tag{1c}$$

$$F_{\kappa\lambda\mu\nu} = F\sqrt{|g|}\,e_{\kappa\lambda\mu\nu}, \quad F^{\kappa\lambda\mu\nu} = Fe^{\kappa\lambda\mu\nu}/\sqrt{|g|}, \tag{1d}$$

∇_μ denotes a covariant derivative and a square bracket around spacetime indices complete anti-symmetrization, $\nabla^\alpha F^2\,\nabla_\alpha F^2$ is $g^{\alpha\beta}\,\nabla_\beta F^2\,\nabla_\alpha F^2$, $K(F)$ is some factor depending on F only (here not on its derivatives), $\mathcal{L}^{\text{SM}}$ is the Lagrange density of the fields of the standard model (SM) of elementary particle physics. Throughout, we use natural units with $c = \hbar = 1$ and take the metric signature $(-+++)$.

Variation over $A_{\lambda\mu\nu}$ gives the Maxwell equations:

$$\nabla_\kappa\left(\frac{R}{16\pi}\frac{dG^{-1}(F)}{dF} + \frac{d\epsilon(F)}{dF} + \right.$$
$$\left. + \frac{1}{8}\frac{dK(F)}{dF}\partial^\alpha F^2\partial_\alpha F^2 - \frac{1}{2}F\nabla^\alpha\left(K(F)\partial_\alpha F^2\right)\right) = 0. \tag{2}$$

From Maxwell Eq. (2) we get

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$$\frac{R}{16\pi}\frac{dG^{-1}(F)}{dF}+\frac{d\epsilon(F)}{dF}+\frac{1}{8}\frac{dK(F)}{dF}\partial^\alpha F^2\partial_\alpha F^2 - \\ -\frac{1}{2}F\nabla^\alpha\left(K(F)\partial_\alpha F^2\right)=\mu, \qquad (3)$$

where μ is the integration constant. It is convenient to set

$$C(F)=F^2K(F), \qquad (4)$$

which gives for Maxwell equations:

$$\frac{R}{16\pi}\frac{dG^{-1}(F)}{dF}+\frac{d\epsilon(F)}{dF}-\frac{1}{2}\frac{dC(F)}{dF}\partial^\alpha F\partial_\alpha F - \\ - C(F)\Box F=\mu. \qquad (5)$$

Variation over the metric $g^{\mu\nu}$ gives the generalized Einstein equations:

$$\frac{1}{8\pi G(F)}\Big(R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}\Big)+\frac{1}{16\pi}F\frac{dG^{-1}}{dF}Rg_{\mu\nu}+ \\ +\frac{1}{8\pi}\Big(\nabla_\mu\nabla_\nu G^{-1}(F)-g_{\mu\nu}\Box G^{-1}(F)\Big)- \\ -\left(\epsilon(F)-F\frac{d\epsilon(F)}{dF}\right)g_{\mu\nu}- \\ -\frac{1}{2}C(F)g_{\mu\nu}\nabla_\alpha F\nabla^\alpha F-\frac{1}{2}F\frac{dC(F)}{dF}\nabla_\alpha F\nabla^\alpha F g_{\mu\nu}+ \\ +C(F)\nabla_\mu F\nabla_\nu F-C(F)\Box F g_{\mu\nu}+T^{(\mathrm{SM})}_{\mu\nu}=0, \qquad (6)$$

which can be simplified using Eq. (5):

$$\frac{1}{8\pi G(F)}\Big(R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}\Big)+ \\ +\frac{1}{8\pi}\Big(\nabla_\mu\nabla_\nu G^{-1}(F)-g_{\mu\nu}\Box G^{-1}(F)\Big)- \\ -\left(\epsilon(F)-\mu F\right)g_{\mu\nu}-\frac{1}{2}C(F)g_{\mu\nu}\nabla_\alpha F\nabla^\alpha F+ \\ +C(F)\nabla_\mu F\nabla_\nu F+T^{(\mathrm{SM})}_{\mu\nu}=0. \qquad (7)$$

For the constant gravitational coupling $G(F)$ these equations are reduced to the corresponding equation in the article [10].

The Einstein equation (7) shows that the contribution of the 4-form field to the gravitating energy-momentum tensor is given by

$$T^{(F)}_{\alpha\beta}=\left(C(F)\nabla_\alpha F\nabla_\beta F-\frac{1}{2}g_{\alpha\beta}C(F)\nabla_\mu F\nabla^\mu F\right)- \\ -g_{\alpha\beta}\left(\epsilon(F)-\mu F\right). \qquad (8)$$

There are two faces of the 4-form field, as follows from Eq. (8): it has the signature of the (pseudo)scalar field and the signature of the conserved quantity, which characterizes the deep quantum vacuum. The former leads to the effective dark matter, while the latter contributes to dark energy. Since the scalar F is not the fundamental scalar, but is the field strength of the 3-form gauge field, the contribution to the vacuum energy from the F-field, $\epsilon(F)-\mu F$, contains the integration constant μ, which serves as the analog of the chemical potential in condensed matter systems with conserved charge.

The obtained equations are applicable for different problems such as: (i) relaxation of the vacuum energy in the expanding Universe; (ii) the internal structure of the black hole including the structure of the singularity; (iii) investigation of topological and non-topological objects; etc.

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