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Vortices in polar and β phases of ^{3}He

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At the moment the most interesting topics in the condensed matter physics are related to topological materials: topological insulators, topological superconductors, Dirac and Weyl topological semimetals, etc. Superfluid phases of liquid ^{3}He [1–3] are the best representatives of the topological matter. Each phase has its unique topological property. Recently, the new phases have been discovered, which are also unique: the polar phase [4, 5] and the β-phase [6]. These superfluid phases were obtained by confinement of liquid ^{3}He in nematic aerogels with nearly parallel strands. The polar phase has Dirac nodal line in the fermionic spectrum in bulk liquid and the flat band (drumhead states) on the surface [7, 8], which is similar to that in the semimetals with nodal lines [9, 10].

In the β phase the superfluid pairing takes place only for single spin projection. Thus the spin degeneracy of the flat band is lifted, and the surface contains the nondegenerate Majorana fermions. Also, similar to the polar phase the β phase in aerogel is robust to disorder owing to the extension of the Anderson theorem to superconductors with columnar defects [7, 11, 12]. Here we consider the vortex states, which appear in the rotating polar and β phases, and their evolution in the process of transformation of the polar phase to β phase.

The spin-triplet p-wave order parameter, $A_{\alpha i} \equiv \mathbf{A}_i$, in the polar phase, in the spin-polarized polar phase, and in the β-phase has the following form:

$$\mathbf{A}_i = \hat{z}_i \left(\Delta_{\uparrow\uparrow}(\hat{\mathbf{x}} + i\hat{\mathbf{y}})e^{i\Phi_\uparrow} + \Delta_{\downarrow\downarrow}(\hat{\mathbf{x}} - i\hat{\mathbf{y}})e^{i\Phi_\downarrow}\right), \quad (1)$$

where $\Delta_{\uparrow\uparrow} = \Delta_{\downarrow\downarrow}$ in the polar phase, $\Delta_{\downarrow\downarrow} < \Delta_{\uparrow\uparrow}$ in the spin-polarized polar phase in the magnetic field along z-axis, and $\Delta_{\downarrow\downarrow} = 0$ in the β-phase.

In rotating vessel there is the competition between the energy of a single-quantum vortex (SQV) and the energy of two half-quantum vortices (HQVs) [13, 14]. In SQV $\Phi_\uparrow = \Phi_\downarrow = \phi$, where ϕ is the azimuthal angle in cylindrical coordinates. In the HQVs either $\Phi_\uparrow = \phi$ and $\Phi_\downarrow = 0$, or $\Phi_\downarrow = \phi$ and $\Phi_\uparrow = 0$:

$$\mathbf{A}_i = e^{i\frac{\phi}{2}} \hat{z}_i \left(\Delta_{\uparrow\uparrow}(\hat{\mathbf{x}} + i\hat{\mathbf{y}})e^{i\frac{\phi}{2}} + \Delta_{\downarrow\downarrow}(\hat{\mathbf{x}} - i\hat{\mathbf{y}}e^{-i\frac{\phi}{2}})\right), \quad (2)$$

$$\mathbf{A}_i = e^{i\frac{\phi}{2}} \hat{z}_i \left(\Delta_{\uparrow\uparrow}(\hat{\mathbf{x}} + i\hat{\mathbf{y}})e^{-i\frac{\phi}{2}} + \Delta_{\downarrow\downarrow}(\hat{\mathbf{x}} - i\hat{\mathbf{y}})e^{i\frac{\phi}{2}}\right). \quad (3)$$

They correspond to the single-quantum vortex in the spin-up component and to the single-quantum vortex in the spin-down component, see also [15]. When two HQVs are combined, they form the SQV (the phase vortex):

$$\mathbf{A}_i = e^{i\phi} \hat{z}_i \left(\Delta_{\uparrow\uparrow}(\hat{\mathbf{x}} + i\hat{\mathbf{y}}) + \Delta_{\downarrow\downarrow}(\hat{\mathbf{x}} - i\hat{\mathbf{y}})\right). \quad (4)$$

The energy of superflow in the cryostat rotating with angular velocity $\mathbf{\Omega} \parallel \hat{\mathbf{z}}$ is:

$$F = \frac{1}{2}\rho_{s\uparrow}(\mathbf{v}_{s\uparrow} - \mathbf{\Omega} \times \mathbf{r})^2 + \frac{1}{2}\rho_{s\downarrow}(\mathbf{v}_{s\downarrow} - \mathbf{\Omega} \times \mathbf{r})^2 + \quad (5)$$

$$+ \rho_{\uparrow\downarrow}(\mathbf{v}_{s\uparrow} - \mathbf{\Omega} \times \mathbf{r})(\mathbf{v}_{s\downarrow} - \mathbf{\Omega} \times \mathbf{r}), \quad (6)$$

where $\mathbf{v}_{s\uparrow} = (\hbar/2m)\nabla\Phi_\uparrow$ and $\mathbf{v}_{s\downarrow} = (\hbar/2m)\nabla\Phi_\downarrow$. Eq. (6) is the Andreev–Bashkin term [16] which mixes spin components and stabilizes the HQVs [17]. The energy of two separated HQVs is proportional to $(\rho_{s\uparrow} + \rho_{s\downarrow})$, while the energy of SQV is proportional to $(\rho_{s\uparrow} + \rho_{s\downarrow} + 2\rho_{\uparrow\downarrow})$. For $\rho_{\uparrow\downarrow} > 0$ the SQV splits into two HQVs, and the vortex lattice splits in two sublattices of spin-up and spin-down vortices in Fig. 1 with densities:

$$n_\uparrow = n_\downarrow = \frac{2m\Omega}{\pi\hbar}. \quad (7)$$

Figure 1a demonstrates the lattice of HQVs in zero magnetic field, where $\Delta_{\downarrow\downarrow} = \Delta_{\uparrow\uparrow}$ and $\rho_{s\downarrow} = \rho_{s\uparrow}$. The elementary cell of the lattice contains two vortices: the spin-up and spin-down HQVs. They have opposite circulations of spin current and their spin currents are compensated.

When the polar phase is spin-polarized by the nonzero magnetic field $\mathbf{H} \parallel \hat{\mathbf{z}}$ in Fig. 1b, the balance between spin-up and spin-down vortices is violated, though their densities are the same as in Eq. (7). Vortices in the spin-down component have smaller superfluid density, $\rho_{s\downarrow} < \rho_{s\uparrow}$. The spin currents of spin-up and spin-down vortices are not compensated leading to the global spin current, which is proportional to $\mathbf{\Omega} \times \mathbf{r}$. This is similar to the mechanism of the formation of the global spin currents discussed in [18, 19].

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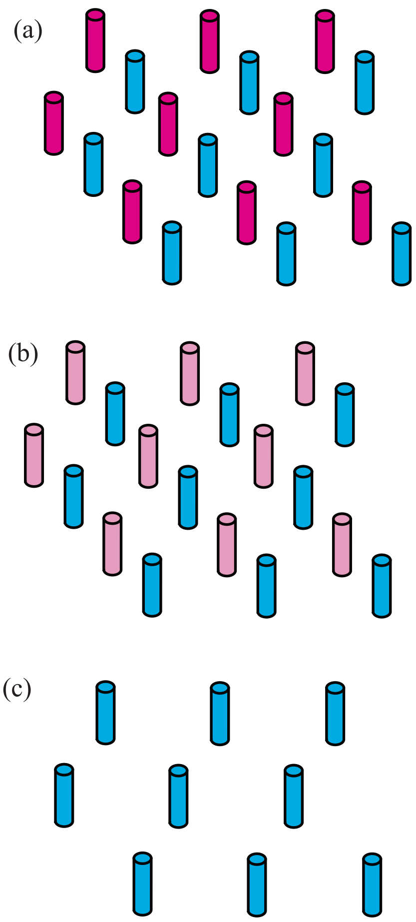

Fig. 1. (Color online) Illustration of vortices in (a) – polar phase, (b) – spin-polarized phase and (c) – β-phase in rotating cryostat

With increasing field the spin-down vortices are gradually evaporated, and spin-down lattice fades away at the transition to the β-phase in Fig. 1c, where $\rho_{s\downarrow} = \rho_{\uparrow\downarrow} = 0$, and only spin-up vortices remain:

$$\mathbf{A}_i = e^{i\phi}\Delta_{\uparrow\uparrow}\hat{z}_i(\hat{\mathbf{x}} + i\hat{\mathbf{y}}). \quad (8)$$

The lattice of these vortices has the same vortex density $n_\uparrow$ in Eq. (7) as in the polar phase.

The pinning of vortices by aerogel strands is strong [13, 14]. This leads to the formation of different vortex glasses [20] and to stabilization of different exotic structures [14, 21], including Bogoliubov Fermi surface [7, 8] and analogs of cosmic walls bounded by strings [22], see review paper [23]. The pinning may produce different routes in the transition from the polar phase to the β-phase under rotation. For example, instead of the two HQVs, the elementary cell in the spin polarized polar phase may contain SQV, which continuously transforms to the singly quantized vortex in the β phase.

In the other possible configuration, the elementary cell contains 3 objects: 2 spin-down HQVs and the spin vortex (SV) with the order parameter:

$$\mathbf{A}_i = \hat{z}_i\left(\Delta_{\uparrow\uparrow}(\hat{\mathbf{x}} + i\hat{\mathbf{y}})e^{i\phi} + \Delta_{\downarrow\downarrow}(\hat{\mathbf{x}} - i\hat{\mathbf{y}})e^{-i\phi}\right). \quad (9)$$

The total energy of three objects is proportional to $(\rho_{s\uparrow} + 3\rho_{s\downarrow} - 2\rho_{\uparrow\downarrow})$. Approaching the β-phase, two spin-down vortices fade away, while the SV continuously transforms to the SQV in Eq. (8).

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