Pis'ma v ZhETF, vol. 116, iss. 1, pp. 60 – 61

Riemann–Cartan gravity with dynamical signature

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Submitted 18 March 2022
Resubmitted 9 May 2022
Accepted 19 May 2022

DOI: 10.31857/S1234567822130092, EDN: ixkptw

Model of Riemann–Cartan gravity with varying signature of metric is considered. The basic dynamical variables of the formalism are vierbein, spin connection, and an internal metric in the tangent space. The corresponding action contains new terms, which depend on these fields. In general case the signature of the metric is determined dynamically. The Minkowski signature is preferred dynamically because the configurations with the other signatures are dynamically suppressed. We also discuss briefly the motion of particles in the background of the modified black hole configuration, in which inside the horizon the signature is that of Euclidean space-time.

Our first basic variable is vierbein e^a_μ, which is matrix 4×4. Metric is composed of vierbein as follows $g_{\mu\nu} = O_{ab}\, e^a_\mu\, e^b_\nu$, the real symmetric matrix O is our second dynamical variable, which plays the role of metric on tangent space. The case of space-time with Minkowski signature corresponds to the choice $O = \mathrm{diag}(1,-1,-1,-1)$ while the case of Euclidean signature is $O = \mathrm{diag}(1,1,1,1)$. The choices $O = \mathrm{diag}(-1,1,1,1)$ and $O = \mathrm{diag}(-1,-1,-1,-1)$ also represent Minkowski and Euclidean signatures correspondingly. The cases $O = \mathrm{diag}(-1,-1,1,1)$ and $O = \mathrm{diag}(1,1,-1,-1)$ represent the signature, which is typically not considered in the framework of conventional quantum field theory. $O(4)$ transformations Ω of vierbein $e^a_\mu \to \Omega^a_b e^b_\mu$ together with rescaling $e^a_\mu \to \Lambda^a_b e^b_\mu$ (where $\Lambda = \mathrm{diag}(\lambda_1,\lambda_2,\lambda_3,\lambda_4)$ with positive λ_i) are able to reduce the general form of matrix O to the six above mentioned canonical forms.

We introduce connection $\omega^a_{\mu b}$ that belongs to algebra of $SL(4,R)$. As well as in conventional case we define the inverse vierbein matrices through $E^\mu_a e^a_\nu = \delta^\mu_\nu$, $E^\mu_a e^b_\mu = \delta^b_a$ while metric with upper indices is defined through $g^{\mu\nu} g_{\nu\rho} = E^\mu_a E^\nu_b \mathcal{O}^{ab} e^c_\nu e^d_\rho O_{cd} = \delta^\mu_\rho$ with $\mathcal{O}$ matrix such that $\mathcal{O}^{ab} O_{bc} = \delta^a_c$. One can construct the following action quadratic in the derivatives of O: $S_O = \int d^4x e\sqrt{O} E^\mu_c E^\nu_f\, (D_\mu O)_{ab}\, (D_\nu O)_{de}\, \alpha^{abc;def}$ with $e = \det e = \frac{1}{4!}\, e^a_\mu e^b_\nu e^c_\rho e^d_\sigma \epsilon_{abcd} \epsilon^{\mu\nu\rho\sigma}$, $\sqrt{O} = \sqrt{\det O}$. Tensor α is to be composed of O. We may represent the above term in the action also through the tetrad components of the derivatives of O: $E^\mu_c D_\mu O_{ab} = O_{ab;c}$. Modified Einstein–Cartan action reads $S_\omega = -m_P^2 \int d^4x e\sqrt{O} \mathcal{R}^a_{\mu\nu b} E^\mu_a E^\nu_d \mathcal{O}^{bd}$. Here $\mathcal{R}$ is curvature of gauge field ω. Besides, we may consider terms quadratic in curvature. In order to classify these terms we introduce first the tetrad components of curvature: $\mathcal{R}_{abcd} = E^\mu_c E^\nu_d O_{ad} \mathcal{R}^d_{\mu\nu b}$. The general form of the action quadratic in curvature has the form: $S_R = \int d^4x e\sqrt{O} \mathcal{R}_{a_1b_1c_1d_1} \mathcal{R}_{a_2b_2c_2d_2} \gamma^{a_1b_1c_1d_1a_2b_2c_2d_2}$, tensor γ is composed of matrices $\mathcal{O}$. Another terms in the action may be composed of the covariant derivative of vielbein $e^a_{\mu;\nu} = D_\nu e^a_\mu$. We define the tetrad components of the derivatives of vierbein as $e_{ab;c} = O_{ad} E^\mu_b E^\nu_c D_\mu e^d_\nu$. There may be several independent terms quadratic in this derivative. Those ones have the form $S_e = \int d^4x e\sqrt{O} e_{a_1b_1;c_1} e_{a_2b_2;c_2} \zeta^{a_1b_1c_1a_2b_2c_2}$. The most general form of tensor ζ is given in our paper. There is also the mixed term $S_{Oe} = \int d^4x\, e\, \sqrt{O}\, e_{a_1b_1;c_1} O_{a_2b_2;c_2} \eta^{a_1b_1c_1a_2b_2c_2}$ with parameters $\eta^{a_1b_1c_1a_2b_2c_2}$. Finally, one may add the trivial cosmological constant term: $S_\lambda = -\lambda \int d^4x e\sqrt{O}$.

Partition function may be written as

$$Z = \int DeDOD\omega e^{-S_O - S_\omega - S_R - S_e - S_{Oe} - S_\lambda}.$$

One can always choose the coefficients in the action $(\alpha_i, \zeta_\sigma, \gamma_\sigma, \eta_\sigma, \lambda)$ in such a way that the action is bounded from below for the case of real positive $\sqrt{\det O}$. Moreover, we require that Euclidean action is positively defined. This allows to define the self-consistent quantum theory. In this theory the fluctuations of fields appear to be exponentially suppressed for real positive $\sqrt{\det O}$. At the same time negative $\det O$ results in the appearance of imaginary unity in the exponent. The corresponding configurations are not exponentially suppressed and dominate over the configurations with

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positive det$O$. This is the way how the signature $(1,-1,-1,-1)$ (or $(-1,1,1,1)$) is distinguished dynamically.

As an illustration of our general construction we considered roughly the modification of the black hole solution, in which outside of the horizon it looks like an ordinary Schwarzshield solution (considered in Gullstrand–Painleve reference frame). Inside of the horizon the expression for the vielbein remains the same as in the ordinary Painleve–Gullstrand black hole, but the matrix O_{ab} changes signature to that of Euclidean space-time. We do not have an intention to consider the given configuration as a real classical solution, buth rather look at it as a toy model of the black hole-like configuration with the signature change. On the background of this configuration the motion of a massive particle is considered briefly. It is worth mentioning that such a configuration may appear at a certain stage of the gravitational collapse, when the singularity appears in the classical solution of Einstein equations close to center of the BH. Then the quantum dynamics comes into play, and the signature change might occur inside the horizon.

This is an excerpt of the article “Riemann–Cartan gravity with dynamical signature”. Full text of the paper is published in JETP Letters journal. DOI: 10.1134/S0021364022601002