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Analog-digital quantum simulation of Dicke model with superconducting circuits

S. V. Remizov[a,b], *A. A. Zhukov*[a,c], *W. V. Pogosov*[a,d], *Yu. E. Lozovik*[e,a,f 1)]

[a]*Dukhov Research Institute of Automatics (VNIIA), 127055 Moscow, Russia*

[b]*V. A. Kotel'nikov Institute of Radio Engineering and Electronics RAS, 125009 Moscow, Russia*

[c]*National Research Nuclear University (MEPhI), 115409 Moscow, Russia*

[d]*Institute for Theoretical and Applied Electrodynamics RAS, 125412 Moscow, Russia*

[e]*Institute of Spectroscopy RAS, 142190 Moscow, Russia*

[f]*National Research University Higher School of Economics, 101000 Moscow, Russia*

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Quantum computation is based on a controllable manipulation with quantum objects, characterized by individual addressability. Due to their outstanding tunability, superconducting quantum circuits can be used not only in quantum computing, but also for the observation of various fundamental phenomena, which are hard to realize in the case of natural systems [1–9]. Superconducting qubits coupled to resonators are also considered as analog simulators of Dicke model. Such systems have been studied using spectroscopic methods, which reveal anticrossings in their spectra [10–12]. Anticrossings are known to appear both for quantum and classical systems provided they are interacting. Thus, it is attractive to consider additional experimental setups with such circuits explicitly based on their properties distinctive for the quantum world.

In this Letter we propose a concept of mixed analog-digital simulation of Dicke model, where quantum logic gates and addressability of qubits are utilized to engineer initial states, which can be both entangled and disentangled. Entangled states are characterized by symmetries and the influence of the symmetry on the subsequent evolution can serve as an unambiguous proof of the quantum nature of an artificial system. The evolution is caused by the Hamiltonian embedded physically into the quantum hardware, which is an essence of analog quantum simulation. We illustrate our suggestion with few examples.

We start with the nonradiant states localized in the qubit subsystem. The localization is based on the excitation blockade in the qubit subsystem due to the quantum entanglement and negative quantum interference [13]. We hereafter numerate qubits in order of increasing their excitation energies. We adopt a notation $|\varphi_m\rangle = \sigma_m^+|\downarrow\ldots\downarrow\rangle$. The simplest example of the nonradiant state is an antisymmetric Bell state $|\Psi_{p,q}^-\rangle$ being defined as $|\Psi_{p,q}^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|\varphi_p\rangle \pm |\varphi_q\rangle\right)$. In order to create such states, it is possible to use the idea realized in [14]. The initial states engineered in [14] are $|\uparrow\rangle$ and $|\uparrow\uparrow\rangle$. They were created through additional control lines for tunable-frequency qubits utilized to excite them in the dispersive regime. By tuning their frequencies, resonant interaction between resonator and the excited qubits has been attained. In a similar way, entangled states can be engineered, although this requires application of a two-qubit entangling gate, such as controlled-not (CNOT) gate, and several single-qubit gates. Using such digital operations, it is possible to engineer even more sophisticated states of the qubit subsystem, which are difficult to construct for purely analog simulator.

Let us consider a dissipative evolution of the system taking into account different imperfections (disorder in qubits excitation energies and decoherence). The whole system is described by the master equation

$$\partial_t \rho(t) - \Gamma[\rho(t)] = -i[H, \rho(t)], \qquad (1)$$

where $\rho(t)$ is a density matrix. The matrix $\Gamma[\rho]$ is $\Gamma[\rho] = \kappa(a\rho a^\dagger - \{a^\dagger a, \rho\}/2) + \sum_j \left(\gamma(\sigma_{j,-}\rho\sigma_{j,+} - \{\sigma_{j,+}\sigma_{j,-}, \rho\}/2) + \gamma_\varphi(\sigma_{j,z}\rho\sigma_{j,z} - \rho)\right)$, where the first term describes an energy dissipation in the cavity, the second one – in each of the qubits, while the third one corresponds to the pure dephasing; κ, γ, and γ_φ are rates of these processes. The Hamiltonian is

1)e-mail: lozovik@mail.ru

$$H = \sum_{j=1}^{L} \epsilon_j \sigma_j^+ \sigma_j^- + \omega a^\dagger a + g \sum_{j=1}^{L} (a^\dagger \sigma_j^- + a \sigma_j^+), \quad (2)$$

where $a^\dagger$ and a correspond to photons, while $\sigma_j^\pm$, σ_j^z are Pauli operators related to the qubits. The number of qubits is $L \lesssim 10$. We consider strong coupling limit, κ, γ, $\gamma_\varphi < g$. We also assume that $g \ll \omega$. The spreading in ϵ_j's is $\sim g\sqrt{L}$ that implies that the disorder starts to be important.

In our computations, we analyze a time evolution of the fidelity F of $|\Psi_{p,q}^\pm\rangle$ with randomly chosen p and q. It can be determined experimentally by measuring qubits in Bell basis. Qualitatively, F describes robustness of the initial state. We focus on the determination of the most destructive mechanisms for the suppression of the expected for the ideal system behavior. We find that the sensitivity of the dynamics to the symmetry of the initial Bell state is strongly suppressed by disorder. However, $F(t)$ for $|\Psi^+\rangle$ contains fast oscillations with frequency $\sim g$ due to Hamiltonian bright eigenstates, which are stronger coupled to the light [15]. Also, envelopes of $F(t)$ remain to be different for two Bell states. For mesoscopic ensembles, these features can be used to distinguish between the dynamics of $|\Psi_{p,q}^-\rangle$ and $|\Psi_{p,q}^+\rangle$. Relaxation $\kappa \sim g$, however, suppresses the fast oscillations, but generally preserves the form of envelopes. Now, if we turn on finite γ_φ and make it of the order of a mean separation between neighboring ϵ_j's, it is able to destroy the difference in envelopes, so that the difference between $F(t)$ is smeared completely. Of course, finite γ also plays a destructive role by simply de-exciting the qubits.

Another interesting feature relevant to the evolution of the closed system is the radiation trapping effect [13]. It occurs provided there are many identical two-level systems interacting resonantly with the single-mode radiation field, while one of these systems is excited, so that the initial state of the system is $|\varphi_m\rangle$. It turns out that the presence of the environment of remaining qubits, though they are in their ground states, must strongly affect dynamics of the particular excited qubit by slowing down its radiative relaxation to the cavity. We suggest that the realization of the radiation trapping effect in artificial qubit-cavity systems can serve for the demonstration of the Dicke physics. We found that this effect is relatively robust to the disorder for the described above range of parameters. It is also robust with respect to the finite κ (up to $\kappa \gtrsim g$), which is related by the domination of dark eigenstates of H [15]. However, the effect is fragile with respect to both the finite γ and γ_φ.

In summary, we suggested an idea of mixed analog-digital simulation of Dicke model with superconducting quantum circuit, where a set of quantum logic gates is used to engineer various initial states of the qubit ensemble, while qubits interact naturally through the photon field of the resonator. The dependence of the dynamics on the symmetry of the initial entangled state can serve as an unambiguous demonstration of a quantum nature of such artificial systems. Various dynamical phenomena related to Dicke physics can be realized.

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1. P. Lähteenmäki, G. S. Paraoanu, J. Hassel, P. J. Hakonen, Proc. Natl. Acad. Sci. U.S.A. **110**(11), 4234 (2013).
2. C. M. Wilson, G. Johansson, A. Pourkabirian , M. Simoen, J. R. Johansson, T. Duty, F. Nori, and P. Delsing, Nature **479**(7373), 376 (2011).
3. D. S. Shapiro, A. A. Zhukov, W. V. Pogosov, and Y. E. Lozovik, Phys. Rev. A **91**, 063814 (2015).
4. A. A. Zhukov, D. S. Shapiro, W. V. Pogosov, and Y. E. Lozovik, Phys. Rev. A **93**, 063845 (2016).
5. S. V. Remizov, A. A. Zhukov, D. S. Shapiro, W. V. Pogosov, and Y. E. Lozovik, Phys. Rev. A **96**, 043870 (2017).
6. Z. H. Peng, S. E. de Graaf, J. S. Tsai, and O. V. Astafiev, Nat. Commun. **7**, 12588 (2016).
7. K. V. Shulga, E. Il'ichev, M. V. Fistul, I. S. Besedin, S. Butz, O. V. Astafiev, U. Hübner, and A. V. Ustinov, Nat. Commun. **9**(1), 150 (2018).
8. A. M. Zagoskin, D. Felbacq, and E. Rousseau, EPJ Quantum Tech. **3**(1), 2 (2016).
9. A. L. Rakhmanov, A. M. Zagoskin, S. Savel'ev, and F. Nori, Phys. Rev. B **77**, 144507 (2008).
10. P. Macha, G. Oelsner, J. M. Reiner, M. Marthaler, S. André, G. Schön, U. Hübner, H. G. Meyer, E. Il'ichev, and A. V. Ustinov, Nat. Commun. **5**(1), 5146 (2014).
11. K. V. Shulga, P. Yang, G. P. Fedorov, M. V. Fistul, M. Weides, and A. V. Ustinov, JETP Lett. **105**(1), 47 (2017).
12. K. Kakuyanagi, Y. Matsuzaki, C. Déprez, H. Toida, K. Semba, H. Yamaguchi, W. J. Munro, and S. Saito, Phys. Rev. Lett. **117**, 210503 (2016).
13. F. W. Cummings and A. Dorri, Phys. Rev. A **28**, 2282 (1983).
14. J. A. Mlynek, A. A. Abdumalikov, C. Eichler, and A. Wallraff, Nat. Commun. **5**, 5186 (2014).
15. A. A. Zhukov, D. S. Shapiro, W. V. Pogosov, and Y. E. Lozovik, Phys. Rev. A **96**, 033804 (2017).