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Perpendicular upper critical magnetic field in a layered *d*-wave superconductor

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The upper critical magnetic field, $H_{c2}(0)$, is one of the most important parameters of the type-II superconductivity since, in the majority of the type-II superconductors, superconducting state is destroyed by magnetic field above $H_{c2}(0)$, $H > H_{c2}(0)$. The Ginzburg–Landau (GL) theory [1] provides an opportunity to understand physical meaning of $H_{c2}(0)$ and to calculate it near transition temperature, $T_c - T_c(H) \ll T_c$. At low temperature, for the first time, the upper critical magnetic field in an isotropic $3D$ superconductor was calculated by Gor'kov [2]. In the whole temperature region, the temperature dependence of $H_{c2}(T)$ was calculated for an isotropic $3D$ superconductor by Werthamer, Helfand, and Hohenberg (WHH) in [3]. The WHH theory has become very popular and physicists have used its results to fit experimental data on $H_{c2}(T)$ in all superconductors, including quasi-one-dimensional (Q1D) and quasi-two-dimensional (Q2D) ones with both s-wave and d-wave electron parings.

In the WHH theory, there exists the main parameter, $\alpha = H_{c2}(0)/(|dH_{c2}/dT|_{T_c} T_c)$, which distinguishes, for example, between clean and dirty superconductors. In a clean $3D$ superconductor, it is equal to $\alpha \approx 0.73$. Bulaevskii recognized that the value of the above mentioned parameter depended on dimensionality of a superconductor and calculated it for perpendicular magnetic field in a s-wave Q2D superconductor [4]. In Q2D s-wave superconductors, the parameter α for the parallel upper critical magnetic field was calculated by us in [5], whereas, in d-wave superconductor [6], we found its angular in-plane oscillations [7]. As to d-wave superconductors in a perpendicular magnetic field, the parameter α was estimated several times [8–10] by approximated method [11], whose accuracy cannot be well controlled, and, as a result, several different values were obtained. In this paper, we show that this parameter, in a perpendicular magnetic field in Q2D case, depends on a nature of superconducting pairing and for d-wave pairing [6] is equal to

$$\alpha_d = H_{c2}(0)/(|dH_{c2}/dT|_{T_c} T_c) \approx 0.629. \quad (1)$$

Our work contains two main messages. The first one is that it is not a good idea to fit every measurement of $H_{c2}(T)$ by the WHH theory, as has been done so far almost in all experimental works. It is necessary to take into account dimensionality and nature of electron pairing in superconductors. The second message is that a comparison of the obtained formula (1) with the experimental data [12] demonstrates that the optimally-doped $YBa_2Cu_3O_{7-\delta}$ is rather good described by either the Bardeen–Cooper–Schrieffer theory or its extension to d-wave superconductivity. Note that below we always disregard quantum effects of electron motion in a magnetic field, which can result in the restoration of superconductivity [13–17] in very high magnetic fields. These effects have been shown so far to be important only in Q1D organic superconductors from chemical family $(TMTSF)_2X$, where $X = ClO_4$ and PF_6, as well as in the heavy fermion triplet superconductor UTe_2 [17].

Let us consider a Q2D superconductor with the following electron spectrum, which is an isotropic one within the conducting plane:

$$\epsilon(\mathbf{p}) = \epsilon(p_x, p_y) - 2t_\perp \cos(p_z c^*), \quad t_\perp \ll \epsilon_F, \quad (2)$$

where

$$\epsilon(p_x, p_y) = \frac{(p_x^2 + p_y^2)}{2m}, \quad \epsilon_F = \frac{p_F^2}{2m}. \quad (3)$$

[Here, m is the effective in-plane electron mass, $t_\perp$ is the integral of overlapping of electron wave functions in a perpendicular to the conducting planes direction, c^* is distance between the conducting planes; ϵ_F and p_F are the Fermi energy and Fermi momentum, respectively;

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$\hbar \equiv 1$.] The perpendicular to the conducting layers magnetic field is applied along $\mathbf{z}$ axis,

$$\mathbf{H} = (0, 0, H), \tag{4}$$

where the vector potential of the field is possible to choose in the form:

$$\mathbf{A} = (0, Hx, 0). \tag{5}$$

To describe electron motion within the conducting plane, we perform the so-called Peierls substitution [1]:

$$p_x = -i\frac{\partial}{\partial x}, \quad p_y = -i\frac{\partial}{\partial y} - \frac{e}{c}A_y. \tag{6}$$

As a result, the electron Hamiltonian in the magnetic field (4) can be represented as:

$$\frac{1}{2m}\left[-\left(\frac{\partial}{\partial x}\right)^2 + \left(-i\frac{\partial}{\partial y} - \frac{e}{c}Hx\right)^2\right]\Psi_\epsilon(x,y) = \epsilon\Psi_\epsilon(x,y). \tag{7}$$

By means of the quasiclassical approach to Eq. (7), it is possible to calculate exactly the electron wave functions and the electron Matsubara's Green functions [18]. Using the Gor'kov's equations [18, 19] for d-wave superconducting gap,

$$\Delta(x, \phi_1) = \sqrt{2}\cos(2\phi_1)\ \Delta(x), \tag{8}$$

it is possible to derive the so-called gap equation for the $\Delta(x)$:

$$\Delta(x) = U\int \frac{dp_y}{|v_x^0(p_y)|}\int_{|x-x_1|>\frac{|v_x^0(p_y)|}{\Omega}} \frac{2\pi T dx_1}{|v_x^0(p_y)|\sinh\left[\frac{2\pi T|x-x_1|}{|v_x^0(p_y)|}\right]} \times 2\cos^2[2\phi_2(p_y)]\cos\left[\frac{\omega_c p_y(x^2-x_1^2)}{v_x^0(p_y)}\right]\Delta(x_1). \tag{9}$$

Analytical solution in the GL region and numerical solution at zero temperature, $T = 0$, of the Eq. (9) gives the result (1). Let us compare the obtained result (1) with the experimental measurements [12]. If we take into account accuracy of the measurements of the perpendicular upper critical magnetic field in [12], we will obtain:

$$\alpha_{\exp} = H_{c2}(0)/(|dH_{c2}/dT|_{T_c}T_c) \approx 0.50 - 0.65, \tag{10}$$

which is in a general agreement with Eq. (1). Although, Eq.(10) clear demonstrates that the 3D WHH coefficient [3] is not applicable to a Q2D case of the $YBa_2Cu_3O_{7-\delta}$, unfortunately, within the accuracy of Eq. (10), it is not possible to distinguish between d-wave and s-wave superconducting pairings. We hope that new more precise measurements of $H_{c2}(0)$ will confirm d-wave nature of supercondcting pairing in the $YBa_2Cu_3O_{7-\delta}$ and that our Eq. (1) will be useful test for some other Q2D superconductors. But what is clear now from Eqs. (1) and (10) is that superconductivity in optimally-doped high-T_c superconductor $YBa_2Cu_3O_{7-\delta}$, at least, satisfactorily obeys either the Bardeen–Cooper–Schrieffer theory [1] or its extension to a d-wave superconducting pairing.

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