

Schwarzschild black hole as accelerator of accelerated particles

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We consider collision of two particles near the horizon of a nonextremal static black hole. At least one of them is accelerated. We show that the energy $E_{c.m.}$ in the center of mass can become unbounded in spite of the fact that a black hole is neither rotating nor electrically charged. In particular, this happens even for the Schwarzschild black hole. The key ingredient that makes it possible is the presence of positive acceleration (repulsion). Then, if one of particles is fine-tuned properly, the effect takes place. This acceleration can be caused by an external force in the case of particles or some engine in the case of a macroscopic body (“rocket”). If the force is attractive, $E_{c.m.}$ is bounded but, instead, the analogue of the Penrose effect is possible.

More explicitly, the black hole metric has the form

$$ds^2 = -f dt^2 + f^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (1)$$

where the horizon is located at $r = r_+$, so $f(r_+) = 0$. We consider pure radial motion with the four-velocity u^μ and four-acceleration a^μ with

$$a_\mu a^\mu \equiv a^2, \quad (2)$$

where by definition $a \geq 0$. The presence of acceleration enables one to have fine-tuned (“critical”) particles, such that the energy

$$E = m \int_{r_+}^{\infty} dr' a(r'). \quad (3)$$

Let particles 1 and 2 move from infinity and collide in some point r_0 . The energy in the center of mass frame

$$\begin{aligned} E_{c.m.}^2 &= -(m_1 u_1^\mu + m_2 u_2^\mu)(m_1 u_{1\mu} + m_2 u_{2\mu}) = \\ &= m_1^2 + m_2^2 + 2m_1 m_2 \gamma, \end{aligned} \quad (4)$$

where $\gamma = -u_{1\mu} u_2^\mu$ is the Lorentz factor of relative motion. It follows from the above equations that

$$\gamma = \frac{X_1 X_2 - P_1 P_2}{m_1 m_2 f}. \quad (5)$$

Here, it is supposed that both particles move in the same direction, P being the radial momentum, $X = E - m \int_r^\infty dr' a(r')$.

If a critical particle 1 collides with a usual particle 2,

$$\gamma \sim \frac{\text{const}}{\sqrt{(r_0 - r_+)}}, \quad (6)$$

where a constant depend on the details of trajectories. Then, taking r_0 as close to r_+ as one likes, we obtain the unbounded growth of γ and $E_{c.m.}^2$ that can be thought of as a counterpart of a similar formula for the Kerr metric was considered. Thus there is a close analogy between our case and the BSW effect near nonextremal black holes. In particular, now the same difficulties persist that forbid arrival of the near-extremal particle from infinity because of the potential barrier typical of any nonextremal black hole. Therefore, either such a particle is supposed to be created already in the vicinity of the horizon from the very beginning or one is led to exploiting scenarios of multiple scattering. What is especially interesting is that the effect under discussion is valid for the Schwarzschild black hole.

Usually, the factor connected with additional forces (like gravitational radiation) are referred to as obstacles to gaining large $E_{c.m.}$. To the extent that such influence can be modeled by some force, backreaction does not spoil the effect. Meanwhile, as we saw now, in our context the presence of the force not only is compatible with the BSW effect but it can be its origin.

If a black hole is surrounded by external electromagnetic fields, we can suppose that the described mechanism promotes high energy collisions near black holes. The Schwarzschild metric and radial motion give us the simplest exactly solvable example but it is quite probable that qualitatively the similar results hold in a more realistic situation as well.

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