Pis'ma v ZhETF, vol. 115, iss. 8, pp. 498 – 499

Quantum turbulence and Planckian dissipation

G. E. Volovik[1)]

Low Temperature Laboratory, Aalto University, P.O. Box 15100, FI-00076 Aalto, Finland

Landau Institute for Theoretical Physics, 142432 Chernogolovka, Russia

Submitted 3 March 2022
Resubmitted 13 March 2022
Accepted 15 March 2022

DOI: 10.31857/S1234567822080067, EDN: fnewwm

The dissipation is called Planckian, when the relaxation time τ is comparable with $\hbar/T$, see e.g. [1–3] and criticism in [4]. In principle, the other types of dissipation can be also called Planckian, when the relaxation time τ is comparable with the other quantum time scales. Then, in general, the Planckian dissipation takes place, when the time scales related to dissipation (Thouless time [5], elastic scattering time $\tau_{\rm el}$, inelastic scattering time $\tau_{\rm inel}$, spin-relaxation time, etc.) are compared with the characteristic energy scale: temperature T, kinetic energy per particle, gap in the energy spectrum, interlevel distance ΔE in mesoscopic systems and in the vortex core, etc. We consider the case, when $\hbar/\tau$ becomes comparable with the interlevel distance in the core of vortices [6]. In this case the transition from super-Planckian to the sub-Planckian dissipation marks the onset of the vortex turbulence in superconductors and fermionic superfluids [7, 8].

In the weak-coupling BCS regime, where the gap $\Delta \ll E_F$, the vortex core contains the chiral branch of the discrete levels. These Caroli-de Gennes-Matricon levels are quantized with either $E_n = (n + 1/2)\,\Delta E$ or $E_n = n\,\Delta E$ [9]. In the latter case the core contains the Majorana fermions [9–11]. The scattering leads to the broadening of the core levels. If the broadening $\hbar/\tau$ becomes comparable with or larger than the minigap ΔE, the energy levels overlap, and the chiral branch becomes continuous. This allows the spectral flow from the negative energy states to the positive energy states, which leads to the spectral flow force acting on the vortex [12]. This phenomenon represents the analog of the axial anomaly. The analogy becomes exact in case of continuous vortices (skyrmions) in ^{3}He-A with Weyl fermionic quasiparticles, where the spectral flow is governed by the Adler–Bell–Jackiw anomaly equation [13]. The spectral flow is the important ingredient of the vortex dynamics experimentally studied in detail in the fully gapped ^{3}He-B and in the chiral ^{3}He-A [13, 14].

The coarse-grained hydrodynamic equation for the velocity of the superfluid component is:

$$\frac{\partial \mathbf{v}_s}{\partial t} + \nabla\mu = \mathbf{v}_s \times \boldsymbol{\omega}_s - \alpha'(\mathbf{v}_s - \mathbf{v}_n) \times \boldsymbol{\omega}_s + + \alpha\hat{\boldsymbol{\omega}}_s \times [\boldsymbol{\omega}_s \times (\mathbf{v}_s - \mathbf{v}_n)]\ ,\ \boldsymbol{\omega}_s = \nabla \times \mathbf{v}_s, \quad (1)$$

where $\mathbf{v}_s$ and $\mathbf{v}_n$ are the superfluid and normal velocities correspondingly, and $\hat{\boldsymbol{\omega}}_s$ is the unit vector of superfluid vorticity $\boldsymbol{\omega}_s$. The first term in the rhs of Eq. (1) comes from the conventional Magnus force acting on vortices. The second term with the parameter α' is also the reactive (nondissipative) force, but it comes from the effect of the chiral anomaly. The third term is the friction force acting on quantized vortices, when they move with respect to the normal bath.

As distinct from the turbulence in classical liquids the turbulence in the two-fluid hydrodynamics is described by 3 Reynolds numbers. One Reynolds number describes the dynamics of the normal component. In ^{3}He due to the high viscosity of the normal component, its Reynolds number is small and the normal flow is laminar. The normal component is typically at rest, i.e. $\mathbf{v}_n = 0$. Then the superfluid dynamics in Eq. (1) contains two dimensionless parameters:

$$1 - \alpha' = \frac{\omega_0^2\tau^2}{1+\omega_0^2\tau^2}\tanh\frac{\Delta}{2T}, \quad (2)$$

$$\alpha = \frac{\omega_0\tau}{1+\omega_0^2\tau^2}\tanh\frac{\Delta}{2T}, \quad (3)$$

where $\hbar\omega_0 = \Delta E$ is the minigap.

The velocity independent superfluid Reynolds number – the ratio between the reactive and dissipative terms – has the direct relation to the Planck dissipation:

$$\mathrm{Re}_v = \frac{1-\alpha'}{\alpha} = \frac{\Delta E}{\hbar/\tau}. \quad (4)$$

[1)] e-mail: grigori.volovik@aalto.fi

In the regime $\mathrm{Re}_v < 1$ the spectral flow along the core levels becomes allowed, and in the limit $\mathrm{Re}_v \ll 1$ the Kopnin spectral force almost completely compensates the conventional Magnus force. In this regime, the dissipation wins over the combined reactive force, and this prevents the turbulence. When $\mathrm{Re}_v > 1$, the energy levels become isolated from each other, the spectral flow is suppressed, the Magnus force wins over dissipation giving rise to the quantum turbulence.

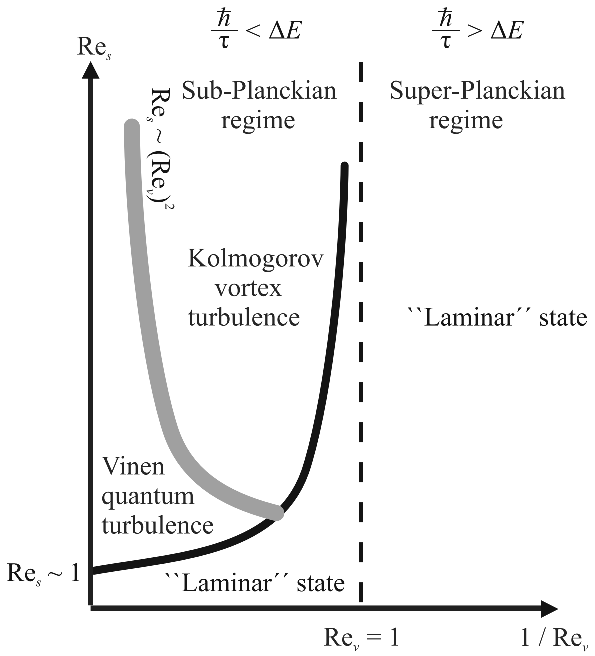

Fig. 1. Phase diagram of the vortex turbulence from [15] in terms of two Planckian parameters, which serve as the quantum Reynolds numbers, Re_v and Re_s, where $1/\mathrm{Re}_v = (\hbar/\tau)/\Delta E$ and $1/\mathrm{Re}_s = \hbar/(mUR)$

The phase diagram of vortex turbulence in Fig. 1 is determined by superfluid Reynolds numbers, Re_v and Re_s, both can be called Planckian. The Reynolds number Re_s is $\mathrm{Re}_s = UR/(\hbar/m)$, where U and R are the parameters of the flow at large scale. This superfluid Reynolds number looks similar to the Reynolds number in classical liquids, $\mathrm{Re} = UR/\nu$, but instead of the kinematic viscosity ν the quantum quantity $\hbar/m$ enters. At $T \sim 0.6T_c$, the transition from the laminar flow to the vortex turbulence takes place, which follows both from experiment and theoretical calculations. Below this transition the vortex turbulence is described by the classical-like Kolmogorov cascade. At very low temperature, when $\mathrm{Re}_s > \mathrm{Re}_v^2$, the dynamics of the individual vortices becomes important. It is the Vinen turbulence [16], which is characterized by the single length scale – the distance between vortices.

In conclusion, the notion of the Planckian dissipation can be extended to the system of the Caroli-de Gennes-Matricon energy levels in the vortex core of superconductors and fermionic superfluids. In this approach the Planck dissipation takes place when the scattering time τ is comparable with the Heisenberg time $t_H = \hbar/\Delta E$. The spectral flow of levels along the chiral branch of the Caroli-de Gennes-Matricon states takes place in the super-Planckian region, i.e. when $\tau < \hbar/\Delta E$, and is absent in the sub-Planckian region, $\tau > \hbar/\Delta E$. As a result, the Planck dissipation separates the laminar flow of the superfluid liquid at $\tau < \hbar/\Delta E$ and the vortex turbulence at $\tau > \hbar/\Delta E$.

This work has been supported by the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (Grant Agreement # 694248).

This is an excerpt of the article "Quantum turbulence and Planckian dissipation". Full text of the paper is published in JETP Letters journal. DOI: 10.1134/S0021364022100344

1. J. Zaanen, arXiv:2110.00961.
2. J. Zaanen, SciPost Phys. **6**, 061 (2019).
3. A. Ataei, A. Gourgout, G. Grissonnanche et al. (Collaboration), arXiv:2203.05035.
4. M. V. Sadovskii, Phys.-Uspekhi **64**, 175 (2021).
5. D. J. Thouless, Phys. Rep. **13**, 93 (1974).
6. C. Caroli, P. G. de Gennes, and J. Matricon, Phys. Lett. **9**, 307 (1964).
7. A. P. Finne, T. Araki, R. Blaauwgeers, V. B. Eltsov, N. B. Kopnin, M. Krusius, L. Skrbek, M. Tsubota, and G. E. Volovik, Nature **424**, 1022 (2003).
8. A. P. Finne, V. B. Eltsov, R. Hanninen, N. B. Kopnin, J. Kopu, M. Krusius, M. Tsubota, and G. E. Volovik, Rep. Prog. Phys. **69**, 3157 (2006).
9. G. E. Volovik, JETP Lett. **70**, 609 (1999).
10. N. B. Kopnin and M. M. Salomaa, Phys. Rev. B **44**, 9667 (1991).
11. D. A. Ivanov, Phys. Rev. Lett. **86**, 268 (2001).
12. N. B. Kopnin, G. E. Volovik, and Ü. Parts, Europhys. Lett. **32**, 651 (1995).
13. T. D. C. Bevan, A. J. Manninen, J. B. Cook, J. R. Hook, H. E. Hall, T. Vachaspati, and G. E. Volovik, Nature **386**, 689 (1997).
14. T. D. C. Bevan, A. J. Manninen, J. B. Cook, H. Alles, J. R. Hook, and H. E. Hall, J. Low Temp. Phys. **109**, 423 (1997).
15. G. E. Volovik, JETP Lett. **78**, 533 (2003).
16. W. F. Vinen, *Progress in Low Temperature Physics III*, ed. by C. J. Gorter, Elsevier, Amsterdam (1961), v. III, p. 1.