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Quantum analysis of fluctuations of electromagnetic fields in heavy-ion collisions

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Non-central AA collisions at high energies can generate a very strong magnetic field perpendicular to the reaction plane [1]. In this letter we perform a quantum analysis of fluctuations of the electromagnetic fields in AA collisions at RHIC and LHC energies based on the fluctuation-dissipation theorem (FDT). This issue is very important in the context of the chiral magnetic effect and charge separation [1, 2] in AA collisions because the fluctuations may partly destroy the correlation between the magnetic field direction and the reaction plane, and can lead to reduction of the **B**-induced observables [3]. Previously the field fluctuations have been addressed by Monte-Carlo (MC) simulation with the Woods-Saxon (WS) nuclear distribution using the classical Lienard-Weichert potentials [3–5]. But the WS nuclear distribution ignores the collective quantum dynamics of the nuclear ground state. The classical treatment of the electromagnetic field may also be inadequate because, similarly to the van der Waals forces [6], it becomes invalid at large distances.

We consider the proper time region $\tau \sim 0.2{-}1\,\mathrm{fm}$ which is of the most interest for the **B**-induced effects in the quark-gluon plasma (QGP). We ignore the electromagnetic fields generated by the induced currents in the QGP fireball after interaction of the colliding nuclei [7]. We consider the right moving and left moving nuclei with velocities $\mathbf{V}_\mathrm{R} = (0, 0, V)$ and $\mathbf{V}_\mathrm{L} = (0, 0, -V)$, and with the impact parameters $\mathbf{b}_\mathrm{R} = (-b/2, 0, 0)$ and $\mathbf{b}_\mathrm{L} = (b/2, 0, 0)$. We take $z_{\mathrm{R,L}} = \pm Vt$. For each nucleus the electromagnetic field is a sum of the mean field and the fluctuating field

$$F^{\mu\nu} = \langle F^{\mu\nu}\rangle + \delta F^{\mu\nu}. \qquad (1)$$

The mean fields $\langle\mathbf{E}\rangle$ and $\langle\mathbf{B}\rangle$ are given by the Lorentz transformation of the Coulomb field in the nucleus rest frame. For two colliding nuclei the mean magnetic field at $\mathbf{r} = 0$ has only y-component.

The contribution of each nucleus to the correlators of the electromagnetic fields in the lab-frame may be expressed via the correlators in the nucleus rest frame. The dominant transverse components of the field correlators can be written as

$$\langle\delta E_i\delta E_k\rangle = \gamma^2\left[\langle\delta E_i\delta E_k\rangle + V^2 e_{3il}e_{3kj}\langle\delta B_l\delta B_j\rangle\right]_{rf}, \qquad (2)$$

$$\langle\delta B_i\delta B_k\rangle = \gamma^2\left[\langle\delta B_i\delta B_k\rangle + V^2 e_{3il}e_{3kj}\langle\delta E_l\delta E_j\rangle\right]_{rf}, \qquad (3)$$

where i, k are the transverse indices and the subscript rf on the right-hand side of (2), (3) indicates that the correlators are calculated in the nucleus rest frame.

In calculations of the rest frame correlators $\langle\delta E_l\delta E_j\rangle$, $\langle\delta B_i\delta B_k\rangle$ (hereafter we drop the subscript rf) with the help of the FDT we follow the formalism of [8] (formulated in the gauge $\delta A^0 = 0$). It allows to relate the time Fourier component of the symmetric correlator of δA_i with that of the retarded Green's function D_{ik}. In the zero temperature limit this relation reads [8]:

$$\langle\delta A_i(\mathbf{r}_1)\delta A_k(\mathbf{r}_2)\rangle_\omega = -\mathrm{sign}(\omega)\mathrm{Im}D_{ik}(\omega, \mathbf{r}_1, \mathbf{r}_2). \qquad (4)$$

At $t \gtrsim 0.2\,\mathrm{fm}$ (in the lab-frame) the distance between the observation point and the center of each nucleus (in its rest frame) is much bigger than the nucleus radius and one can treat each nucleus as a point like dipole described by the dipole polarizability $\alpha(\omega)$. The field fluctuations are described by correction to the retarded Green's function proportional to the dipole polarizability [8]:

$$\Delta D_{ik}(\omega, \mathbf{r}_1, \mathbf{r}_2) = -\omega^2 D^v_{il}(\omega, \mathbf{r}_1, \mathbf{r}_A)\alpha(\omega) \times \times D^v_{lk}(\omega, \mathbf{r}_A, \mathbf{r}_2), \qquad (5)$$

with D^v_{ik} the vacuum Green's function ($\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$)

$$D^v_{ik}(\omega, \mathbf{r}_1, \mathbf{r}_2) = \frac{e^{\mathrm{i}\omega r}}{r}\left[-\delta_{ik}\left(1 + \frac{\mathrm{i}}{\omega r} - \frac{1}{\omega^2 r^2}\right) + + \frac{x_i x_k}{r^2}\left(1 + \frac{3\mathrm{i}}{\omega r} - \frac{3}{\omega^2 r^2}\right)\right]. \qquad (6)$$

The function $\alpha(\omega)$ reads [6]:

$$\alpha(\omega) = \frac{1}{3}\sum_s\left[\frac{|\langle 0|\mathbf{d}|s\rangle|^2}{\omega_{s0} - \omega - \mathrm{i}\delta} + \frac{|\langle 0|\mathbf{d}|s\rangle|^2}{\omega_{s0} + \omega + \mathrm{i}\delta}\right], \qquad (7)$$

where $\mathbf{d} = \left(eN\sum_p \mathbf{r}_p - eZ\sum_n \mathbf{r}_n\right)/A$ is the dipole operator. At $\omega > 0$ the imaginary part of $\alpha(\omega)$ is connected with the dipole photoabsorption cross section

$$\sigma_\mathrm{abs}(\omega) = 4\pi\omega\mathrm{Im}\alpha(\omega). \qquad (8)$$

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For heavy nuclei the dipole strength is dominated by the giant dipole resonance (GDR) [9]. It appears as a broad peak in $\sigma_{\rm abs}$ at $\omega \sim 14$ MeV. We parametrize the dipole polarizability for ^{197}Au and ^{208}Pb nuclei by a single GDR state

$$\alpha(\omega) = c\left[\frac{1}{\omega_{10}-\omega-i\Gamma/2}+\frac{1}{\omega_{10}+\omega+i\Gamma/2}\right]. \quad (9)$$

By fitting the data on the photoabsorption cross section from [10] for ^{197}Au and from [11] for ^{208}Pb we obtained the following values of the parameters: $\omega_{10} \approx 13.6$ MeV, $\Gamma \approx 4.38$ MeV, $c \approx 18.2$ GeV^{-2} for ^{197}Au, and $\omega_{10} \approx 13.3$ MeV, $\Gamma \approx 3.72$ MeV, $c \approx 18.93$ GeV^{-2} for ^{208}Pb. Using these parameters we calculated the fluctuations of the nuclear dipole moment. From (7), (9) one can obtain

$$\langle 0|\mathbf{d}^2|0\rangle = \frac{3}{\pi}\int_0^\infty d\omega \mathrm{Im}\alpha(\omega) = \frac{6c}{\pi}\mathrm{arctg}\,(2\omega_{10}/\Gamma)\,. \quad (10)$$

This formula gives $\langle 0|\mathbf{d}^2|0\rangle \approx 1.91$ fm^2 and $\langle 0|\mathbf{d}^2|0\rangle \approx 2.02$ fm^2 for ^{197}Au and ^{208}Pb, respectively. The classical MC calculation with the WS nuclear density gives for these nuclei the values $\langle \mathbf{d}^2\rangle \approx 9.89$ fm^2 and $\langle \mathbf{d}^2\rangle \approx 10.39$ fm^2. Thus, we see that the classical treatment overestimates the dipole moment squared by a factor of ~ 5.

At the center of the plasma fireball the fluctuations of the direction of the magnetic field are dominated by the fluctuations of the component B_x that vanishes without fluctuations. In Fig. 1 we show our quantum and classical results for t-dependence of the ratio $\langle \delta B_x^2\rangle^{1/2}/\langle B_y\rangle$ at $x = y = 0$ for several impact parameters for Au+Au collisions at $\sqrt{s} = 0.2$ TeV and Pb+Pb collisions at $\sqrt{s} = 2.76$ TeV. This figure shows that the quantum treatment gives $\langle \delta B_x^2\rangle^{1/2}/\langle B_y\rangle$ smaller than the classical one by a factor of $\sim 5-8$ for RHIC and by a factor of $\sim 13-27$ for LHC.

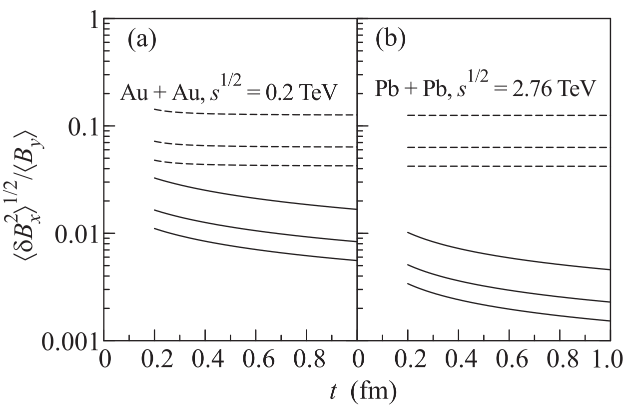

Fig. 1. The t-dependence of the ratio $\langle \delta B_x^2\rangle^{1/2}/\langle B_y\rangle$ at $\mathbf{r} = 0$ for Au+Au collisions at $\sqrt{s} = 0.2$ TeV (a) and for Pb+Pb collisions at $\sqrt{s} = 2.76$ TeV (b) for the impact parameters $b = 3$, 6 and 9 fm (from top to bottom). Solid lines are for quantum calculations, dashed lines for classical MC calculations with the WS nuclear density

In the event-by-event measurements the orientation of the reaction plane is extracted from the elliptic flow in the particle distribution (it is often called the participant plane), and it fluctuates around the real reaction plane. Calculations of the fluctuations of the direction of the magnetic field relative to the participant plane require a joint analysis of the field fluctuations and of the fluctuations of the initial entropy deposition that control the fluctuations of the orientation of the participant plane in the hydrodynamical simulations of AA collisions. The initial entropy distribution is sensitive to the long range fluctuations of the nuclear density. Besides the nuclear fluctuations related to the GDR there are other collective nuclear modes [9] such as the giant monopole resonance and the giant quadrupole resonance that may also be important for the participant plane fluctuations. It is of great interest for the event-by-event hydrodynamic simulations of AA collision to study their effect as well.

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