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Four-fold anisotropy of the parallel upper critical magnetic field in a pure layered *d*-wave superconductor at $T = 0$

A. G. Lebed[+*1)], *O. Sepper*[+]

[+]*Department of Physics, University of Arizona, 1118 E. 4-th Street, Tucson, AZ 85721, USA*

[*]*L. D. Landau Institute for Theoretical Physics, Russian Academy of Sciences, 117334 Moscow, Russia*

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Since the discovery of unconventional d-wave superconductivity in high-temperature superconductors [1], physical consequences of d-wave electron pairing have been intensively investigated. One of such physical properties is a four-fold symmetry of the parallel upper critical magnetic field in these quasi-two-dimensional (Q2D) superconductors [2–5]. From the beginning, it was recognized that the four-fold anisotropy of the parallel upper critical magnetic field disappears in the Ginzburg–Landau (GL) region (see, for example, book [6]) and has to be calculated as a non-local correction to the GL results [3, 4]. Another approach was calculation of the parallel upper critical magnetic field at low temperatures and even at $T = 0$ [2, 7–9] using approximate method [10], which was elaborated for unconventional superconductors with closed electron orbits in an external magnetic field. Note that Q2D conductors in a parallel magnetic field are characterized by open electron orbits, which makes the calculations [2, 7–9] to be unappropriate.

The goal of our article is to suggest an appropriate method to calculate the parallel upper critical magnetic field in a Q2D d-wave superconductor. For this purpose, we explicitly take into account almost cylindrical shape of its Fermi surface (FS) and the existence of open electron orbits in a parallel magnetic field. We use the Green's functions formalism to obtain the Gor'kov's gap equation in the field. As an important example, we numerically solve this integral equation to obtain the four-fold anisotropy of the parallel upper critical magnetic field in a $d_{x^2-y^2}$-wave Q2D superconductor with isotropic in-plane FS. In particular, we demonstrate that the so-called supercondcting nuclei at $T = 0$ oscillate in space in contrast to the previous results [2, 7–9].

Below, we consider a layered superconductor with the following Q2D electron spectrum, which is an isotropic within the conducting plane:

$$\epsilon(\mathbf{p}) = \epsilon(p_x, p_y) - 2t_\perp \cos(p_z c^*), \quad t_\perp \ll \epsilon_F, \qquad (1)$$

where

$$\epsilon(p_x, p_y) = \frac{(p_x^2 + p_y^2)}{2m}, \quad \epsilon_F = \frac{p_F^2}{2m}. \qquad (2)$$

[Here, m is the effective in-plane electron mass, $t_\perp$ is the integral of overlapping of electron wave functions in a perpendicular to the conducting planes direction; ϵ_F and p_F are the Fermi energy and Fermi momentum, respectively; $\hbar \equiv 1$.] At the beginning, the parallel magnetic field is assumed to be applied along $\mathbf{x}$ axis,

$$\mathbf{H} = (H, 0, 0), \qquad (3)$$

where vector potential of the field is convenient to choose in the form:

$$\mathbf{A} = (0, 0, Hy). \qquad (4)$$

Electron motion within the conducting plane is supposed to be free (2), therefore, we can make the following substitutions in the electron energy (1) and (2):

$$p_x \to -i\left(\frac{\partial}{\partial x}\right), \quad p_y \to -i\left(\frac{\partial}{\partial y}\right), \qquad (5)$$

whereas, for the perpendicular electron motion, we can perform the so-called Peierls substitution:

$$p_z c^* \to p_z c^* - \left(\frac{\omega_c}{v_F}\right) y, \quad \omega_c = \frac{e v_F c^* H}{c}. \qquad (6)$$

As a result, the electron Hamiltonian in the magnetic field (3) can be represented as:

$$\hat{H} = -\frac{1}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) - 2t_\perp \cos\left(p_z c^* - \frac{\omega_c}{v_F} y\right). \quad (7)$$

1)e-mail: lebed@email.arizona.edu

By means of the quasi-classical approach [11] to the Hamiltonian (7), it is possible to find electron Green's functions in a magnetic field. Then, using linearized Gor'kov's gap equation for unconventional non-uniform superconductivity [12, 13], it is possible to obtain for $d_{x^2-y^2}$ electron pairing,

$$U(\phi, \phi_1) = g \cos(2\phi) \cos(2\phi_1), \quad (8)$$

the following gap equation at $T = 0$:

$$\Delta_\alpha(y) = g \int_d^\infty \frac{dz}{z} \left\langle J_0 \left\{ \frac{2t_\perp \omega_c}{v_F^2} [z(2y + z \sin\phi_1)] \right\} \times \right.$$
$$\left. \times [1 + \cos(4\alpha)\cos(4\phi_1)] \Delta_\alpha(y + z \sin\phi_1) \right\rangle_{\phi_1}, \quad (9)$$

where now angle α is the angle between the magnetic field and $\mathbf{x}$ axes in $(\mathbf{x}, \mathbf{y})$-plane; $< ... >_{\phi_1}$ means average over angle ϕ_1. For further solution of Eq. (9), it is useful to introduce new convenient variables:

$$\tilde{z} = \frac{\sqrt{2t_\perp \omega_c}}{v_F} z, \quad \tilde{y} = \frac{\sqrt{2t_\perp \omega_c}}{v_F} y. \quad (10)$$

[Here, at $T = 0$ we define the upper critical magnetic field in the framework of the Landau theory of the second order phase transitions, as it is done, for example, in [12, 13]. Therefore, we disregard the possible appearance of the so-called quantum phase transitions.] In new variables Eq. (9) can be written as follows

$$\Delta_\alpha(\tilde{y}) = g \int_d^\infty \frac{d\tilde{z}}{\tilde{z}} \left\langle J_0[\tilde{z}(2\tilde{y} + \tilde{z} \sin\phi_1)] \times \right.$$
$$\left. \times [1 + \cos(4\alpha)\cos(4\phi_1)] \Delta_\alpha(\tilde{y} + \tilde{z} \sin\phi_1) \right\rangle_{\phi_1}. \quad (11)$$

Below, we solve integral Eq. (11) numerically. The typical example of its solution for the superconducting nucleus, $\Delta_\alpha(\tilde{y})$, is shown in Fig. 1 of [11], where it oscillates and changes its sign with the changing coordinate $\tilde{y}$. These oscillations are consequences of the open nature of electron trajectories in a parallel magnetic field for the Q2D electron spectrum (1), (2). As we have shown, they result in the appearance of the oscillating Bessel function in the gap Eq. (11). This typical behavior of the superconducting nuclei is in a sharp contrast with the calculations of the four-fold anisotropy at $T = 0$, which were done before [2, 7–9]. The reason for that is the fact that the previous calculations didn't take into account open nature of electron orbits in Q2D conductors in a parallel magnetic field. Numerically calculated from Eq. (11) angular dependence of the parallel upper critical magnetic field of a Q2D $d_{x^2-y^2}$ superconductor is shown in Fig. 1, where there is a sharp peak at $\alpha = 0$ and a shallow minima at $\alpha = \pm 45^0$. The calculated magnitude of the four-fold anisotropy is $[H_{c2}(0^0) - H_{c2}(45^0)]/H_{c2}(22.5^0) = 0.13$, which is higher than that reported before [2]. In addition, from Fig.1, it is clear that the calculated by us anisotropic term is not of a pure $\cos(4\alpha)$ form as was stated in the all previous calculations [2, 7–9].

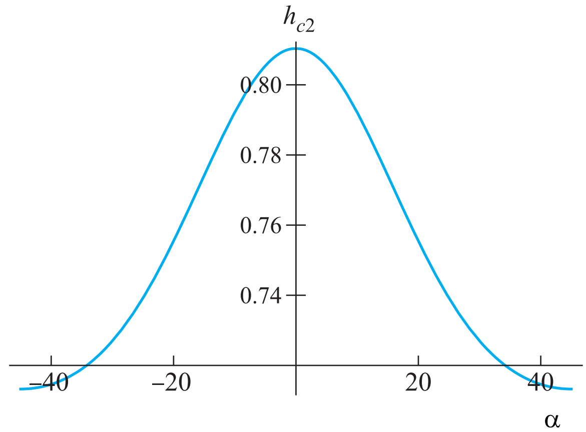

Fig. 1. (Color online) Angular dependence of the ratio $h_{c2}(\alpha) = H_{c2}(\alpha)/H_{c2}^{GL}(0)$, where $H_{c2}(\alpha)$ is the direction dependent parallel upper critical magnetic field at $T = 0$ and $H_{c2}^{GL}(0)$ is the Ginzburg–Landau parallel upper critical magnetic field slope at $T = 0$

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