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Negative temperature: further extensions

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The authors of recent paper [1] suggest that negative absolute temperatures are consistent with equilibrium thermodynamics. All the thermodynamic properties, such as thermometry, thermodynamics of cyclic transformations, ensemble equivalence, fluctuation-dissipation relations, response theory and the transport processes, can be reformulated to include the negative temperatures. In [1] the condensed matter systems are discussed, such as the subsystem of nuclear spins with inverse spin population. The states with negative temperature has been experimentally studied in detail [2], and even the magnetic phase transitions occurring at negative temperature have been detected.

The equilibrium thermodynamics at $T < 0$ is only possible, if the environment also has negative temperature. Otherwise, the heat will be transversed from the negative temperature system to the environment, and the whole system will relax to the conventional state with $T > 0$. Here we show that the negative temperature states are also possible for the quantum vacuum in the relativistic quantum field theories. And if this vacuum state fills the whole Universe, this vacuum becomes thermodynamically stable in agreement with the conjecture in [1].

The relativistic vacuum with inverse population can be obtained by the PT symmetry operation, where P and T are space and time reversal transformations correspondingly. When the gravitational tetrads are taken into account, the discrete PT symmetry of spacetime acquires new formulations, since the sign of the tetrad field becomes also important [3–7]. In particular, if the tetrads are composite objects, which are formed as bilinear combinations of the fermionic operators [8–10], the parity P and the time reversal T operations are formed as combinations of the original more fundamental symmetry operators, $P = P_c P_s$ and $T = T_c T_s$. Here P_c and T_c refer to pure coordinate transformations, $P_c \mathbf{r} = -\mathbf{r}$ and $T_c t = -t$, while P_s and T_s refer to the corresponding transformations of Dirac or Weyl spinors [11]. In this approach the Lorentz transformation also represents the combination of the fundamental symmetry operations, $L = L_c L_s$. These symmetries are spontaneously broken to their diagonal subgroup, $L_c \times L_s \to L$, when the gravitational tetrad emerges as the order parameter of the phase transition.

In this paper we consider the extension of the PT symmetry to thermodynamics, where the PT operation also changes the sign of temperature. We apply this extension to the Dirac vacuum.

The Universe with negative temperature is obtained using the Dirac picture of the quantum vacuum. The conventional Dirac vacuum represents an infinite sea of particles with negative energy. Let us note that the infinite energy of the Dirac vacuum does not necessarily produce the huge contribution to the cosmological constant. For example, in the so-called q-theory, the relevant vacuum energy, which enters the Einstein equations in the form of the cosmological constant, is determined by the infrared thermodynamics [12, 13], rather than by the ultraviolet cut-off. In the fully equilibrium Minkowski vacuum without matter and without external environment the thermodynamic vacuum energy is nullified without any fine tuning. This follows from the thermodynamic Gibbs–Duhem relation, which is valid for any quantum vacuum state, including the non-relativistic vacua – the ground states in condensed matter. That is why there is no need for the spurious renormalization of the infinite mass of the filled states to zero.

The PT transformation leads to the mirror Dirac vacuum, where all the positive energy states are occupied and the negative energy states are empty [6]. The thermodynamic energy of this vacuum remains zero, but the thermal states in the background of this "false" vacuum with inverse population are characterized by negative temperature. This is similar to what happens in the subsystem of nuclear spins in condensed matter with inverse spin population [1, 2, 14]. In this spin subsystem there is an upper limit to all allowed energy states. On the contrary, in the case of Dirac vacua, the energy is unbounded both from below and from above.

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At first glance the state with the negative temperature is unstable. However, it is unstable only in case if there is the normal environment – the thermal bath with positive temperature. If there is no external environment, i.e. this mirror vacuum occupies the whole universe, this isolated vacuum becomes stable in spite of its negative temperature.

In the relativistic physics, the energy is unbounded. The negative and positive energy branches of fermionic states are symmetric with respect to zero energy. Due to this symmetry the isolated mirror relativistic vacuum, in which all the positive energy states are occupied and the negative energy states are empty, has exactly the same physics as the conventional Dirac vacuum in the "normal" Universe. Though the matter (excitations) in this mirror Universe has negative energy, and the thermodynamic states are characterized by the negative temperature, the inhabitants of the mirror Universe would think that they live in the normal Universe with positive energies for matter and positive temperature. It is only with respect to our Universe their temperature and energies are negative. But with respect to their Universe it is our Universe, which is "false" and which is described by negative temperature.

Recently there was suggestion to extend our Universe beyond the Big Bang (BB) using the analytic continuation of the radiation-dominated epoch across the BB singularity [4, 5]. The same analytic continuation, but which also takes into account the thermodynamics of the matter field, suggests that the temperature on the two sides of the BB has different sign [15]. From the point of view of the pre-BB Universe, our post-BB Universe has negative temperature.

So, in the symmetry transformations, one must take into account not only the vacuum states, but also the thermodynamics in the background of the vacuum. The PT transformation is accompanied by $T \to -T$. While the energy and temperature of matter change sign, the entropy remains positive, $S(E) = S(-E)$.

There can be another situation: when both the temperature and entropy are negative, while energy remains positive. This takes place for thermodynamics of the white holes. The negative entropy of the white hole has been obtained in three different ways of calculations [16, 17], where the quantum tunneling from the black hole to the white hole with the same mass M was exploited. Considering the quantum tunneling as thermodynamic fluctuation, and expressing it in terms of the total change of the entropy in this process, one finds the entropy and temperature of the white hole: $S_{\rm WH}(M) = -S_{\rm BH}(M)$ and $T_{\rm WH}(M) = -T_{\rm BH}(M)$.

In the mirror Universe, the black hole partner is also the black hole. This mirror black hole has negative mass, the negative Hawking temperature, but the positive entropy, $T_{\rm BH}(-M) = -T_{\rm BH}(M)$ and $S_{\rm BH}(-M) = S_{\rm BH}(M)$. The same is with the white hole. Its mirror partner in the mirror Universe is the white hole with opposite temperature: $T_{\rm WH}(-M) = -T_{\rm WH}(M) = T_{\rm BH}(M)$ and $S_{\rm WH}(-M) = S_{\rm WH}(M) = -S_{\rm BH}(M)$.

In conclusion, the PT symmetry is extended to include the reversal of temperature. The PT operation applied to the Dirac quantum vacuum leads to the mirror Dirac vacuum, where all the positive energy states are occupied and the negative energy states are empty. Such vacuum is thermodynamically stable, while matter in the background of this vacuum is described by negative temperature. The thermodynamics in the two Dirac vacua are equivalent, being (anti)symmetric with respect to the reversal of temperature. The PT symmetry also connects the black hole thermodynamics in the two vacua. This demonstrates that the equilibrium thermodynamics with $T < 0$ discussed in [1] can be realized in quantum field theory.

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