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Chiral torsional effect

Z. V. Khaidukov[+*], *M. A. Zubkov* [×1)]

[+]*Institute for Theoretical and Experimental Physics, 117259 Moscow, Russia*

[*]*Moscow Institute of Physics and Technology, 141700 Dolgoprudny, Russia*

[×]*Physics Department, Ariel University, 40700 Ariel, Israel*

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In the present paper we propose the new non-dissipative transport effect – the chiral torsional effect. Namely, we will discuss the emergence of axial current in the presence of torsion. It will be shown that this effect is intimately related to the chiral vortical effect [1]. In conventional general relativity [2, 3] torsion vanishes identically, it appears only in its various extensions. The background (non-dynamical) gravity with torsion emerges, in particular, in certain condensed matter systems. For example, elastic deformations in graphene and in Weyl semimetals induce the effective torsion experienced by the quasiparticles [4, 5]. In ^{3}He-A torsion appears dynamically when motion of the superfluid component is non-homogeneous.

We are considering the model of massless Dirac fermions, which serves as the low energy approximation for the electronic excitations in Weyl semimetals and also for the fermionic excitations in ^{3}He-A. For simplicity we will restrict ourselves by one flavor of Dirac fermions, and do not take into account the anisotropy of the Fermi velocity. In order to take it into account we should simply rescale the coordinates in an appropriate way.

Following [6, 7] we will calculate here the response of the axial current $j^{5\mu}(x) = \bar{\psi}(x)\gamma^\mu\gamma^5\psi(x)$ to torsion in terms of the Wigner transformation of the Green function. We consider the case of vanishing spin connection and the nontrivial torsion encoded in the vielbein $e^a_\mu(x) = \delta^a_\mu + \delta e^a_\mu(x)$. The inverse vielbein is denoted here by $E^\mu_a = \delta^\mu_a + \delta E^\mu_a$, it obeys equation $e^a_\mu(x)E^\mu_b(x) = \delta^a_b$, which gives $\delta e^a_b(x) \approx -\delta E^a_b$. Momentum operator is to be substituted by $p_a \to (\delta^b_a + \delta E^b_a(x))p_b \approx (p_a - \delta e^b_a(x)p_b)$. Product of the coordinate-depending vielbein and momentum operator should be symmetrized in order to lead to the Hermitian Hamiltonian of the quasipaticles[2)]. The term $\delta e^b_i p_b = A_i$ to a certain extent is similar to the gauge field (see also [8, 9]). Suppose, that $G(p)$ is the two point fermion Green function in momentum space for the system of Dirac spinors. Following the same steps as those of [7] in the leading order in the derivative of the vielbein we obtain the following expression for the axial current:

$$j^{5k} = \frac{i}{2} T^a_{ij} \int \frac{d\omega d^3p}{(2\pi)^4} p_a \mathrm{Tr}\, G\partial_{p_i} G^{-1} \partial_{p_j} G \gamma^5 \partial_{p_k} G^{-1},$$
$$T^a_{ij} = \partial_i e^a_j - \partial_j e^a_i. \qquad (1)$$

The obtained above expression is divergent and requires regularization. Below we present the results of our calculations made using the two complementary methods: via the zeta-regularization and via the sum over the Matsubara frequencies with the contribution of vacuum subtracted. In both cases the results coincide.

$$j^{5k}_{\mathrm{medium}} = -\frac{T^2}{24}\epsilon^{0kij} T^0_{ij}. \qquad (2)$$

Let us consider the particular non-relativistic system (having the emergent relativistic symmetry at low energies) in the presence of macroscopic motion with velocity $\mathbf{v}$. When $\mathbf{v}$ is constant or slowly varying, the one-particle Hamiltonian is given by $\hat{H} = \hat{H}_0 + \mathbf{pv}$. Here $\hat{H}_0$ is the Hamiltonian in the reference frame accompanying the motion. For the case of emergent massless Dirac fermions $\hat{H}_0 = \gamma^0 \sum_{k=1,2,3} \gamma^k p_k$. Then the combination $p_0 - \hat{H}$ may be written as $\gamma^0 E^\mu_a \gamma^a p_\mu$ with the inverse vierbein $E^0_0 = 1$, $E^k_a = \delta^k_a$, $E^0_k = -v_k$, $E^0_a = 0$ for $a, k = 1, 2, 3$. Therefore, we are able to treat velocity of the macroscopic motion as the components of the vielbein: $e^0_k = v_k$, $k = 1, 2, 3$. Correspondingly, in the presence of rotation torsion appears with the nonzero components $T^0_{ij} = \partial_i v_j - \partial_j v_i = 2\epsilon_{0ijk}\omega^k$, where $\boldsymbol{\omega}$ is vector of angular velocity. Equation (2) gives

$$j^{5k}_{\mathrm{medium}} = -\frac{T^2}{24}\epsilon^{0kij} T^0_{ij} = \frac{T^2}{6}\omega^k, \qquad (3)$$

1)e-mail: zubkov@itep.ru, on leave of absence from Institute for Theoretical and Experimental Physics, B. Cheremushkinskaya 25, 117259 Moscow, Russia

2)In this form gravity emerges in graphene and Weyl semimetals.

which is nothing but the conventional expression for the chiral vortical effect (see also [10]).

We see, that the vacuum contribution to the axial current is divergent in the presence of background torsion. This prompts that the quantum field theory in the presence of background gravity with torsion is ill-defined. Presumably, there are the aspects of the theory, in which it depends on regularization. In the other words, suppose, that the microscopically different real physical systems have naively the same continuum low energy effective field theory. In the presence of emergent torsion these effective theories become different. In particular, we expect the appearance of such differences between the low energy description of electrons in Weyl semimetals (in the presence of elastic deformations) and the description of the quasiparticles in the ^{3}He-A superfluid (in the presence of the in-homogeneous motion of the superfluid component). This difference may, possibly, be related to the appearance of the Nieh–Yan-like term [11] in the chiral anomaly for the solids and its absence for ^{3}He-A. Notice, that experiments with vortices in ^{3}He-A prompt that the chiral anomaly in this superfluid does not contain the Nieh–Yan term (see [12] and references therein). At the same time the recent theoretical consideration of Weyl semimetals indicates the appearance of this term in the chiral anomaly for these materials [13].

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