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Two-impurity scattering in quasi-one-dimensional systems[1)]

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In quasi-one-dimensional systems with low concentration of impurities the quantization of transverse electronic motion is essential and the conductivity demonstrates van Hove singularities when the Fermi level E_F approaches a bottom of some transverse quantization subband E_N. In our previous work [1, 2] we have demonstrated that for the case of a conducting tube of radius R with weak disorder potential present on its surface, the scattering at the central part of each singularity is suppressed by single impurity non-Born effects. However, single-impurity treatment of scattering breaks down at $|\varepsilon| \sim \varepsilon_{\min} = (n/\pi)^2$, where $\varepsilon = 2m^* R^2(E_F - E_N)$, m^* is effective electron mass, $n = n_2(2\pi R)^2$ is dimensionless concentration of point-like repulsing impurities. n and dimensionless scattering amplitude λ are assumed to be small: $n, \lambda \ll 1$. For simplicity, in the present paper we consider only the case of repulsing impurities $\lambda > 0$ and develop a theoretical description of multi-impurity effects in resistivity for $|\varepsilon| \lesssim \varepsilon_{\min}$. We show that these effects are effectively reduced to just two-impurity ones.

Scattering rate τ_{mk}^{-1} for state with longitudinal momentum k in an m-th subband of transversal quantization is related to corresponding self-energy $\Sigma_{mk}(\varepsilon)$: $\tau_{mk}^{-1} = -2\mathrm{Im}\ \{\Sigma_{mk}\}$. The current-carrying states from ("nonresonant") subbands with $m \neq N$ are semiclassical, therefore the self-energies are formally additive:

$$\Sigma_{mk} = \sum_i \Sigma_{mk}^{(i)}, \quad \Sigma_{mk}^{(i)} \equiv \Sigma^{(i)}\left(E = \varepsilon_m + k^2/2m^*\right).$$

Our aim is to account for all scattering processes within the resonant subband ($m = N$) exactly while for nonresonant subband ($m \neq N$) processes perturbative treatment is sufficient. For perturbative scattering amplitude we have:

$$\tilde{V}_{m_1,m_2}^{(i)} = V_{m_1,m_2}^{(i)} + V_{m_1,N}^{(i)} G_\varepsilon(z_i, z_i) V_{N,m_2}^{(i)} \equiv$$
$$\equiv \frac{\tilde{\lambda}_i}{\pi^2} e^{i\phi_i(m_1 - m_2)}, \quad \tilde{\lambda}_i = \lambda\left\{1 + \frac{\lambda}{\pi^2} G_\varepsilon(z_i, z_i)\right\}. \quad (1)$$

Here $G_\varepsilon(z_i, z_i)$ is the exact multi-impurity Green function of a strictly one-dimensional problem. In order to take into account multiple scattering, we solve the following Dyson equation:

$$\frac{\tilde{\Lambda}^{(i)(\mathrm{ren})}}{\pi^2} = \frac{\tilde{\lambda}^{(i)}}{\pi^2} + \frac{\tilde{\lambda}^{(i)}}{\pi^2} g_\varepsilon(0) \frac{\tilde{\Lambda}^{(i)(\mathrm{ren})}}{\pi^2},$$
$$g_\varepsilon(0) = \sum_{m \neq N} g_\varepsilon^{(m)}(0) \approx -i\pi^2, \quad g_\varepsilon^{(m)}(0) = -\pi i \varepsilon_m^{-1/2}, \quad (2)$$

where $g_\varepsilon^{(m)}(0)$ is the free one-dimensional Green function in the m-th subband. The solution of (2) reads:

$$\tilde{\Lambda}_i^{(\mathrm{ren})} = \lambda(q_i^{-1} + 1 + i\lambda)^{-1}, \quad (3)$$
$$q_i = -\left[(\lambda/\pi^2) G_\varepsilon(z_i, z_i)\right]^{-1} - 1. \quad (4)$$

In order to proceed we need to evaluate $G_\varepsilon(z_i, z_i)$. One-dimensional Green function satisfies the following Schroedinger equation:

$$\left\{-\frac{1}{(2\pi)^2}\frac{d^2}{dz^2} + U(z) - \varepsilon\right\} G(z, z_i) = -\delta(z - z_i), \quad (5)$$
$$U(z) = \lambda/\pi^2 \sum_j \delta(z - z_j). \quad (6)$$

However, for $|\varepsilon| \ll \varepsilon_{\mathrm{nB}}$ one can show that it is enough to consider only 3 impurities:

$$U(z) \to \overline{U}(z) = \lambda/\pi^2 \sum_{j=i,i\pm1} \delta(z - z_j). \quad (7)$$

Taking into account more distant impurities leads to only small corrections to $\mathrm{Re}\, q_i$ and, at the same time,

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to dramatic suppression of $\operatorname{Im} q_i$. Therefore, for q_i we have: $q_i = q_i^{(+)} + q_i^{(-)}$, where

$$q_i^{(\pm)} \approx (k/4\lambda)\cot k\left[L_i^{\pm} + 1/4\lambda\right], \quad k = 2\pi\sqrt{\varepsilon},$$
$$L_i^{(+)} = z_{i+1} - z_i, \quad L_i^{(-)} = z_i - z_{i-1}. \tag{8}$$

Averaging over impurities positions, for resistivity $\rho(\varepsilon)$ we arrive at the following result:

$$\frac{\rho}{\rho_0} = -\frac{1}{\lambda^2}\operatorname{Im}\langle\Lambda^{(\mathrm{ren})}\rangle_{L^{(\pm)}} =$$
$$= \int_0^\infty \frac{\exp\{-n(L^{(+)} + L^{(-)})\}n^2 dL^{(+)}dL^{(-)}}{([q(L^{(+)}) + q(L^{(-)})]^{-1} + 1)^2 + \lambda^2}, \tag{9}$$

where $\rho_0 = (4\pi/e^2E_F)n(\lambda/\pi)^2$ is resistivity away from van Hove singularity. In principle, (9) together with (8) solve our problem: what is left is only to perform a double integration in (9) (see numerical results at Fig. 1). Below we do it analytically in different energy domains.

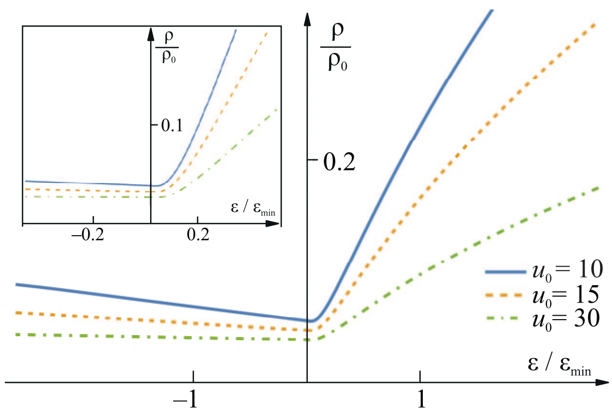

Fig. 1. (Color online) Plot of the total resistivity $\rho(\varepsilon)$ for $\lambda = 0.2$ (main plot) and $\lambda = 0.05$ (inset). In both cases three values of $u_0 = \lambda/n$ are used: $u_0 = 10, 15, 30$

For $\varepsilon > 0$ quasistationary states confined between pairs of adjacent inpurities are present in the resonant subband, and for not very low ε the principal contribution to $\rho(\varepsilon)$ comes from resonant scattering at these states. The corresponding resonance condition is $kL = \pi p$, where $p = 1, 2, \ldots$ and L is either $L_i^{(+)}$ or $L_i^{(-)}$. As a result, we obtain:

$$\frac{\rho_{\mathrm{res}}}{\rho_0} = \frac{\pi n}{2\lambda^2}\sum_{p=1}^{\infty} e^{-nL_p} = \frac{\pi n}{2\lambda^2}\left[\exp\left(\frac{n}{2\sqrt{\varepsilon}}\right) - 1\right]^{-1}. \tag{10}$$

However, $\rho_{\mathrm{res}}(\varepsilon)$ vanishes at $\varepsilon \to 0$ and the finite contribution to $\rho(\varepsilon = 0)$ has non-resonant character. The most important nonresonant contribution ρ_{twin} comes from anomalously small $L_i^{(+)}$ or $L_i^{(-)}$: $L^{(\pm)} \sim 1/\lambda \ll 1/n$. For ρ_{twin} we have:

$$\frac{\rho_{\mathrm{twin}}}{\rho_0} \approx 2n\int_0^\infty \frac{e^{-nL}dL}{(4\lambda L + 2)^2} = \frac{n}{4\lambda}. \tag{11}$$

Why the scattering at twin impurities is dominant at low energy? There is no special enhancement for the twin impurities scattering at low ε, but single-impurity scattering amplitude $\Lambda_i^{(\mathrm{ren})}$ is suppressed by non-Born effects for $\varepsilon \to 0$ [2]. This screening effect is, however, gradually destroyed, as a pair of impurities approach each other.

However, at the first glance this observation is counter-intuitive since the closer impurities are, the more their pair resembles a solitary "composite impurity", scattering at which is expected to be suppressed. The resolution to this paradox is as follows. Let us consider transitions between states from $m, m' \neq N$ bands due to scattering at a twin pair. In this case, the scattering cross-section component that describes coherent scattering at 2 impurities constituting the pair is proportional to $e^{ik_{mm'}L}$, where $L = |z_i - z_j|$ and typical momentum transfer $k_{mm'}$ in a multi-channel system is large: $k_{mm'} \sim N \gg 1$. This contribution vanishes after averaging over L and, therefore, twin pair of impurities could be thought of as a "coherent" object for the processes within resonant subband but it is "incoherent" for scattering processes between states from current-carrying nonresonant subbands.

To conclude, we have studied the behavior of $\rho(\varepsilon)$ in a tube in the vicinity of a van Hove singularity. We have shown that in the range of energies $-(1/4)(\varepsilon_{\min}\varepsilon_{\mathrm{nB}})^{1/2} < \varepsilon < \varepsilon_{\min}\ln^{-2}\lambda$ the resistivity is dominated by scattering at rare "twin" pairs of close defects. The predicted effect is characteristic for multichannel systems, it can not be observed in strictly one-dimensional one.

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